Demand Uncertainty and the Choice of Business Model in the Semiconductor Industry

Yingyi Tsai and Ching-Tang Wu*

In this paper, we provide another reason that may explain the wide adoption of outsourcing approach in the semiconductor industry. We show the fab-lite business model of outsourcing wafer fabrication to foundries is optimal in the presence of demand uncertainty. This is because outsourcing helps the integrated device manufacturer (largely the brand-producing firm) to lower its cost of capital investment in the case of low demand and to improve its capacity allocation in the case of high demand.

Keywords: Outsourcing, Capital investment, Uncertainty

JEL Classification: D24, D40, D81, E22, L23

I. Introduction

In the semiconductor industry, one of the most striking changes precipitated by rapid technological progress has been wide adoption of the fab-lite business model. Fab-lite refers to integrated device

* Assistant Professor, Department of Applied Economics, National University of Kaohsiung, Nan-Tzu Dist., Kaohsiung 811, Taiwan. (Tel) +886-7-591-9189, (E-mail) yytsai@nuk.edu.tw; Associate Professor, Department of Applied Mathematics, National Chiao Tung University, Hsinchu 30050, Taiwan, (Tel) +886-3-513-1220, (E-mail) ctwu@math.nctu.edu.tw, respectively. We thank the Editor, anonymous referees, Justin Yifu Lin, CY Lu, Preston McAfee, Abhinay Muthoo, Roderick James McCrorie, and Hao Wang for their valuable suggestions and comments. We would also like to thank the seminar participants at the China Center for Economic Research (CCER) for providing important comments. The authors are greatly indebted to the National Science Council, Taiwan for the research grant (#NSC 92-2115-M-39-002-).
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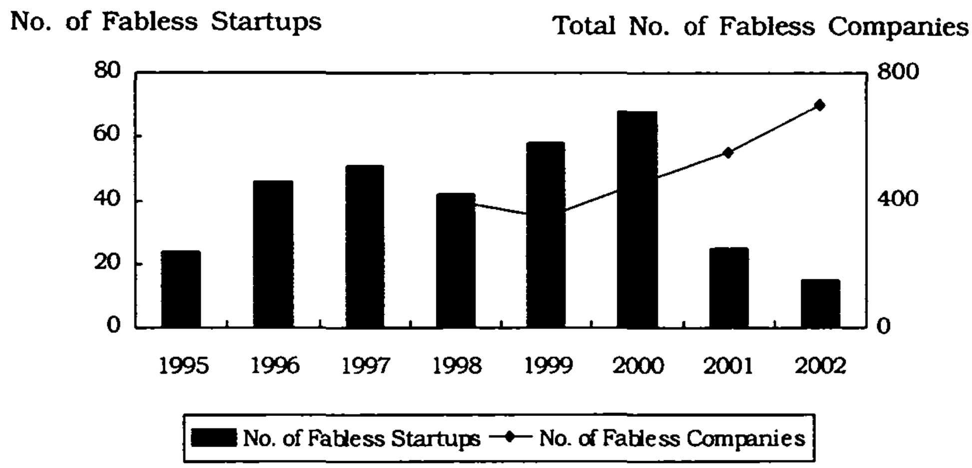
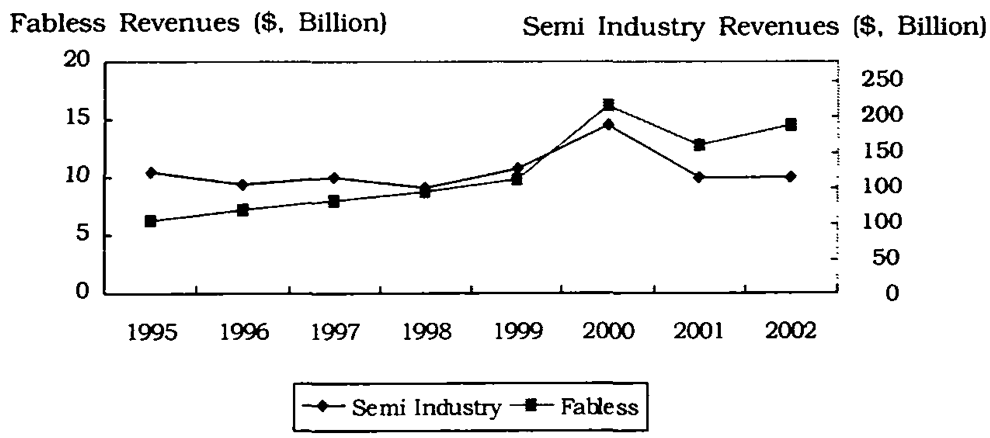


Source: Fabless Semiconductor Association, 2003.

FIGURE 1
GROWTH OF THE FABLESS COMPANIES



Source: Fabless Semiconductor Association, 2003.

FIGURE 2
REVENUE GROWTH OF THE FABLESS COMPANIES

manufacturers or vertically integrated firms with a corporate strategy bent toward utilizing a fabless approach.[1] Figure 1 illustrates the trend of growing number of the fabless semiconductor companies since 1995. And Figure 2 shows the increasing revenue growth of the fabless companies in the semiconductor industry.

Specialization and economies of scale have been identified as the major factors explaining this change. The usual argument is that a fab-lite model allows the fabless companies and integrated device manufacturers to focus on new product development by using efficiently its in-house resources (or facilities) on the one hand, and permits both front-end foundry (for wafer fabrication) and back-end foundry (for packaging and testing) to spread the costs of capital investment over different contracts on the other.

