On the Stability of Heller's Coordination Failure Model

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This paper reconsiders Heller's multisector Cournot-Nash model (Heller 1986) hat is known as one of the representative models of coordination failure. In Heller's model, there are multiple Pareto-anked equilibria if the price elasticity of demand is highly inelastic near the subsistence level. This paper analyzes the stability of those equilibria and shows that the inferior equilibrium is unstable while the superior equilibrium is stable. Therefore, the coordination failure does not emerge in Heller's model.

Keywords: Coordination failure, Cournot, Stability

JEL Classification: D43, D58, E12

I. Introduction

This paper reconsiders Heller's multisector Cournot-Nash model (Heller 1986) from the stability aspect. The model is known as one of the representative models of coordination failure models whose characteristic is that there exist multiple, Pareto-ranked equilibria (Romer 1996, Cooper 1999). In particular, the multisector Cournot-Nash models have been presented by Hart (1982) and Heller (1986). In their models, a firm of each sector behaves strategically as an oligopolist taking as given the position of the industry demand curve,

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determined by the level of activity in other sectors, and the output level of other firms in their own sector. "A major objective of both essays is to provide an explanation of how unemployment might be sustained in general equilibrium even if there is perfect price flexibility and rational demand perceptions Heller (1986)." Hart (1982) introduces unproduced good and imposes assumptions to assure that there is a unique equilibrium for any level of unproduced good. Hence there are no coordination inefficiencies in his model. On the other hand, Heller considered a framework different from Hart (1982), so that coordination inefficiencies possible occur by the existence of multiple equilibria. The key condition of Heller (1986) is that the price elasticity of demand is highly inelastic when consumption is near the subsistence level.

Like Cooper and John (1988) and Romer (1996), it seems reasonable to focus on the stable symmetric Nash equilibria in discussion of coordination failure. However, Heller (1986) disregards the strategic interactions among firms in each sector and thus the stability of Cournot adjustment process. In fact, he discusses the stability of equilibria:

> These equilibria are stable under experimentation. Suppose a manager of a firm ... contemplated a unilateral expansion away from a low-level equilibrium. ... The manager therefore chooses to remain at his initial equilibrium strategy. - Heller (1986)

However it implies that each player does not deviate from the equilibrium alone since it is not profitable for himself. This is merely explanation of Nash equilibrium.

This paper uncovers the strategic interactions among firms in Heller's model and shows that there exist two partial Cournot-Nash equilibria, one is stable and another is unstable under the Cournot adjustment process. Moreover, we show that the inferior general Cournot-Nash equilibrium consists of the unstable lower output partial equilibrium in each sector. Therefore, the coordination failure does not emerge in Heller's model.[1]

In Section 2, we present the Heller model and analyze the partial

[1] Although Kawai and Minagawa (2006) analyze the strategic interrelation and the macroeconomic implication of Heller's model, it is also disregarded the stability.

Cournot-Nash equilibria. In Section 3, we analyze the general Cournot-Nash equilibria. Section 4 is devoted to our conclusion.

II. The Heller Model

In this section, we present Heller's model. There are two sectors, say sectors 1 and 2. In each sector there are m firms, each of which employs n households to produce a homogeneous final consumption good. Each household works only in one sector and consumes only the good of the other sector. The households working in sector 1 are called as type 1 households and those working in sector 2 called as type 2 households.

A. Households

Suppose that the type 1 households have the utility function u_1 defined by

$$u_1(c_2, l_1)=\beta_c \log(c_2-\underline{c})+\beta_l \log l_1, \text{ where } \beta_c>0,\ \beta_l>0, \tag{1}$$

where c_2, l_1, β_c, and β_l denote consumption amount of good 2, the leisure of type 1, and the discount factors of consumption and of leisure, respectively. Notice that there is the subsistence level of consumption $\underline{c}$ (>0) in this function. This function is called Stone-Geary type or the Linear Expenditure System (LES).

The type 1 representative household solves the following problem.

$$\begin{aligned} \max_{c_2, l_1}\ & \beta_c \log(c_2-\underline{c})+\beta_l \log l_1, \\ s.t.\ & p_2 c_2 \leq w_1 L_1^s + r_1, \\ & l_1=\bar{L}-L_1^s, \end{aligned} \tag{2}$$

where $\bar{L}$, L_1^s, p_2 and w_1 denote the initial endowment of labor common to all households, the labor supply of type 1 household, the price of good 2, and the wage of type 1 labor, respectively. The profits of good 1 firms are equally distributed to the type 1 households. Thus the dividend received by type 1 household is denoted by $r_1 \equiv 1/n \sum_{i=1}^{m} \pi_1^i$ is the profit of the ith firm of sector 1.

