Left-side Relatively Strong Increases in Risk and Their Comparative Statics

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This paper proposes a new concept, a left-side relatively strong increase in risk (L-RSIR) order that extends the definition of a relatively strong increase in risk (RSIR) order. We show that for the class of linear payoffs, one can obtain an appealing comparative statics result for L-RSIR shifts by imposing additional restrictions on the risk preferences of a risk-averse decision maker.

Keywords: Left-side relatively strong increases in risk, Prudence, Relatively strong increases in risk

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I. Introduction

In most economic models, there is an element of uncertainty about the payoff (or outcome). Given a decision model, an important comparative statics question is to find a set of changes in a cumulative distribution function (CDF) or probability distribution function (PDF) which is sufficient for signing the effect on the choice variable. Since Rothschild and Stiglitz (1970, 1971) developed

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a definition of an increase in risk, several researchers have provided general comparative statics statements dealing with the effects of various types of changes in the distribution of the random variable.

There are recent papers concerning the subsets of Rothschild -Stiglitz (R-S) increases in risk which are obtained from restricting the changes in distribution of the random parameter. Meyer and Ormiston (1985) introduced the concept of a 'strong increase in risk' (SIR) as a subset of R-S increases in risk, which is a direct generalization of probability mass transfers involved in the introduction of risk. An SIR is defined by imposing restrictions on the difference between the initial and the final CDF. A further generalization of an SIR is given by Black and Bulkley (1989) who introduced the concept of a 'relatively strong increase in risk' (RSIR). Dionne, Eeckhoudt, and Gollier (1993a, 1993b) considered a 'relatively weak increase in risk' (RWIR) which is larger than the set of RSIR shifts. The RSIR and the RWIR shifts use a ratio approach as restrictions on changes in PDF and their comparative static analysis is carried out for the set of risk-averse agents.

Recently, Ryu and Kim (2004) introduced a 'left-side strong increase in risk' (L-SIR) which extends an SIR. Dionne and Mounsif (1996) analyzed the newsboy problem where even though they allow more general changes in R-S increases in risk, their main results are established only for some limited intervals. Their comparative statics analyses are applied to risk-averse decision makers with non-negative third derivative of utility functions ($u''' \geq 0$).

In this paper we define a left-side relatively strong increase in risk (L-RSIR) order that extends the subsets of R-S increases in risk and provide its desirable comparative statics properties with L-RSIR shifts for the class of linear payoffs. The L-RSIR order imposes monotonicity restrictions on the ratio of a pair of PDFs. We refer to the relaxed version of the RSIR shifts as L-RSIR ones. This implies that an RSIR is extended to an L-RSIR. Note that as an RSIR extends the set of SIR shifts, an L-RSIR extends the set of L-SIR shifts.

This paper is organized as follows. In section II, we present a model in which a decision-maker maximizes the expected utility of the outcome variable depending on a choice variable and a random variable and define the subset of R-S increases in risk we term an L-RSIR. We also give numerical and graphical examples. Section III provides a comparative statics result for the L-RSIR shifts and indicates how our general result applies to specific economic models. Finally, section IV contains concluding remarks.

II. The Decision Model and Definitions

Following Meyer and Ormiston (1985), we use the model employed by Kraus (1979) and Katz (1981). The decision maker maximizes the expected utility by choosing the optimal value of the choice variable α for a given distribution of the random variable x. Formally, the economic agent's decision problem is to choose α to maximize $Eu[z(x,\alpha)]$, where z is a real valued function and u is the von Neumann-Morgenstern utility function.

We assume that utility function $u(z)$ is three times differentiable with $u'(z)\geq 0$, $u''(z)\leq 0$ and $u'''(z)\geq 0$: the set of decision makers includes the set of decreasing absolute risk aversion (DARA) and the concept of 'prudence' $(-u'''/u''>0)$ introduced by Kimball (1990). Note that the term 'prudence' is a precautionary saving motive and is meant to suggest the propensity to prepare and forearm oneself in the face of uncertainty.