In this paper, we argue that demand uncertainty can be a reason for outsourcing in the fast changing industry like semiconductor. Setting aside as explanations for outsourcing of cost advantage (Abraham and Taylor 1996; Feenstra and Hanson 1996) and corporate strategy (Deavers 1997; Shy and Stenbacka 2003), we focus on the effect for outsourcing of demand uncertainty. A basic underlying idea is that, in the presence of demand uncertainty, outsourcing renders a firm the flexibility to balance a trade-off between having in-house facilities shortage while demand unexpectedly surges and excess capacities otherwise. Although this assumption appears intuitively simple, it allows us to explore the rationale of outsourcing that can provide important insights into the use of fab-lite approach in a fast changing sector like semiconductor. The main feature of this analysis is how the presence of outsourcing opportunity under demand uncertainty will affect a brand-producing firm's choices of business model and in-house capital investment. We will show, in the presence of demand uncertainty, that an outsourcing (or non-integrated) business model is optimal. By allowing for lower in-house

[1] "Fabless" refers to the business methodology of outsourcing the manufacturing of silicon wafers. Fabless companies focus on the design, development and marketing of their products and form alliances with foundries, or silicon wafer manufacturers. And "Integrated Device Manufacturer (IDM)" refers to a class of semiconductor companies that owns an internal silicon fab or, alternatively, the fabrication of wafers is integrated into its business. Nonetheless, even IDMs may undertake some outsourcing activities. "Foundry" is a service organization that caters to the processing and manufacturing of silicon wafers. It typically develops and owns the process technology or partners with another company for it.

capital investment, outsourcing renders higher profit than it otherwise would if the option to outsource is not available. Moreover, we also explore the employment implications of outsourcing with a production function incorporating both labor and capital. Our results show that low-wage need not drive an increase in the growing outsourcing activities. Hence, the general belief based upon cost advantage of low wage in explaining outsourcing is yet to be further studied. Throughout, in our discussion of outsourcing, we assume that only the manufacturing segment of end product is outsourced, that outsourcing market is competitive, namely, there is a mass of subcontractors competing for contracts, and that outsourcing is at a brand-producing firm's disposal, but with a setup cost.

The paper is organized as follows. After presenting the basic model in the next section, we characterize the optimal capital investment under uncertainty with no outsourcing as a benchmark. In section III, we present results on the choices of business model in equilibrium, and explore the implications for these decisions of demand uncertainty. Section IV studies optimal capital investment under the business models of integrated and non-integrated production with demand uncertainty. In section V, we discuss our results and the role and applicability of the various assumptions that we make on production technology and distribution of random variable, thus providing some informal defense for interpreting our model as a qualitative characterization of reality. And we conclude in section VI.

II. The Basic Model

Consider a brand-producing firm facing an inverse demand[2] of

$$P=XY^{-\varepsilon}, \tag{1}$$

where P is the price, Y total production, $\varepsilon \in (0, 1)$ an elasticity parameter, and X denotes an exogenous, absolutely continuous positive bounded random variable on a complete probability space

[2] This formulation implies the brand-producing firm alone faces overall demand shocks. For similar characterization, see, Caballero (1991) for demand at the individual level facing a single competitive firm; and Pindyck (1993) for industry-wide demand shock facing a large number of equal-sized firms.

(Ω, F, P).[3]

An important message that emerges from Equation (1) suggests the brand price consists of two components: Product design (for brand features) and basic manufacture (for total supply). Hence, if we interpret X as brand-specificity,[4] and Y the manufacturing segment for the brand, then this inverse demand function captures the impact on price of brand quality and quantity. Alternatively, Equation (1) characterizes a price reflecting the "qualitative" aspect of market demand owing to the indefinite outcome of "product innovation".[5]

Production of Y requires the use of labor (L) and capital (K) by a linearly homogeneous technology $Y=K^{1-\beta}L^{\beta}$, where $\beta \in (0, 1)$ denotes the share of L. To make the point that capital investment is irreversible and not easily expandable,[6] we assume production facilities must be installed before actual production can take place. Hence, the brand-producing firm first decides the amount of capital investment K $(K \geq 0)$, it then chooses, upon the realization of actual market demand, labor employment L $(L \geq 0)$. The cost to capital and labor is denoted by γ and w, respectively.

In the presence of demand uncertainty, the brand-producing firm's problem is whether to adopt an integrated business model - in which case the firm produces Y using its own facilities to serve the market demand; or a non-integrated (or outsourcing) model - in which case market demand is served with in-house production Y and, possibly, the purchase of y from other firms in the primary market.

In order to make our point in a manner as simple as possible, we assume the brand-producing firm withholds to itself the design of product, and decides whether to produce in-house or to purchase from an independent specialist firm the manufacturing segment of end product. Hence, in the present model we define outsourcing to mean that the brand-producing firm purchases the basic manufacturing component instead of carrying out the production of such

[3] The assumption of a bounded X suggests market demand should not tend to infinite.

[4] Alternatively, X consists of the variations in consumer taste, the changes in technology, and even a changing market environment.

[5] See Levhari and Peles (1973) for a justification of this characterization on "product innovation".

