Group Size and Heterogeneity in the Private Provision of Public Goods

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The traditional approach to voluntary provision of pure public goods typically models the factors of group heterogeneity and group size in a piecemeal fashion and fails to explain salient empirical observations. Integrating both factors into a single model, we examine how they interact with each other to determine the structure of Nash equilibrium, and show that this model is indeed useful in making realistic predictions.

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I. Introduction

While being recognized as important factors characterizing the structure of the Nash equilibrium for private provision of the public good, the two factors of group size and heterogeneity have not been analyzed in an integrated fashion in the literature. For example, Bergstrom *et al.* (1986) only deals with the effect of group heterogeneity (caused by income redistribution) on the structure of Nash equilibrium, whereas Andreoni (1988) focuses on the effect of group size on the set of free riders and the aggregate level of public goods (see also Fries *et al.* (1991), McGuire and Shrestha

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(2003)).

In particular, Andreoni (1988) shows that, as the group/economy size grows to infinity, only the most "generous" type of consumers will contribute. Noting that such prediction is in sharp contrast with the salient fact that the vast majority of people do not free ride, many authors have explored other (non-altruistic) motives for making contributions to public goods, such as social status (Hollander 1990, Shrestha and Cheong 2004), joint production (Cornes and Sandler 1984), warm glow (Andreoni 1990), inequity aversion (Fehr and Schmidt 1999), and so forth. Such efforts to model additional motives for giving have formed an interesting and meaningful strand of literature.

In this paper, we show that the predictive power of the traditional pure public good approach can be significantly enhanced without the introduction of additional motives, simply by incorporating both the group size and group heterogeneity factors into a single framework. The proposed model allows us to meaningfully isolate and analyze the effect of each factor on the Nash equilibrium outcome by controlling the other factor. This advantage is compared to Andreoni's (1988) model in which the effect of group size is not explicitly distinguished from that of group heterogeneity.

The proposed model is useful in producing predictions that are indeed consistent with many observed phenomena unlike the existing models based on the pure public good approach. For example, we can explain why only one consumer may contribute even in a small group/economy as reported by Kindleberger (1986) in the international public good cases, and why a large proportion of population may contribute even in a large economy as documented by many researchers of charitable donations in the United States, such as Boskin and Feldstein (1977) and Reece and Zieschang (1985).

This paper is organized as follows. Section II first develops a model incorporating group size and group heterogeneity, and then analyzes their effects on the structure of the resulting Nash equilibrium. Concluding remarks are made in Section III.

II. Group Size and Group Heterogeneity

A. The Model

Consider an economy consisting of a finite but arbitrary I different 'types' of consumers (indexed by $i=1,2,\cdots,I$) with their own preferences and income endowments (see Definition 2 below for the technical definition of 'types'). Each consumer of type i allocates his endowment (denoted by w_i) between the consumption of private good (x_i) and the contribution (g_i) to the public good (G). Each consumer's preference for the public and private goods is represented by a continuous, twice-differentiable and strictly quasi-concave utility function: $U_i=U_i(x_i,G)$. For simplicity, we assume that the relative price of the public good is unity, and so the budget constraint of each consumer of type i is written as $x_i+g_i=w_i$.

Following existing studies, such as Fries *et al.* (1991) and McGuire and Shrestha (2003), we define group size or economy size by the number of replications made for the original economy. It is noted that such replications do not change the relative distribution of preferences and incomes of the consumers. More specifically,

Definition 1

Group Size - Group size being equal to n means there are n consumers of each type in the economy. Put differently, it is the number of replications of the original economy consisting of a set of I different types of consumers.

Under the Nash assumption, each consumer of type i chooses the level of g_i given the total contribution by other consumers (denoted by G_{-i}), in order to maximize U_i subject to the budget constraint. The first-order conditions of this maximization problem yield

$$G(n)=f_i\,(w_i+G_{-i}(n)) \quad \text{for} \quad i\in C \quad \text{and} \tag{1}$$

$$G(n)\geq f_j\,(w_j+G_{-j}(n)) \quad \text{for} \quad j\notin C \tag{2}$$

where $C=\{i\,|\,i$ is a type of the consumers who contribute to $G\}$. In fact, Equations (1) and (2) generalize Equations (2) and (3) in

Bergstrom *et al.* (1986, p. 33) to the cases of varying group size. As in their model, if both private and public goods are normal, there exists a unique Nash equilibrium.