Since the payoff function is restricted to be linear in the random variable, it may be expressed as $z(x,\alpha)\equiv z_0+\alpha x$, where z_0 is an exogenous constant, α is a decision variable, and x is a random variable. To simplify the discussion, we will consider the case where $z_x(x,\alpha)\geq 0$. Combined with $u'(z)\geq 0$, this assumption indicates that higher values of the random variable are preferred to lower values. The opposite case where $z_x(x,\alpha)\leq 0$ can be handled with proper modifications. Note that primes on $u(\cdot)$ are used to denote derivatives while subscripts with other functions denote partial derivatives.

In order to analyze the effects of an increase in riskiness of a random variable, we assume that $F(x)$ is an initial cumulative distribution function (CDF) of x with the finite interval $[x_2, x_3]$ and $G(x)$ the final distribution with the finite interval $[x_1, x_4]$, where $x_1\leq x_2\leq x_3\leq x_4$. Each distribution has its associated probability density function (PDF) $f(x)$ and $g(x)$, respectively.

The decision problem is expressed as

$$\alpha_F\in\arg\max_{\alpha} Eu(z_0+\alpha_F x)=\int_{x_2}^{x_3}u(z_0+\alpha_F x)dF(x).$$

The necessary and sufficient condition for the choice of α_F to maximize expected utility is

$$\int_{x_2}^{x_5} u'(z_0+\alpha_F x)x dF(x)=0. \tag{1}$$

It is well-known that α_F has the same sign as $E_F(x)=\int_{x_2}^{x_5} x dF(x)$ and is positive (see Dionne, Eeckhoudt, and Gollier (1993a), and Eeckhoudt and Gollier (1995)). Therefore, we assume that $E_F(x)$ is positive. In order to prove $\alpha_F \geq \alpha_G$ for a specified change in the PDF (or CDF) from f (or F) to g(or G), it is sufficient to show that, for all $x \in [x_1, x_4]$,

$$Q(\alpha_F)=\int_{x_1}^{x_4} u'(z_0+\alpha_F x)x d[F(x)-G(x)] \geq 0. \tag{2}$$

The question we consider is what changes in distribution satisfy the condition (2). Black and Bulkley (1989) introduced the concept of a 'relatively strong increase in risk' (RSIR) which generalizes a 'strong increase in risk' (SIR) proposed by Meyer and Ormiston (1985). They use a ratio approach as restrictions on changes in PDF and their comparative statics analysis is carried out for the set of risk-averse agents.

Definition 1

$G(x)$ represents a relatively strong increase in risk from $F(x)$ (denoted by G RSIR F) if

(a) $\int_{x_1}^{x_6} [G(x)-F(x)]dx=0$ and

(b) For all points in the interval $[x_3, x_4]$, $f(x) \geq g(x)$ and for all points outside this interval $f(x) \leq g(x)$ where $x_1 \leq x_2 \leq x_3 \leq x_4 \leq x_5 \leq x_6$, $[x_1, x_6]$ being the supports of x under $G(x)$ and $[x_2, x_5]$ being the supports under $F(x)$ and

(c) $f(x)/g(x)$ is non-decreasing in the interval $[x_2, x_3)$ and

(d) $f(x)/g(x)$ is non-increasing in the interval $(x_4, x_5]$.

Conditions (a) and (b) are sufficient for $G(x)$ to represent an R-S increase in risk from $F(x)$. That is, these conditions impose the restrictions that the two distributions have the equal mean, two

PDFs cross only twice and probability mass is transferred from points within the interval (x_3, x_4) to points lying outside this interval. Conditions (c) and (d) restrict the extent to which probability mass can be transferred to any one value in the tails of $F(x)$ relative to any other.

By relaxing the restrictions imposed to the right-side of the point m, we introduce a 'left-side relatively strong increases in risk' (L-RSIR) that is a less stringent type of R-S increases in risk than an RSIR. Below we give a formal definition of an L-RSIR which is sufficient to sign the effect on α given the assumptions about $u(z)$ and $z(x, \alpha)$ stated above.