[6] Abel, Dixit, Eberly, and Pindyck (1996) and Dixit and Pindyck (1998) argued that "expandability" of capital investment in the future gives rise to call option while investigating the relations between optimal investment and uncertainty.

component at its own facility, given an identical technology. We further assume, for simplicity, a unitary marginal cost for each unit of the manufacturing component. Thus, with the possibility to outsource basic production, the brand-producing firm faces a cost structure of

$$(C+y)\,\mathbf{I}_{\{y>0\}}(y)=\begin{cases} C+y, & \text{if } y>0, \\ 0, & \text{if } y=0, \end{cases}$$

where $\mathbf{I}_{\{y>0\}}$ is the indicator function of $\{y>0\}$, the price per unit outsourcing output is normalized to one, and C represents the setup costs incurred in establishing an outsourcing partnership with the suitable subcontractor or in monitoring the contracts.[7]

Hence, the brand-producing firm's total production cost is

$$TC(K, L, y)=\begin{cases} wL+\gamma K, & \text{if } y=0 \quad \text{(in-house production)} \\ wL+\gamma K+(C+y), & \text{if } y>0 \quad \text{(outsourcing production)} \end{cases} \tag{2}$$

The choices of business model and capital investment are made in the context of uncertainty. Market demand condition is not known until the firm enters actual production, given the chosen production mode (*cf.* Sandmo 1971; Pindyck 1988). This timing reflects that outsourcing can serve as a device of mitigating the gap between unexpected demand shock and in-house production constraints underlying in capital investment, and that any *ex post* adjustment is not possible since it is costly to alter the decisions over business model or capital investment in the light of new market information.

Using Equations (1) and (2), the brand-producing firm's profit is given by

[7] Grossman and Helpman (2002) provide an intuitive justification for this formulation since "⋯ there are fixed costs associated with ⋯ searching for a potential supplier". Further, this characterization of total outsourcing cost is similarly captured by Shy and Stenbacka (2003), who modeled outsourcing in terms of a trade-off between the "make-or-buy" decision, except that we consider here a unitary marginal cost per unit outsourcing output.

$$\pi(K, L, y) = \begin{cases} \pi^N(K, L) = X(K^{1-\beta}L^{\beta})^{1-\varepsilon} - wL - \gamma K, & \text{in-house} \\ \pi_C^O(K, L, y) = X(K^{1-\beta}L^{\beta} + y)^{1-\varepsilon} - wL - \gamma K - (C+y), & \text{outsourcing} \end{cases} \quad (3)$$

Equation (3) highlights the problem facing a brand-producing firm in the presence of demand uncertainty, that is, it has to balance a trade-off between an irreversible capital investment with possible idle capacity and the option of avoiding such investment but having to incur a cost for outsourcing partnership and even paying the subcontractor a premium.

Suppose a business model of integrated production (alternatively, outsourcing production is not possible) is chosen. This characterization corresponds to a standard model of optimal capital investment under uncertainty in which a monopolist facing uncertain future demand chooses the amount of capital investment. The following proposition characterizes the optimal capital investment in a model of integrated production.

Proposition 1

If it chooses an integrated production mode, then the brand-producing firm raises its capital investment, K^N, with a greater expected market demand, i.e.,

$$K^N = H_{\beta,\varepsilon}\left(E\left[X^{\frac{1}{1-\beta(1-\varepsilon)}}\right]\right)^{\frac{1-\beta(1-\varepsilon)}{\varepsilon}},$$

for some positive constant $H_{\beta,\varepsilon}$.

Proof: See Appendix A. □

Proposition 1 implies that higher expected market demand will lead to greater capital investment. Thus, the driving force for a high demand deserves careful investigation. Successful quality improvement or new product features provide an example. Levhari and Peles (1973) showed formally, in a deterministic setting, that quality improvement (as a form of product innovation) is able to raise market demand. This question of interpretation is important as it bears upon the issue of the nature of uncertainty. Indeed, if we interpret X as an indefinite outcome of product innovation, the brand-producing firm will increase its capital investment when it expects to successfully deliver new invention (or improve upon product quality) even it is not possible to outsource. Further, the above result also suggests that the brand-producing firm will increase its capital investment as the market demand becomes more volatile.[8]

III. Choices of Business Model

Proposition 2 below establishes the conditions under which the business model of non-integrated production (implying the possibility to outsource) is chosen in the presence of demand uncertainty.

Proposition 2

For any C and K, there exists a critical $X_C^*(K)$ such that **(i)** $y^O(K)>0$ if $X>X_C^*(K)$, and **(ii)** $y^O(K)=0$ if $X\leq X_C^*(K)$, where

$$X_C^*(K)=\begin{cases}\dfrac{1}{1-\varepsilon}\left(\dfrac{\beta}{w}\right)^{\frac{\beta\varepsilon}{1-\beta}}K^{\varepsilon}, & \text{if } C=0,\\ \sup\{X:\ \pi_C^O(K,L^O(K),y^O(K))\leq\pi^N(K,L^N(K))\}, & \text{if } C>0.\end{cases}$$

Proof: See Appendix B. □

Proposition 2 implies that outsourcing takes place only when the realized market demand is sufficiently large, and that the setup cost of outsourcing has a decisive impact on firm's choice of business model. Figure 3 illustrates the role of the outsourcing setup cost in affecting the outsourcing amount y.

Appropriately interpreted, a sufficiently small setup cost implies a negligibly low price of contracting with a compatible supplier. Thus, the outsourcing firm is able to work with subcontractor(s) without much difficulty whenever the realized market demand exceeds its

[8] Given two positive random variables X_1 and X_2, each with probability distribution μ_1 and μ_2. Using Föllmer and Schied (2002), we know that if μ_1 is uniformly preferred over μ_2, and X_1 and X_2 have the same mean, there exists a "mean preserving spread" Q such that $\mu_2=\mu_1 Q$. Since we have obtained, for given X_1 and X_2, that the relative optimal capital investments K_1^N and K_2^N exhibiting $K_1^N\leq K_2^N$. Hence, it follows that the brand-producing firm will increase the capital investment as X becomes "riskier", *i.e.*, more volatile.