The problem of type 2 households has a symmetric structure. That is,

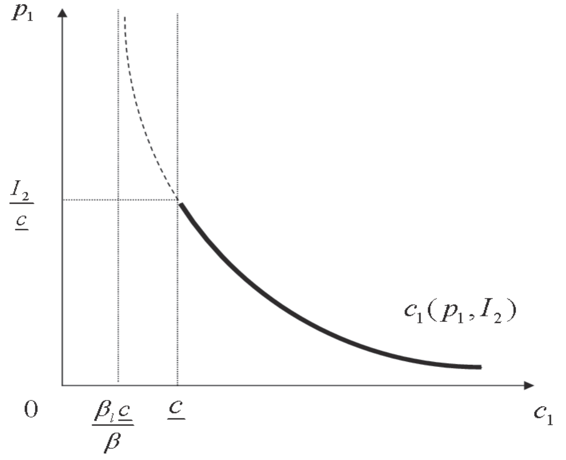

FIGURE 1
INDUSTRY DEMAND CURVE IN SECTOR 1

$$\begin{aligned} &\max_{c_1, l_2} \ \beta_c \log(c_1 - \underline{c}) + \beta_l \log l_2, \\ &s.t. \ \ p_1 c_1 \leq w_2 L_2^s + r_2, \\ &\qquad l_2 = \bar{L} - L_2^s, \end{aligned} \tag{3}$$

Notice that β_c and β_l are common across types.

For convenience, we consider the problem of type 2. The first order condition is

$$\frac{\beta_c l_2}{\beta_l (c_1 - \underline{c})} = \frac{p_1}{w_2} \ . \tag{4}$$

The demand function of type 2 becomes

$$c_1(p_1, \ I_2) = \begin{cases} \frac{\beta_l \underline{c}}{\beta} + \frac{\beta_c I_2}{\beta p_1}, & \text{if } 0 < p_1 < \frac{I_2}{\underline{c}}, \\ 0, & \text{if } p_1 \geq \frac{I_2}{\underline{c}}, \end{cases} \tag{5}$$

where $\beta = \beta_c + \beta_l$, where $I_2 = w_2 \bar{L} + r_2$ (see Figure 1).

For simplicity, without loss of generality, suppose that $n=1$. Then, the inverse demand function of market 1 can be written as

$$p_1(c_1,\ I_2)=\frac{\beta_c I_2}{\beta c_1-\beta_l \underline{c}}\ ,\quad \text{when}\quad c_1>\underline{c}\ . \tag{6}$$

The inverse of price elasticity of demand becomes

$$\frac{Ep_1(c_1,\ I_2)}{Ec_1}=\frac{\beta c_1}{\beta c_1-\beta_l \underline{c}}. \tag{7}$$

Similarly, the above equations for type 1 can be derived by changing the subscript number.

B. Partial Cournot-Nash Equilibrium

There are m firms within a sector. For simplicity, suppose that the production function is identical to all firms and assume that $m=2$. We define the i th firm production function in sector j as

$$y_j^i=\alpha L_j^{di},\ \alpha>0,\ i=1,\ 2\quad \text{and}\quad j=1,\ 2, \tag{8}$$

where y_j^i and L_j^{di} denote the production and the labor demand of the i th firm of sector j, respectively. Taking as given the subjective inverse demand function (6) and the outputs of the other firm within the sector, it maximizes its profit. Then, the profit function of the ith firm in sector 1 can be expressed as

$$\pi_1^i(y_1^i,\ y_1^j;\ I_2)=\left[p_1(Y_1;\ I_2)-\frac{w_1}{\alpha}\right]y_1^i\quad \text{for } i,\ j=1,\ 2,\ i\neq j. \tag{9}$$

where $Y_1=y_1^1+y_1^2$. The first order condition of firm i of sector 1 is

$$p_1\left(1-\frac{y_1^i}{Y_1}\cdot\frac{Ep_1}{Ec_1}\right)=\frac{w_1}{\alpha}\quad \text{for } i=1,\ 2. \tag{10}$$