Definition 2

$G(x)$ represents a left-side relatively strong increase in risk from $F(x)$ (denoted by G L-RSIR F) if

(a) $\int_{x_1}^{x_4} [G(x)-F(x)]dx=0$ and

(b) $\int_{x_1}^{y} [G(x)-F(x)]dx\geq 0$ for all $y\in[x_1, x_4]$ and

(c) there exists a point $m\in[x_2, x_4]$ such that $F(x)\leq G(x)$ for all $x\in [x_1, m)$ and $F(x)\geq G(x)$ for all $x\in[m, x_4]$ and

(d) there exists a point $k\in[x_2, m]$ such that a non-decreasing function $h:[x_2, k) \rightarrow [0,1]$ such that $f(x)=h(x)g(x)$ for all $x\in[x_2, k)$ and $f(x)\geq g(x)$ for all $x\in[k, m]$.

Numerical example: Consider the following two random variables with probability density functions $f(x)$ and $g(x)$, respectively;

$$f(x)=\frac{1}{2}x+\frac{7}{8} \text{ for } -\frac{7}{4}\leq x\leq -\frac{5}{4},\ \frac{4}{25}x+\frac{9}{20} \text{ for } -\frac{5}{4}\leq x\leq 0,$$

$$-\frac{4}{15}x+\frac{9}{20} \text{ for } 0\leq x\leq\frac{3}{4},\ -\frac{1}{2}x+\frac{5}{8} \text{ for } \frac{3}{4}\leq x\leq 1,\ \frac{1}{2}x-\frac{3}{8} \text{ for}$$

$$1\leq x\leq\frac{5}{4},\ \frac{16}{5}x-\frac{15}{4} \text{ for } \frac{5}{4}\leq x\leq\frac{11}{8},\ -\frac{16}{5}x+\frac{101}{20} \text{ for}$$

$$\frac{11}{8}\leq x\leq\frac{3}{2},\ -x+\frac{7}{4} \text{ for } \frac{3}{2}\leq x\leq\frac{7}{4},\ \text{and } g(x)=\frac{1}{4} \text{ for } -2\leq x\leq 2.$$

Note that $F(x)$ and $G(x)$ cross at the point $m\,(x=0)$ and

$\int_{-2}^{0} f(x)dx = F(0) = 1/2 = G(0) = \int_{-2}^{0} g(x)dx$. Note also that multiple crossing between $f(x)$ and $g(x)$ occurs in the right-hand side of $m\,(x=0)$. After some calculations, we can determine that $f(x)$ and $g(x)$ satisfy the following conditions of the Definition 2:

(a) $\int_{-2}^{2} [G(x) - F(x)]dx = 0$ and

(b) $\int_{-2}^{y} [G(x) - F(x)]dx \geq 0$ for all $y \in [-2, 2]$ and

(c) $F(x) \leq G(x)$ for all $x \in [-2, 0)$ and $F(x) \geq G(x)$ for all $x \in [0, 2]$ and

(d) $f(x) \leq g(x)$ and $h'(x) = 2 > 0$ for all $x \in [-\frac{7}{4}, -\frac{5}{4})$ and $f(x) \geq g(x)$ for all $x \in [-\frac{5}{4}, 0]$.

Conditions (a) and (b) define R-S increases in risk. Condition (c) imposes the restriction that the two CDFs cross only once at the point m. Condition (d) implies that, to the left-side of the point m, an L-RSIR requires the same restriction used by Black and Bulkley who define an RSIR. Now to the right-side of the point m, there is no restriction on the number of times of crossing between the PDFs $f(x)$ and $g(x)$, nor is there any monotonicity restriction on the ratio of the two PDFs. Only the required restriction on the right-side of m is that $F(x) \geq G(x)$. Therefore, an RSIR implies an L-RSIR.

The L-RSIR shift can be divided into two shifts: one for the left and the other for the right-side of the point m. The shift for the left-side is understood as 'F dominant over G in the first-degree stochastic dominance (FSD) sense' and the shift for the right-side is understood, in the opposite direction, as 'G dominant over F in the FSD sense'.

Figure 1 illustrates an example of a left-side relatively strong increase in risk and a case where the restriction on the interval $x \in [m, x_4)$ necessary to obtain a relatively strong increase in risk is not met. This implies that an L-RSIR is obtained from an RSIR by relaxing the restrictions imposed to the right-side of the point m.