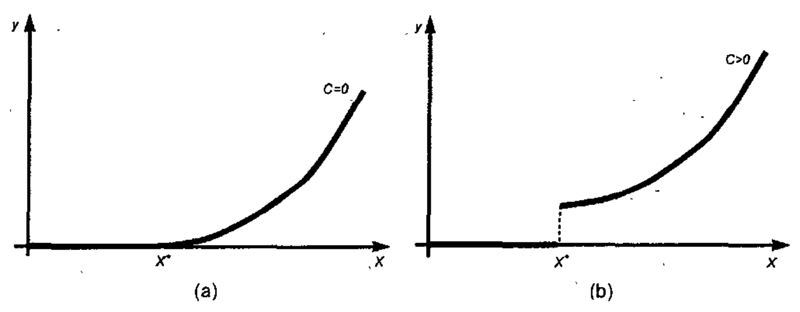


FIGURE 3
THE IMPACT OF C ON y

in-house capacity limit. This explains the continuous curve characterizing y and X as shown in Figure 3(a). An increase in the setup cost implies it becomes more costly to engage in outsourcing activities. The brand-producing firm now faces a trade-off between gains from outsourcing with rising setup cost and losses due to capacity shortage when market demand surges. The discontinuity of y at $X_C^*(K)$ in Figure 3(b) implies the outsourcing firm will not use outside resources unless the gains from outsourcing exceeds the setup cost at the marginal level.

IV. Optimal Capital Investment under Uncertainty

In the presence of uncertainty, the brand-producing firm chooses the capital investment to maximize its expected profit conditional on the available information. Thus, the brand-producing firm faces, depending upon whether it is possible to outsource or not, an optimal capital investment problem of

$$\max_{K\geq 0}\begin{cases} E[\pi^N(K, L^N(K))], & \text{in-house} \\ E[\pi_C^O(K, L^O(K), y^O(K))\mathbf{I}_{\{X>X_C^*(K)\}} + \pi^N(K, L^N(K))\mathbf{I}_{\{X\leq X_C^*(K)\}}], & \text{outsourcing} \end{cases} \quad (4)$$

Proposition 3 characterizes the firm's optimal capital investment under demand uncertainty with two models of integrated and non-integrated production.

Proposition 3

In equilibrium, the capital investment levels with outsourcing and without, denoted by K_C^O and K^N respectively, satisfy the following properties: **(1)** K_C^O increases in C. **(2)** K_C^O tends to K^N as $C \rightarrow \infty$.

Proof: See Appendix C. □

Proposition 3 suggests a brand-producing firm will reduce its capital investment when the option of outsourcing is available. Nevertheless, it raises such investment for a higher setup cost of outsourcing. In the extreme, as the cost of outsourcing becomes prohibitively high, the amount of capital investment under outsourcing approximates that in the absence of outsourcing. This result could therefore be interpreted as searching for the "compatible" partner in the outsourcing relationship. We have, thus, identified the conditions under which capital investment are chosen in both business models of integrated and non-integrated production.

V. Discussion

Sections V-*A* and V-*B* state, and comment upon, some of the main properties of the equilibrium results as described in Propositions 2 and 3. Section V-*C* compares the total outputs under the two modes of integrated production and non-integrated one. In section V-*D* we argue that our results are robust to modification in the technology of the basic production and in the distribution of the random variable.

A. Implications for Outsourcing Choices of Uncertainty

To explore the implications for choices of business models of demand uncertainty, we further investigate the properties of $X_C^*(K)$.

Lemma 1

(1) $X_C^*(K)$ is strictly increasing both in C and in K. **(2)** For any $C \geq 0$, $X_C^*(K)$ decreases in w and is independent of γ.

Proof: See Appendix D. □

Lemma 1 provides an important insight into debates over the nature of outsourcing. If we interpret $X_C^*(K)$ as the firm boundary, then Part (1) of Lemma 1 implies that idiosyncratic investment and industry-specific characteristics, such as capital- and/or labor-intensity for production and knowledge content in the product, and their interactions play a critical role in determining the outsourcing choices. Indeed, a low setup cost implies the brand-producing firm need not devoting much effort while monitoring the existing (or establishing for) outsourcing relationship. Hence, the boundary beyond which it chooses for outsourcing is low as the setup cost drops. Further, the critical value of this boundary is also affected by factor price. Part (2) of Lemma 1 suggests the higher the wage rate, the lower the boundary beyond which outsourcing occurs. Appropriately interpreted, the results characterize the growing off-shoring of basic production abroad from the developed economies subsequent to an increase in their domestic wages, in particular, under demand uncertainty. Notice, nevertheless, that outsourcing boundary is independent of capital price since it was incurred (a sunk cost) prior to actual production.

B. Implications for Job Losses of Outsourcing

The following Lemma characterizes the conditions for factor employment, given the chosen business model and optimal capital investment.

Lemma 2

(1) If $(\gamma/1-\beta)^{1-\beta}(w/\beta)^{\beta}\geq 1$, then $K_O^O=0$, $L_O^O=0$ and $y^O=(X(1-\varepsilon))^{1/\varepsilon}$. Moreover, $K_C^O=K^N$ for all C if and only if $X=0$ *a.s.*; and if $\mathbb{P}(X>0)>0$, $K_O^O<K^N$. **(2)** If $(\gamma/1-\beta)^{1-\beta}(w/\beta)^{\beta}<1$ and

$$X^{\frac{1}{1-\beta(1-\varepsilon)}}\leq\frac{1-\beta}{\gamma}\left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}}E\,[X^{\frac{1}{1-\beta(1-\varepsilon)}}]\quad a.s.,\tag{5}$$

then $K_C^O=K^N$ for all C. Conversely, if (5) does not hold almost everywhere, $K_O^O<K^N$.

Proof: See Appendix E. □

The above results can provide an important insight into the issue of job losses in the presence of outsourcing. Indeed, Lemma 2 highlights that the extent of outsourcing is determined crucially by the marginal cost of in-house production relative to the price of purchasing from the specialized subcontractor. Part (1) reflects that the brand-producing firm may simply purchase the basic manufacturing component and not enters the primary market at all if using own facilities is relatively costly. And Part (2) characterizes a firm's choice under situations when there is substantial divergence between the expected market demand and the realized one. This result offers a justification as to why brand-producing firm does not opt for complete outsourcing and retains some in-house production capacities (or capital investment) even though the outsourcing opportunity is available.