From Equations (1) and (2), we define an individual-specific critical value, G_i^0, which is an implicit solution to $G_i^0 = f_i(w_i + G_i^0)$, such that when $G_i^0 \leq G(n)$, $g_i(n) = 0$ and when $G_i^0 > G(n)$, $g_i(n) > 0$. G_i^0 is specific to each individual consumer and computed from his income and preference. Noting that, given n, G_i^0 uniquely determines whether i is a contributor or a free rider, Andreoni and McGuire (1993) call it i's 'free-rider-inducing supply.' In addition to the concept of free-rider-inducing supply, we also make use of the concept of 'isolation contribution' defined as an individual's contribution, g_i, when $G_{-i} = 0$. Clearly from Equation (2), it follows that $g_i|_{G_{-i}=0} = f_i(w_i)$. It is also obvious that $f_i(w_i)$ and G_i^0 are positively correlated; that is, the higher the free-rider-inducing supply of a consumer, the higher will be his isolation contribution. Both G_i^0 and $g_i|_{G_{-i}} = 0$ are heuristic concepts, but they are nevertheless instrumental in our analysis. We now define the 'types' of consumers as follows.

Definition 2

Consumer Types - Consumers i and j are considered to be of different types if and only if $G_i^0 \neq G_j^0 \, \forall i \neq j$.

Notice that if all the consumers have identical preferences [income], the income endowments [preferences] solely determine the types of consumers. Next, we rank the consumers by their types such that $j > k$ if and only if $G_j^0 < G_k^0$. Therefore, it follows that $G_1^0 > G_2^0 > \cdots > G_I^0$.

Lemma 1

The Nash equilibrium contribution of an individual of type i is positive, that is, $g_i(n) > 0$, if and only if $G_i^0 > G(n)$. Furthermore, if $G_i^0 \geq G_j^0$ and $g_j > 0$, then $g_i > 0 \, \forall i \leq j$.

Proof: The first part of this lemma follows directly from the definition of G_i^0. By construction, it is an implicit solution to $G_i^0 = f_i(w_i + G_i^0)$, such that when $G_i^0 \leq G(n)$, $g_i(n) = 0$ and when $G_i^0 > G(n)$, $g_i(n) > 0$. The second part can be proved as follows. If $G_i^0 \geq G_j^0$ and g_j

>0, then from the first part of this lemma $G_j^0 > G(n)$. Now from transitivity and that $G_1^0 > G_2^0 > \cdots > G_j^0$, it follows that $G_i^0 > G(n)$ and hence $g_i > 0 \forall i \leq j$.

Let C be the set of contributors in the economy. Applying an inverse demand function $\phi_i(\cdot)$ on both sides of Equation (1) and summing over all $i \in C$, we obtain the following result, (which extends Equation (4) in Bergstrom *et al.* (1986) to include the cases of varying group size):

$$\sum_{i \in C} \phi_i(G(n)) - (c-1)G(n) = \sum_{i \in C} w_i, \tag{3}$$

where c is the number of types of contributors, which may be simply called the size of C.

Equation (3) relates the group size and group heterogeneity in the determination of the structure of Nash equilibrium. As will be shown later, the size of C shrinks with group size or group heterogeneity. Then it follows that a given size of C (and, for that matter, a given Nash level of G) can be obtained either with a large but more homogeneous group or a small but more heterogeneous group.

B. Group Size and Nash Equilibrium

In this section, we examine how the size of C and the Nash equilibrium G change with n, given the group heterogeneity.

Lemma 2

As n increases, the Nash equilibrium G monotonically increases but the size of C monotonically decreases.

Proof: Without loss of generality, assume that the consumers of type j that are in original C become free riders as a result of an increase in group size from n to $(n+1)$. Then from Lemma 1, $G_j^0 > G(n)$ and $G_j^0 \leq G(n+1)$, where $G(n+1)$ is the Nash equilibrium level of G with the new contributing set $C' = \{i \mid i < j\}$ and group size $= (n+1)$. Invoking transitivity, we obtain $G(n+1) > G(n)$, which then implies that the equilibrium $G(n)$ increases with n.