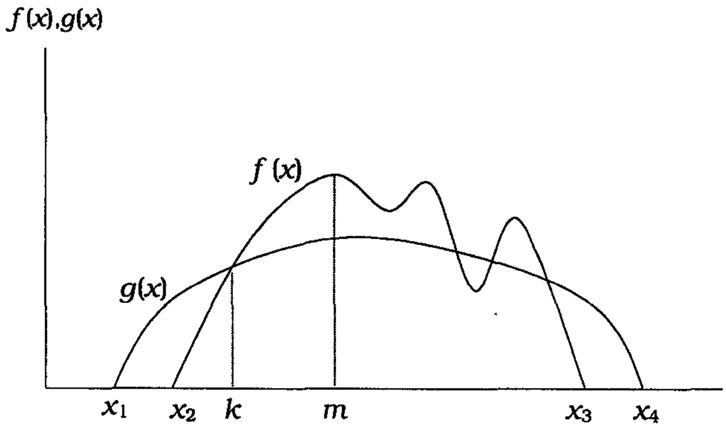

FIGURE 1

G L-RSIR F

III. Comparative Statics Analysis

In this section, we provide comparative statics results concerning the L-RSIR order. The following comparative statics result indicates that, when $z_{xx}=0$, one can further extend the subset of R-S increases in risk at the cost of adding an additional restriction on the risk preferences of decision makers.

Theorem

Suppose that then α_F and α_G maximize $Eu[z(x,\alpha)]$ under $F(x)$ and $G(x)$, respectively. For all risk-averse decision makers with $u''' \geq 0$, $\alpha_F \geq \alpha_G$ if

(a) G L-RSIR F and

(b) $z_{xx}=0$ and $z_{\alpha x} \geq 0$

Proof: Using the payoff function $z(x,\alpha)$ in (1), let x^* be the value of x satisfying $z_\alpha(x,\alpha_F)=0$, and assume that x^* exists on the interval $[x_2, x_3]$. With the points k and m in Definition 2, where $x_2 \leq k \leq m \leq x_3$, we consider the following three cases:

Case (i): $x_2 \leq x^* \leq k$.

Consider the sign of the expression $\int_{x_1}^{k} z_\alpha(f-g)dx$. First, assume that $\int_{x_1}^{k} z_\alpha(f-g)dx \geq 0$, and rewrite $Q(\alpha_F)$ in (2) as

$$Q(\alpha_F)=\int_{x_1}^{k} u'(z)z_\alpha(f-g)dx+\int_{k}^{m} u'(z)z_\alpha(f-g)dx+\int_{m}^{x_4} u'(z)z_\alpha(f-g)dx.$$

Using these assumptions and the L-RSIR definition, we have

$$Q(\alpha_F)\geq u'[z(x^*,\alpha_F)]\int_{x_1}^{k} z_\alpha(f-g)dx+u'[z(m,\alpha_F)]\int_{k}^{m} z_\alpha(f-g)dx \\ +\int_{m}^{x_4} u'(z)z_\alpha(f-g)dx. \tag{3}$$

Adding and subtracting $u'[z(m,\alpha_F)]\int_{x_1}^{k} z_\alpha(f-g)dx$ on the right-hand -side (RHS) of (3) gives

$$Q(\alpha_F)\geq \{u'[z(x^*,\alpha_F)]-u'[z(m,\alpha_F)]\}\int_{x_1}^{k} z_\alpha(f-g)dx \\ +u'[z(m,\alpha_F)]\int_{x_1}^{m} z_\alpha(f-g)dx+\int_{m}^{x_4} u'(z)z_\alpha(f-g)dx. \tag{4}$$

Since $u'(z)$ is non-increasing and $\int_{x_1}^{k} z_\alpha(f-g)dx \geq 0$, the first term on the RHS of (4) is non-negative. Integrating the second term in (4) by parts and using the assumption $z_{xx}=0$, we obtain

$$u'[z(m,\alpha_F)]\int_{x_1}^{m} z_{\alpha x}[G(x)-F(x)]dx=u'[z(m,\alpha_F)]z_{\alpha x}\int_{x_1}^{m} [G(x)-F(x)]dx. \tag{5}$$

Again using integration by parts, the third term in (4) is equal to

$$q(\alpha_F)=\int_{m}^{x_4} u''(z)z_x z_\alpha[G(x)-F(x)]dx+\int_{m}^{x_4} u'(z)z_{\alpha x}[G(x)-F(x)]dx. \tag{6}$$