C. Total Output under Integrated and Non-integrated Production.

Theorem 1: In equilibrium, there exists $Y_C^*(K_C^O) \geq X_C^*(K_C^O)$ such that

(1) if $X > Y_C^*(K_C^O)$, then $Y_C^O + y^O > Y^N$.

(2) if $X \leq Y_C^*(K_C^O)$, then $Y_C^O + y^O \leq Y^N$.

***Proof*:** Using the results in Appendices A-E, it is straightforward to verify this result. □

Intuitively, greater uncertainty implies a higher required return on the use of outside resources if outsourcing is adopted. Hence, when the realized market demand is sufficiently high, the possibility to outsource provides the firm an avenue for profit increases by raising total output.

D. Comments on Production Technology and the Distribution of X

How would the results obtained in this paper change if we consider a general function of the basic production? Would the results change if a different specification for demand uncertainty is employed? We now sketch an argument that establishes the outcome is, in fact, unaffected: That is, the equilibrium choices of business model and capital investment (in an alternative setting) are identical to the equilibrium ones (in the present setting).

Clearly, the brand-producing firm decides the business model and, simultaneously, the capital investment. In the presence of demand uncertainty, the adjustment mechanism made available to the firm consists of the variable input of labor and the opportunity to outsource, if chosen. This suggests that capital investment, once installed, is independent from the variations in market demand. It is, therefore, evident that our results are robust to any modification of production technology that involves the use of capital and labor so long as the assumption of irreversibility and in-expandability for capital are retained.

We have investigated the issue of uncertainty by characterizing X as random variable. It may, however, be reasonable (in some contexts) to explore different distribution of this variable. In that case one can still define the critical value of $X_C^*(K)$ and the optimal capital investment. With such a change in the setting that defines the firm profit, our results are unaffected. Propositions 1-3 still describe the equilibrium choices.[9]

VI. Conclusion

In this paper, we show that outsourcing provides brand-producing firms with increased flexibility in adjusting their resources as new information about demand conditions become available. This argument can easily be extended to encompass even anticipated demand shifts, such as seasonal factors. Thus, outsourcing may well reflect an effort to deal with non-perfectly positively correlated anticipated demand variations.

Assessing the importance of demand uncertainty in explaining the wide adoption of the outsourcing business model would require the following: First, evaluation of the importance of demand uncertainty in various industries; second, examination of the extent to which random demand components are correlated across industries; and third, investigation for the importance of production networks and adjustment costs. Although some industry studies seem to confirm both the presence of a market idiosyncratic uncertainty as well as the presence of inflexibility in capital investment, the issue at hand still begs for a more rigorous empirical analysis.

Finally, it is important to note that the interaction between demand uncertainty and such factors as cost considerations, consumer

[9] However, with this change, the arguments and proofs are lengthier and restrictive.

preferences and strategic interactions - none of which are dealt with in the present paper - may yield important new insights. In particular, the welfare implications of outsourcing can be formally addressed once consumer utility is incorporated. Thus, analyses of these interactions feature high in our research agenda.

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Appendix A

Proof of Proposition 1

We derive the optimal capital investment using the backward induction. Upon the realization of market demand, for given $K \geq 0$, the firm chooses an optimal labor employment to maximize

$$\pi^N(K, L) = X(K^{1-\beta}L^{\beta})^{1-\varepsilon} - wL - \gamma K.$$

It is east to verify that the optimal labor employment $L^N(K)$ exists using the first and second order conditions. Hence, the optimal choice of labor implies a firm profit of

$$\pi^N(K, L^N(K)) = G_{\beta,\varepsilon} X^{\frac{1}{1-\beta(1-\varepsilon)}} K^{\frac{(1-\beta)(1-\varepsilon)}{1-\beta(1-\varepsilon)}} - \gamma K, \tag{A.1}$$

where $G_{\beta,\varepsilon}$ is a positive constant depending on β and ε.

The firm then decides on its capital investment to maximize the expected profit, *i.e.*,

$$\max_{K \geq 0} E[\pi^N(K, L^N(K))] = G_{\beta,\varepsilon} [X^{\frac{1}{1-\beta(1-\varepsilon)}}] K^{\frac{(1-\beta)(1-\varepsilon)}{1-\beta(1-\varepsilon)}} - \gamma K. \tag{A.2}$$

Using standard techniques to analyze the maximization problem of Equation (A.2), it is easy to verify that the first- and second-order conditions of (A.2) are satisfied, and, thus, gives us the required results. Notice that the direction of change of optimal capital investment under uncertainty depends only on the convexity effect, *i.e.*, for any $0 < \beta < 1$, K^N is convex in X (*cf.* Hartman 1972).