We prove the second part of the lemma by contradiction. Suppose, on the contrary, the size of C increases with n. Without loss of generality, let additional consumers of type j, who were not in original C, now start contributing after the group size increases from n to $(n+1)$. Then from Lemma 1 $G_j^0 > G(n+1)$, where $G(n+1)$ is the Nash equilibrium level of G with the new contributing set $C' = \{i \mid i \leq j\}$ and group size$=(n+1)$. From the first part of the lemma, we have $G(n+1) > G(n)$. Now, by transitivity, $G_j^0 > G(n)$ - a contradiction to the assumption that $j \notin C$, the original contributing set. Hence $C(n)$ shrinks monotonically with n.[1]

Intuitively, C shrinks with n because G increases with n. As G increases, more and more consumers will be better off by free riding on the contribution made by others. Although the set C shrinks with n, there remains at least one type of consumers in the set because it is always true that $G(n) > 0$. From Lemma 1, it follows that the last remaining type of consumers in C is the one with the highest G_i^0, that is, G_1^0. For all other types, we derive the following result.

Lemma 3

For each type of consumers i, except the one with the highest G_i^0, there exists a finite number of replications or a group size n_i, such that for all $n \geq n_i$, $g_i(n) = 0$. The level of n_i that implicitly solves $G(n_i) = G_2^0$ is the highest among such n_i's.

Proof: Suppose for a given group size n, consumers of arbitrary type i are contributors. Then from Lemma 1 $G_i^0 > G(n)$. As n increases, from Lemma 2 $G(n)$ increases but G_i^0 is unaffected (note that G_i^0 is independent of n). That means there must exists some critical value of $n = n_i$ (that is specific to consumers of type i) for which $G_i^0 = G(n_i)$ and hence $g_i(n) = 0$ for all $n \geq n_i$. Similarly, as n increases the maximum value that $G(n)$ can approach is G_2^0 (because $G(n)$ must be less than G_1^0; otherwise everybody will be free riders). This implies that the highest value of n_i for which $G_i^0 = G(n_i)$ is n_2.

Elaborating on this result, we can readily generate the following proposition, which implies that only one type of consumers may

[1] For alternative proofs of this lemma, see Fries *et al.* (1991, Proposition 1, p. 152), McGuire and Shrestha (2003) and Andreoni (1988, Theorem 1.1, p. 66).

contribute in a sufficiently large economy.

Proposition 1

For an economy with I types of consumers with each type having a strictly quasi-concave utility function $U_i(x_i, G)$, where $i = 1, 2, \cdots, I$, there exists a finite number n_c such that the replication of the original economy for n_c or more times will result in all free riders but the type of consumers with the highest free-rider-inducing supply (G_1^0). By definition, n_c is $\max\{n_i\}$, where $G(n_i) \geq G_i^0 \ \forall i > 1$, and, in fact, it is obtained when $i = 2$; that is, $n_c = n_2$.

Because Proposition 1 directly follows from Lemma 3, we omit the proof but discuss the economic intuition of the proposition. As the group size continues to grow (by replication), $G(n)$ continues to increase and consequently more and more types of consumers find their G_i^0 to fall below $G(n)$. Once $G_i^0 < G(n)$, the consumers of type i become better off as they quit contributing but free ride on the contribution from others. With the increase in n, this cascade of free riding proceeds from one type to another type of consumers in the G_i^0 ranks until only one type of consumers, that is, the consumers with the highest G_i^0 (that is G_1^0) remain contributing. In a practical sense, this is the type of consumers with the strongest demand for the public good.

It follows from Proposition 1 and Lemma 1 that $G(n_c) < G_1^0$ (but $G(n_c) \geq G_i^0 \forall i \neq 1$) and thus only the consumers of type 1 contribute when $n \geq n_c$. At the same time, Lemma 2 implies that $G(n)$ increases as the group size increases. However, $G(n)$ is bounded by a finite maximum value $G(n)_{max}$, which must be lower than G_1^0 since otherwise type 1 consumers (and so everybody) would become free riders too. Although Proposition 1 is closely in line with Theorem 1.1 of Andreoni (1988, p. 66) and Proposition 1 of Fries *et al.* (1991, p. 152), this proposition is distinguished from the previous results in that it explicitly identifies the most 'generous' type of consumers to be the ones with the highest free-rider-inducing supply, G_1^0.

As in Andreoni's Theorem 1 (1988), it also follows from Proposition 1 that, if all consumers have identical preferences, only the wealthiest (type of) consumers will remain as contributors as n continues to increase beyond n_c. On the other hand, when all consumers have identical endowments (*i.e.*, $w_i = w \forall i$), only the consumers with the strongest desire for the public good will continue to contribute as n increases above n_c.