Since z_α is positive and $[G(x)-F(x)]$ is non-positive for all $x \in [m, x_4]$, the first term in (6) is non-negative. Thus, we have

$$q(\alpha_F)\geq \int_{m}^{x_4} u'(z)z_{\alpha x}[G(x)-F(x)]dx. \tag{7}$$

Because $u'(z)z_{\alpha x}$ is non-increasing and $[G(x)-F(x)]$ is non-positive for all $x \in [m, x_4]$, we have

$$q(\alpha_F)\geq u'[z(m,\alpha_F)]z_{\alpha x}\int_{m}^{x_4} [G(x)-F(x)]dx. \tag{8}$$

Hence, from (5) and (8), the second and third term in (4) can be written as

$$\begin{aligned} &u'[z(m,\alpha_F)]\int_{x_1}^{m} z_\alpha(f-g)dx+\int_m^{x_4} u'(z)z_\alpha(f-g)dx \\ &\geq u'[z(m,\alpha_F)]z_{\alpha x}\int_{x_1}^{x_4}[G(x)-F(x)]dx=0. \end{aligned} \tag{9}$$

Therefore, $Q(\alpha_F)$ is non-negative.

Second, assuming that $\int_{x_1}^{k} z_\alpha(f-g)dx \leq 0$, we rewrite $Q(\alpha_F)$ in (2) as

$$\begin{aligned} Q(\alpha_F) \geq &\int_{x_1}^{k} u'(z)z_\alpha(f-g)dx+u'[z(m,\alpha_F)]\int_k^m z_\alpha(f-g)dx \\ &+\int_m^{x_4} u'(z)z_\alpha(f-g)dx. \end{aligned} \tag{10}$$

Adding and subtracting $u'[z(m,\alpha_F)]\int_{x_1}^{k} z_\alpha(f-g)dx$ on the RHS of (10) gives

$$\begin{aligned} Q(\alpha_F) \geq &\int_{x_1}^{k} u'(z)z_\alpha(f-g)dx-u'[z(m,\alpha_F)]\int_{x_1}^{k} z_\alpha(f-g)dx \\ &+u'[z(m,\alpha_F)]\int_{x_1}^{m} z_\alpha(f-g)dx+\int_m^{x_4} u'(z)z_\alpha(f-g)dx. \end{aligned}$$

From (9) and the assumption $\int_{x_1}^{k} z_\alpha(f-g)dx \leq 0$, we obtain

$$Q(\alpha_F) \geq \int_{x_1}^{k} u'(z)z_\alpha(f-g)dx=-\int_{x_1}^{x_2} u'(z)z_\alpha g dx+\int_{x_2}^{k} u'(z)z_\alpha(f-g)dx. \tag{11}$$

The first term on the RHS of (11) is non-negative. By applying the condition (d) in Definition 2, the second term can be written as

$$\int_{x_2}^{k} u'(z)z_\alpha(f-g)dx=\int_{x_2}^{k} u'(z)z_\alpha(1-\frac{1}{h})fdx \geq [1-\frac{1}{h(x^*)}]\int_{x_2}^{k} u'(z)z_\alpha fdx.$$

Since $h(x^*) \leq 1$ and $\int_{x_2}^{k} u'(z)z_\alpha fdx \leq 0$ by the first-order condition, $Q(\alpha_F)$ is non-negative.

Case (ii): $k \leq x^* \leq m$.

We can write $Q(\alpha_F)$ in (2) as

$$Q(\alpha_F)=\int_{x_1}^{x^*} u'(z)z_\alpha(f-g)dx+\int_{x^*}^{m} u'(z)z_\alpha(f-g)dx+\int_m^{x_4} u'(z)z_\alpha(f-g)dx.$$

Using the given assumptions and the L-RSIR definition, we have

$$Q(\alpha_F) \geq u'[z(x_3, \alpha_F)] \int_{x_1}^{x^*} z_\alpha(f-g)dx + u'[z(m, \alpha_F)] \int_{x^*}^{m} z_\alpha(f-g)dx$$
$$+ \int_{m}^{x_4} u'(z) z_\alpha (f-g)dx. \tag{12}$$