Appendix B

Proof of Proposition 2

If outsourcing is possible ($y \geq 0$), then the firm chooses, for given K, an optimal labor employment to solve

$$\max_{L,y\geq 0} \pi_C^O(K, L, y), \tag{A.3}$$

To investigate (A.3), we first consider an optimization problem of "definite outsourcing" (*i.e.*, $y>0$), that is,

$$\max_{L,y\geq 0} \tilde{\pi}_C^O(K, L, y)=X(K^{1-\beta}L^{\beta}+y)^{1-\varepsilon}-wL-\gamma K-(C+y). \tag{A.4}$$

Clearly, the relation between $\pi_C^O(K, L, y)$ and $\tilde{\pi}_C^O(K, L, y)$ is given by

$$\pi_C^O(K, L, y)= \begin{cases} \tilde{\pi}_C^O(K, L, 0)+C\geq \tilde{\pi}_C^O(K, L, 0), & \text{for } y=0, \\ \tilde{\pi}_C^O(K, L, y), & \text{for } y>0. \end{cases} \tag{A.5}$$

The optimal solution to (A.5) exists if

$$\frac{\partial \tilde{\pi}_C^O}{\partial L}=X(1-\varepsilon)(K^{1-\beta}L^{\beta}+y)^{-\varepsilon}\beta K^{1-\beta}L^{\beta-1}-w=0,$$

$$\frac{\partial \tilde{\pi}_C^O}{\partial y}=X(1-\varepsilon)(K^{1-\beta}L^{\beta}+y)^{-\varepsilon}-1=0.$$

This implies that optimal L and y, in equilibrium, must be given by

$$\tilde{L}^O(K)=\left(\frac{\beta}{w}\right)^{\frac{1}{1-\beta}}K, \tag{A.6}$$

$$\tilde{y}^O(K)=(X(1-\varepsilon))^{\frac{1}{\varepsilon}}-\left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}}K. \tag{A.7}$$

Further, it is easy to verify the second-order condition is satisfied. Using (A.7), we see that $\tilde{y}^O(K)\geq 0$ if and only if $X\geq\tilde{X}(K):=(1/(1-\varepsilon))(\beta/w)^{(\beta\varepsilon/(1-\beta))}K^{\varepsilon}$. This implies the value of X plays a crucial role in determining the optimal solution of (A.4). Thus, we separate our discussion into two cases: $\{\omega\in\Omega: X(\omega)\leq\tilde{X}(K)\}$ and $\{\omega\in\Omega: X(\omega)>\tilde{X}(K)\}$.

First, on the set $\{\omega\in\Omega: X(\omega)\leq\tilde{X}(K)\}$, we have $\tilde{y}^O(K)\leq 0$. Hence, the optimal solution to equation (A.4) occurs on the boundary $y=0$. This coincides with the case of no outsourcing, and, thus, the optimal solution of (A.3) is $(L^O(K), y^O(K))=(L^N(K), 0)$.

Second, on the set $\{\omega\in\Omega: X(\omega)\leq\tilde{X}(K)\}$, an interior solution holds and $(\tilde{L}^O(K), \tilde{y}^O(K))$ is the optimal solution to (A.4). Note, however, from (A.5) that

$$\pi_C^O(K,\ L,\ 0)=\tilde{\pi}_C^O(K,\ L,\ 0)+C.$$

We, therefore, compare if

$$\pi_C^O(K,\ \tilde{L}^O(K),\ \tilde{y}^O(K))>\pi^n(K,\ L^N(K)).$$

Since the monopolist involves outsourcing if and only if the payoff generated from doing so is greater than it otherwise would have been. For fixed K, define by $F_C(X, K)$ the payoff differences between the two outsourcing regimes, *i.e.*,

$$F_C(X, K)=\pi_C^O(K,\ \tilde{L}^O(K),\ \tilde{y}^O(K))-\pi^N(K,\ L^N(K))>0. \tag{A.8}$$

A straightforward calculation shows, for any $X>\tilde{X}(K)$, that $(\partial F_C/\partial X)>0$. This implies $F_C(\cdot, K)$ is strictly increasing to infinite on $(\tilde{X}(K),\ \infty)$. Furthermore, notice that

$$F_C(\tilde{X}(K), K)=-C.$$

It follows that if $C=0$, $F_0(X, K)>F_0(\tilde{X}(K), K)=0$ for all $X>\tilde{X}(K)$, and if $C>0$, $F_C(\tilde{X}(K), K)<0$. Since $F_C(\cdot, K)$ is strictly increasing in X to ∞ on $(\tilde{X}(K), \infty)$, we know that $F_C(X, K)=0$ has a unique solution for $X>\tilde{X}(K)$. And, the solution is given by

$$X_C^*(K)=\sup\{X: F_C(X,\ K)\leq 0\}. \tag{A.9}$$

Thus, we have established that $F_C(X, K)>F_C(X_C^*(K), K)=0$ for all $X>X_C^*(K)$. This result suggests that, given $K\geq 0$, the firm will engage in outsourcing $(y^O(K)>0)$ when X is sufficiently large (*i.e.*, $X>X_C^*(K)$).

Appendix C

Proof of Proposition 3

Due to Proposition 2, we write the monopolist's expected profit as

$$G_C(K) := E[\pi_C^O(K, L^O(K), y^O(K))\, \mathbf{I}_{\{X > X_C^*(K)\}} + \pi^N(K, L^N(K))\, \mathbf{I}_{\{X \le X_C^*(K)\}}] \quad \text{(A.10)}$$

To prove Proposition 2, we proceed in the following four steps.

Step 1. The existence of K_0^O. Given $C=0$, differentiating (A.10) with respect to K, we have

$$G'_0(K) = (1-\beta)\left(\frac{\beta}{w}\right)^{\frac{\beta(1-\varepsilon)}{1-\beta(1-\varepsilon)}} (1-\varepsilon)^{\frac{1}{1-\beta(1-\varepsilon)}} K^{\frac{-\varepsilon}{1-\beta(1-\varepsilon)}} U_0(K), \quad \text{(A.11)}$$

where

$$\begin{aligned} U_0(K) = E\Big[&(X^{\frac{1}{1-\beta(1-\varepsilon)}} - X_0^*(K)^{\frac{1}{1-\beta(1-\varepsilon)}})\, \mathbf{I}_{\{X \le X_0^*(K)\}}\Big] \\ &+ X_0^*(K)^{\frac{1}{1-\beta(1-\varepsilon)}} \left(1 - \left[\left(\frac{\gamma}{1-\beta}\right)^{1-\beta} \left(\frac{w}{\beta}\right)^{\beta}\right]^{\frac{1}{1-\beta}}\right). \end{aligned} \quad \text{(A.12)}$$

Thus, we separate our discussion of (A.12) into two cases:

(i) $((\gamma/(1-\beta))^{(1-\beta)}(w/\beta)^{\beta}) \ge 1$. Since the first term in (A.12) is strictly negative for $K>0$, $G'_0(K)<0$ for any $K>0$. This suggests $G_0(K)$ has a global maximum at $K=0$, *i.e.*, $K_0^O=0$.