It is worthwhile to note that Andreoni's (1988) analysis does not explicitly describe how the number of consumers increases - whether by the replications of the original economy as analyzed in this paper or simply by the addition of more consumers. If the increase were due to the additional consumers of different types, group heterogeneity as well as group size would be affected in our terms. Therefore, it remains to be answered in Andreoni's (1988) paper which factor (group size or group heterogeneity) indeed drives his main result that, in an infinitely large economy, only the most "generous" type of consumers will contribute. In the next section, we address the effect of group heterogeneity on the structure of Nash equilibrium, which is, in fact, comparable to that of group size.

C. Group Heterogeneity and Nash Equilibrium

This section deals with group heterogeneity relative to the concepts of free-rider-inducing supply and isolation contribution. The following result shows that not only the group size (as noted by Andreoni (1988)) but also the group heterogeneity is an important determinant of the structure of the Nash equilibrium.

Proposition 2

(a) If a redistribution of income results in a completely homogenous group such that $G_i^0=G^0 \forall i$, every consumer will contribute regardless of the group size; (b) If there exists a type of consumers, say type k, such that their isolation contribution is higher than free-rider-inducing supply of any other type such that $f_k(w_k)\geq G_j^0$ for all $j\neq k$, then only this type of consumers will contribute in the Nash equilibrium.

***Proof*:** Since $G(n)>0$, it must be true that $G_i^0>G(n)$ for at least one i. Therefore, the assumption of $G_i^0=G^0\forall i$ implies $G_i^0>G(n)\forall i$, which proves Part (a). To prove Part (b), first note from Equations (1) and (2) that $f_k(w_k)$ is the Nash equilibrium G when only type-k consumers are contributors, that is, $f_k(w_k)=G$. Now consider $f_k(w_k)\geq G_j^0\forall j\neq k$. It follows then that $G_j^0\leq G\forall j\neq k$, which confirms that all but type-k consumers are free riders.

It is noted that both Propositions 1 and 2(b) address the situation where only one type of consumers contribute; the former is related to the number of potential contributors approaching infinity whereas the latter is based on the relative magnitude of isolation contributions in the light of group heterogeneity.[2] Obviously, the sole contributors in the limiting case, denoted as type-k consumers, are indeed the consumers of type 1, whose free-rider-inducing supply is the highest among types. We also observe that the Nash level of G in this single contributor case is the maximum level of G obtainable such that $G_j^0 \leq \text{Max } G < G_1^0$ for all $j \neq 1$.

In fact, Proposition 2 seems to be consistent with empirical findings in the literature. Among other things, let us take an example of an international public good. Kindleberger (1986) notes that peace, which is considered an international public good, has often been provided by a single dominant world power, such as Pax Romana or Pax Britannica. The exploration of Mars is also carried out by a single country, the United States. Since the number of countries is finite, Andreoni's (1988) model clearly fails to explain these casual observations. In the context of the proposed model, however, one can explain that the isolation contribution to the international public good is large for the United States relative to other countries, leading it to be the sole contributor in the end. When there is only one type of contributors and hence the Nash equilibrium level of G assumes its maximum value close to G_1^0, the rest of the consumers will be better off by free riding. Put differently, free riding will allow these consumers to enjoy the highest levels of G and x possible in this situation.

III. Concluding Remarks

This paper develops a simple model for the privately provided public good, in which both economy size and consumer heterogeneity can be considered together, unlike the traditional models focusing on either one of the factors. The major advantage of combining the two factors in a single model is the ability to explain the empirical observations that are not consistent with the

[2] In the context of international public goods, Shrestha and Feehan (2003) have derived a similar result, but under a more restrictive assumption of homothetic preferences.

predictions from the existing models based on the pure public good approach. For example, the heterogeneity factor helps understand why many consumers may contribute even in a large economy, which is indeed the case in reality but not in line with the theoretical result in the existing literature.

In particular, Propositions 1 and 2, which are proposed as the main results in this paper, demonstrate how different sets of contributors might occur depending on the distributions of consumers' free-rider-inducing supply and isolation contribution in addition to the group size. An important implication of these results is that it is not the pure public good approach itself that is to blame for the failure to make realistic predictions, which triggered a new strand of literature on additional motives to contribute, such as warm glow and social status. Rather, a share of the blame goes to the way the previous models are structured. In this sense, this paper may be deemed to be a step toward enriching the traditional paradigm and enhancing its predictive power in a more general setting.

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