Adding and subtracting $u'[z(m, \alpha_F)] \int_{x_1}^{x^*} z_\alpha(f-g)dx$ on the RHS of (12) gives

$$Q(\alpha_F) \geq \{u'[z(x_3, \alpha_F)] - u'[z(m, \alpha_F)]\} \int_{x_1}^{x^*} z_\alpha(f-g)dx$$
$$+ u'[z(m, \alpha_F)]\} \int_{x_1}^{m} z_\alpha(f-g)dx + \int_{m}^{x_4} u'(z) z_\alpha(f-g)dx. \tag{13}$$

From (9), the second and third terms on the RHS of (13) are non-negative. Thus, we have

$$Q(\alpha_F) \geq \{u'[z(x_3, \alpha_F)] - u'[z(m, \alpha_F)]\} \int_{x_1}^{x^*} z_\alpha(f-g)dx.$$

Integrating by parts, we obtain

$$\int_{x_1}^{x^*} z_\alpha(f-g)dx = \int_{x_1}^{x^*} z_{\alpha x}[G(x) - F(x)]dx = z_{\alpha x} \int_{x_1}^{x^*} [G(x) - F(x)]dx \geq 0$$

since $z_{\alpha x}$ is non-negative and does not depend on x, and $\int_{x_1}^{t} \{G(x) - F(x)]dx \geq 0$ for all $t \in [x_1, x_4]$. Thus, by the assumption $u''(z) \leq 0$, $Q(\alpha_F)$ is non-negative.

Case (iii): $m \leq x^* \leq x_3$.

Integrating by parts, $Q(\alpha_F)$ can be written as

$$Q(\alpha_F) = \int_{x_1}^{x_4} [u''(z) z_x z_\alpha + u'(z) z_{\alpha x}][G(x) - F(x)]dx.$$

Note that, when the assumption of $u''' \geq 0$ is used, $u''(z) z_x z_\alpha + u'(z) z_{\alpha x}$ is positive and non-increasing in x on the interval $[x_1, x^*]$, and it has a maximum at $x = x^*$ on the interval $[x^*, x_4]$ since $u''(z) z_x z_\alpha$ is always non-positive and $u'(z) z_{\alpha x}$ is non-increasing in x. Since $m \leq x^*$, we have

$$[u''(z) z_x z_\alpha + u'(z) z_{\alpha x}]|_{x<m} \geq [u''(z) z_x z_\alpha + u'(z) z_{\alpha x}]|_{x=m} \geq [u''(z) z_x z_\alpha + u'(z) z_{\alpha x}]|_{x>m}.$$

By the L-RSIR definition which implies $G(x) - F(x) \geq 0$ for all $x \in [x_1, m]$ and $G(x) - F(x) \leq 0$ for all $x \in [m, x_4]$, we have the following inequality,

$$Q(\alpha_F) \geq [u''(z)z_x z_\alpha + u'(z)z_{\alpha x}]\,|_{x=m} \int_{x_1}^{x_4} [G(x) - F(x)]dx = 0.$$

Q.E.D.

When the payoff function is linear in the random variable, our result reveals a trade-off between the restrictions on the risk preferences of decision makers and the admissible set of changes in the distribution of the random variable. Compared with the result for RSIR shifts, the comparative statics result for L-RSIR shifts contains a larger set of changes in distribution and a more restrictive set of assumptions about the decision maker with an additional assumption such as ($u''' \geq 0$).

While the linearity assumption ($z_{xx}=0$) restricts the set of the economic decision problems to which our result is applicable, linear payoffs prevail in many economic environments such as these analyzed by Sandmo (1971), Feder (1977), Rothschild and Stiglitz (1971), Fishburn and Porter (1976), Paroush and Kahana (1980), Dionne, Eeckhoudt and Gollier (1993a), and Eeckhoudt and Gollier (1995). In particular, the linearity model includes the standard portfolio model, the problem of hiring workers, the optimal behavior of a competitive firm with constant marginal costs, and the coinsurance problem.