(ii) $((\gamma/(1-\beta))^{(1-\beta)}(w/\beta)^{\beta}) < 1$. Due to

$$U'_0(K) = M_1 K^{-\frac{(1-\beta)(1-\varepsilon)}{1-\beta(1-\varepsilon)}} \left(P[X \ge X_0^*(K)] - \left[\left(\frac{\gamma}{1-\beta}\right)^{1-\beta} \left(\frac{w}{\beta}\right)^{\beta}\right]^{\frac{1}{1-\beta}} \right),$$

with a positive constant M_1. Using the fact that $U_0(0)=0$, we see that $U_0(K)>0$ if K is small enough. Moreover, we have established that $U_0(K) \to -\infty$ if $K \to \infty$. Therefore, there exists a unique K_0^O such that $U_0(K_0^O)=0$, which suggests $G'_0(K_0^O)=0$. Hence, $G_0(K)$ has a global maximum at $K_0^O \in (0, \infty)$.

Step 2. The existence of K_C^O for general C. Differentiating (A.10) with respect to K, we have

$$G'_C(K)=M_2 K^{-\frac{\varepsilon}{1-\beta(1-\varepsilon)}} U_C(K), \quad \text{(A.13)}$$

where M_2 is a positive constant and

$$U_C(K)=U_0(K)+E\left[\left(X^{\frac{1}{1-\beta(1-\varepsilon)}}-X_0^*(K)^{\frac{1}{1-\beta(1-\varepsilon)}}\right)\mathbf{I}_{\{X_0^*(K)<X<X_C^*(K)\}}\right]. \quad \text{(A.14)}$$

Recall $F_C(X_C^*(K), K)=0$, and the results obtained in Step 1, we have $U_0(K)\to-\infty$ as $K\to\infty$. Moreover, notice $E\{X^{(1/\varepsilon)}\}<\infty$, the second term in (A.14) is strictly positive and bounded. Thus, $U_C(K)<0$ as K large enough. Together with $U_C(0)\geq 0$, we have established that there exists a zero of $U_C(K)$, which is also global maximum of $G_C(K)$. This proves the existence of K_C^O for general C.

Step 3. The monotonicity of K_C^O. From (A.13) we have, for fixed K, $C_1>C_2\geq 0$,

$$G'_{C_1}(K)-G'_{C_2}(K)=M_2K^{-\frac{\varepsilon}{1-\beta(1-\varepsilon)}}(U_{C_1}(K)-U_{C_2}(K)).$$

Recall, from Lemma 1, that $X_0^*(K)<X_{C_2}^*(K)<X_{C_1}^*(K)$. This implies

$$U_{C_1}(K)-U_{C_2}(K)=E\left[\left(X^{\frac{1}{1-\beta(1-\varepsilon)}}-X_0^*(K)^{\frac{1}{1-\beta(1-\varepsilon)}}\right)\mathbf{I}_{\{X_{C_2}^*(K)<X<X_{C_1}^*(K)\}}\right]>0.$$

Hence

$$G'_{C_1}(K)>G'_{C_2}(K), \quad \text{(A.15)}$$

for all K. Following the results obtained in Step 2 we know $G_{C_2}(K)$ has at least one local maximum (which occurs at the points such that $G'_{C_2}(K)=0$). For simplicity, we assume that $G_{C_2}(K)$ has two local maxima: At K_1 and at K_2 with $K_1<K_2$ (For the case with one and n local maxima, we may use the similar argument.).

(i) $K_{C_2}^O=K_1=0$. Clearly $K_{C_1}^O\geq 0=K_{C_2}^O$.

(ii) $K_{C_2}^O=K_1>0$, *i.e.*, $G_{C_2}(K_1)\geq G_{C_2}(K_2)$. Due to (20) and since $G'_{C_2}(K)>0$ for all $K<K_1$, $G'_{C_1}(K)>G'_{C_2}(K)\geq 0$ for all $K\leq K_1$. Following Step 2, we know that $G'_{C_1}(K)=0$ has at least one solution. Together with $G'_{C_1}(K)>0$ for all $K<K_1$, we see that all the zeros of

$G'_{C_1}(K)$ is larger than K_1, *i.e.*, $K^O_{C_1} > K_1 = K^O_{C_2}$.