Now, we give a specific example which provides an appropriate application of Theorem. In the standard coinsurance problem, the payoff function is given by the final wealth $z(y, \alpha) = W_0 - \lambda\mu - (1-b)(y - \lambda\mu)$, where y is the amount of random loss, μ the expected loss, b coinsurance rate, $b\lambda\mu$ the insurance premium, and W_0 the initial wealth. This payoff function is equivalent to $z(x, \alpha) = z_0 + \alpha x$ when $z_0 = W_0 - \lambda\mu$, $\alpha \equiv -(1-b)$ and $x \equiv y - \lambda\mu$. If we limit the discussion to private insurance contracts, the coinsurance rate b belongs to the interval [0,1]. Then, by definition, α is non-positive and belongs to the interval $[-1, 0]$. Hence, $z_{\alpha x} = 1 > 0$ and $z_{\alpha x x} = 0$. Therefore, applying Theorem, an L-RSIR shift causes risk-averse firms with $u''' \geq 0$ to increase the coinsurance rate.

IV. Conclusions

This paper has proposed the concept of a left-side relatively strong increase in risk (L-RSIR) order that is less restrictive than the RSIR order for changes in the distribution as a subset of Rothschild-Stiglitz (R-S) increases in risk. We explore the trade-offs among the changes in CDF of the random parameter, the structure of the payoff function, and assumptions about risk attitudes in obtaining an appealing comparative statics result.

Our result shows that more general subset of R-S increases in risk is allowed in the comparative statics analysis by restricting the payoff function to be linear in the random variable and by limiting our analysis to risk-averse decision makers with non-negative third derivative of utility functions. This implies that one can further extend the subset of R-S increases in risk, but uses somewhat stronger restriction on the structure of the decision model and the set of decision makers.

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References

Black, J. M., and Bulkley, G. "A Ratio Criterion for Signing the Effect of an Increase in Uncertainty." *International Economic Review* 30 (No. 1 1989): 119-30.

Dionne, G., Eeckhoudt, L., and Gollier, C. "Increases in Risk and Linear Payoffs." *International Economic Review* 34 (No. 2 1993a): 309-19.

________. "Relatively Weak Increases in Risk and Their Comparative Statics." *Economics Letters* 41 (No. 3 1993b): 269-72.

Dionne, G., and Mounsif, T. "Investment under Demand Uncertainty: The Newsboy Problem Revisited." *The Geneva Papers on Risk and Insurance Theory* 21 (No. 2 1996): 179-89.

Eeckhoudt, L., and Gollier, C. "Demand for Risky Assets and the Monotone Probability Ratio Order." *Journal of Risk and Uncertainty* 11 (No. 1 1995): 113-22.

Feder, G. "The Impact of Uncertainty in a Class of Objective Functions." *Journal of Economic Theory* 16 (No. 2 1977): 504-12.

Fishburn, P. C., and Porter, B. "Optimal Portfolio with One Safe and One Risky Asset: Effects of Changes in Rate of Return and Risk." *Management Science* 22 (No. 9 1976): 1064-73.

Katz, E. "A Note on a Comparative Statics Theorem for Choice under Risk." *Journal of Economic Theory* 25 (No. 2 1981): 318-9.

Kimball, M. "Precautionary Saving in the Small and the Large." *Econometrica* 58 (No. 1 1990): 53-73.

Kraus, M. "A Comparative Statics Theorem for Choice under Risk." *Journal of Economic Theory* 21 (No. 3 1979): 510-7.

Meyer, J., and Ormiston, M.B. "Strong Increases in Risk and Their Comparative Statics." *International Economic Review* 26 (No. 2 1985): 425-37.

Paroush, J., and Kahana, N. "Price Uncertainty and the Cooperative Firm." *American Economic Review* 70 (No. 1 1980): 212-6.

Rothschild, M., and Stiglitz, J. "Increasing Risk: I. A Definition." *Journal of Economic Theory* 2 (No. 3 1970): 225-43.

_________. "Increasing Risk: II. Its Economic Consequences." *Journal of Economic Theory* 3 (No. 1 1971): 66-84.

Ryu, S. and Kim, I. "Left-Side Strong Increases in Risk and Their Comparative Statics." *Theory and Decision* 57 (No. 1 2004): 59-68.

Sandmo, A. "On the Theory of the Competitive Firm under Price Uncertainty." *American Economic Review* 61 (No. 1 1971): 65-73.