(iii) $K^O_{C_2} = K_2$, *i.e.*, $G_{C_2}(K_1) < G_{C_2}(K_2)$. Due to

$$G_{C_2}(K_2) = G_{C_2}(K_1) + \int_{K_1}^{K_2} G'_{C_2}(K)dK,$$

we have

$$\int_{K_1}^{K_2} G'_{C_2}(K)dK > 0. \tag{A.16}$$

Suppose that the global maximum of $G_{C_1}(K)$, $K^O_{C_1}$, is less than $K^O_{C_2} = K_2$. Using (A.15) we know $G'_{C_1}(K) > G'_{C_2}(K) > 0$ for all $K < K_1$. Thus, there exists a solution to $G'_{C_1}(K) = 0$, denoted by $K^O_{C_1}$, between K_1 and K_2. Furthermore, since $G'_{C_1}(K_2) > G'_{C_2}(K_2) = 0$ and $G'_{C_1}(K) < 0$ as K large enough (see Step 2), $G_{C_1}(K)$ has at least one local maximum larger than K_2, say $\bar{K}$. Because of (A.15) and (A.16), we get

$$\begin{aligned} G_{C_1}(\bar{K}) &= G_{C_1}(K^O_{C_1}) + \int_{K^O_{C_1}}^{\bar{K}} G'_{C_1}(K)dK > G_{C_1}(K^O_{C_1}) + \int_{K^O_{C_1}}^{K_2} G'_{C_1}(K)dK \\ &> G_{C_1}(K^O_{C_1}) + \int_{K^O_{C_1}}^{K_2} G'_{C_2}(K)dK > G_{C_1}(K^O_{C_1}) + \int_{K_1}^{K_2} G'_{C_2}(K)dK \\ &> G_{C_1}(K^O_{C_1}), \end{aligned}$$

which clearly contradicts the result that $K^O_{C_1}$ is the global maximum of $G_{C_1}(K)$. This implies the global maximum for $G_{C_1}(K)$ must occur at the place larger than K_2, *i.e.*, $K^O_{C_1} > K_2 = K^O_{C_2}$.

Hence, $K^O_{C_1} > K^O_{C_2}$ for $C_1 > C_2$. In other words, K^O_C is increasing in C.

Step 4. Using Equation

$$\pi^O_C(K, L^O(K), y^O(K)) = \pi^O_C(K, L^O(K), y^O(K))\mathbf{I}_{\{X > X^*_C(K)\}} + \pi^N(K, L^N(K))\mathbf{I}_{\{X \le X^*_C(K)\}},$$

and note that $X^*_C(K) \to \infty$ as $C \to \infty$, it is easy to verify that $G_C(K) \to \mathrm{E}[\pi^N(K, L^N(K))]$ as $C \to \infty$. Further, using Step 3, we know that K^O_C is increasing in C, thus $K^O_C \le K^n$ for all C.

Appendix D

Proof of Lemma 1

(1) If $C=0$, then $\partial X_0^*(K)/\partial K=(\varepsilon/(1-\varepsilon))(\beta/w)^{\beta\varepsilon/(1-\beta)}K^{\varepsilon-1}>0$; and if $C>0$, using (A.9) and note that $F_C(X_C^*(K), K)=0$, and that $F_C(X, K)$ is continuous in K, we know $X_C^*(K)$ must satisfy

$$X^{\frac{1}{\varepsilon}}\varepsilon(1-\varepsilon)^{\frac{1-\varepsilon}{\varepsilon}}+\left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}}(1-\beta)K-C$$
$$=\left(\frac{\beta(1-\varepsilon)}{w}\right)^{\frac{\beta(1-\varepsilon)}{1-\beta(1-\varepsilon)}}(1-\beta(1-\varepsilon))X^{\frac{1}{1-\beta(1-\varepsilon)}}K^{\frac{(1-\beta)(1-\varepsilon)}{1-\beta(1-\varepsilon)}}. \tag{A.17}$$

Differentiating (A.17) with respect to K and C, respectively, we have both $\partial X_C^*(K)/\partial K$ and $\partial X_C^*(K)/\partial C$ are strictly positive for any $K>0$, implying that $X_C^*(K)$ is strictly increasing both in K and C.

(2) Using (A.17), it is easy to verify that both of $\partial X_0^*(K)/\partial w$ and $\partial X_C^*(K)/\partial w$ are strictly negative since $A<1$.

Appendix E

Proof of Lemma 2

(1) $(\gamma/(1-\beta))^{1-\beta}(w/\beta)^{\beta}\geq 1$. Using the results contained in Proposition 1 and Step 1 in the proof of Proposition 3, we know if $(\gamma/(1-\beta))^{1-\beta}(w/\beta)^{\beta}\geq 1$, $K^N=K_0^O=0$ if and only if $X\equiv 0$ *a.s.*

(2) $(\gamma/(1-\beta))^{1-\beta}(w/\beta)^{\beta}<1$. As shown in the Step 1 in the proof of Proposition 3, it is easy to show that if $(\gamma/(1-\beta))^{1-\beta}(w/\beta)^{\beta}<1$, the equation $G_0'(K)=0$ has two different solutions 0 and K_0^O, and

$$G_0'(K)\begin{cases}>0, & \text{if } K\in(0, K_0^O),\\ <0, & \text{if } K>K_0^O.\end{cases} \tag{A.18}$$

Thus, $K^N=K_0^O$ if and only if

$$G'_0(K^N) = \frac{\gamma}{E\left[X^{\frac{1}{1-\beta(1-\varepsilon)}}\right]} E\left[\left((X_0^*(K^N))^{\frac{1}{1-\beta(1-\varepsilon)}} - X^{\frac{1}{1-\beta(1-\varepsilon)}}\right)\mathbf{I}_{\{X \geq X_0^*(K^N)\}}\right] = 0.$$

This implies that $K^N = K_0^O$ if and only if

$$\left(X_0^*(K^N)\right)^{\frac{1}{1-\beta(1-\varepsilon)}} - X^{\frac{1}{1-\beta(1-\varepsilon)}}\Big)\mathbf{I}_{\{X \geq X_0^*(K^N)\}} = 0, \quad a.s.,$$

which is equivalent to

$$X^{\frac{1}{1-\beta(1-\varepsilon)}} \leq X_0^*(K^N)^{\frac{1}{1-\beta(1-\varepsilon)}} = \frac{1-\beta}{\gamma}\left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}} E\left[X^{\frac{1}{1-\beta(1-\varepsilon)}}\right] \quad a.s.$$

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