From (10) and (7), the response function of the ith firm in sector 1 becomes,

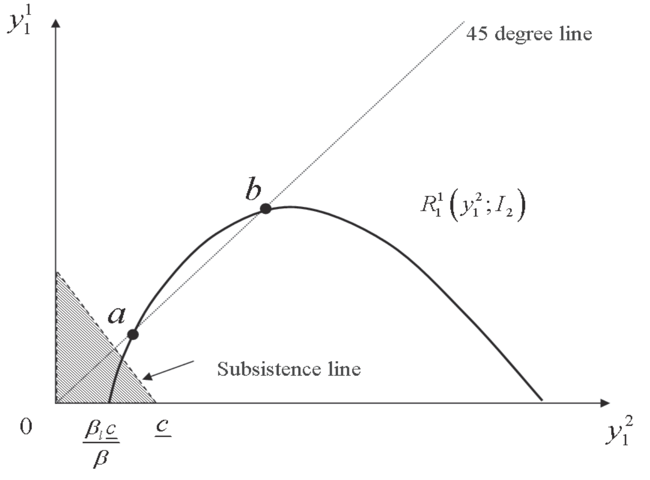

FIGURE 2
REACTION CURVE OF FIRM 1 IN SECTOR 1

$$y_1^i = R_1^i(y_1^i;\ I_2) = -\left(y_1^j - \frac{\beta_l \underline{c}}{\beta}\right) + \sqrt{\frac{\alpha\beta_c I_2}{\beta w_1}\left(y_1^j - \frac{\beta_l \underline{c}}{\beta}\right)}, \tag{11}$$

for i, $j=1,\ 2,\ i \neq j$.

The reaction curve for the first firm in sector 1 is shown in Figure 2. The subsistence line is defined as $y_1^1 + y_1^2 = \underline{c}$. The derived reaction curves are effective above the line.

The point a and b in Figure 2 are the partial symmetric Cournot-Nash equilibria. We examine whether these two equilibria satisfy the second order condition under the sufficient condition in Lemma 2 of Heller (1986). This Heller's condition is that $E\eta/Ec < m-1$, where $\eta \equiv Ep(c)/Ec$. In the present model, it becomes $E\eta/Ec < 1$ since $m=2$. In fact, from (7), we obtain

$$\frac{E\eta}{Ec} = -\frac{\beta_l \underline{c}}{\beta c - \beta_l \underline{c}} < 1.$$

so that Heller's condition holds in our model.

Now, we can establish the following proposition.

***Proposition* 1.** If $\beta_l > \beta_c$ and $I_2 > 8w_1\beta_l \underline{c}/\alpha\beta_c$, then there are two partial symmetric Cournot-Nash equilibria (PSCNE) in each sector.

***Proof*:** Consider the case of sector 1. Solving (11) for y_1^i, $i=1, 2$, we obtain,

$$y_1^* = \frac{\beta_l \underline{c}}{2\beta} + \frac{\alpha\beta_c I_2 w_1^{-1} \pm \sqrt{\alpha\beta_c I_2 w_1^{-1}(\alpha\beta_c I_2 w_1^{-1} - 8\beta_l \underline{c})}}{8\beta}, \tag{12}$$

where y_1^* is a symmetric solution for sector 1. If $\alpha\beta_c I_2 w_1^{-1} > 8\beta_l \underline{c}$, then there are two solutions. Let the higher solution and lower solution denote $\bar{y}_1^*$ and $\underline{y}_1^*$ respectively. If $\beta_l > \beta_c$, then $\underline{y}_1^* > \underline{c}/2$. Thus, the total amounts of production of sector 1 in both solutions are greater than the subsistence level $\underline{c}$. Therefore there exist two PSCNE in sector 1. A similar argument can be applied to the case of sector 2. *Q.E.D.*

We are now in a position to introduce the Cournot adjustment process, which is displayed as

$$\left.\begin{aligned} \dot{y}_1^1 &= R_1^1(y_1^2; I_2) - y_1^1, \\ \dot{y}_1^2 &= R_1^2(y_1^1; I_2) - y_1^2, \end{aligned}\right\} \text{(c)}$$

Then, we have

***Proposition* 2.** Suppose that there exist two PSCNE in each sector. Then the PSCNE of a higher production is stable and that of a lower production is unstable under the Cournot adjustment process (c).

***Proof*:** Consider the case of sector 1. Taking the derivative of (11) with respect to y_1^j, we obtain

$$\frac{dR_1^i(y_1^j; I_2)}{dy_1^j} = -1 + \frac{1}{2}\left(\frac{\alpha\beta_c I_2}{\beta w_1}\left(y_1^j - \frac{\beta_l \underline{c}}{\beta}\right)\right)^{-\frac{1}{2}} \frac{\alpha\beta_c I_2}{\beta w_1}. \tag{13}$$

Substituting (12) into (13), we can obtain,

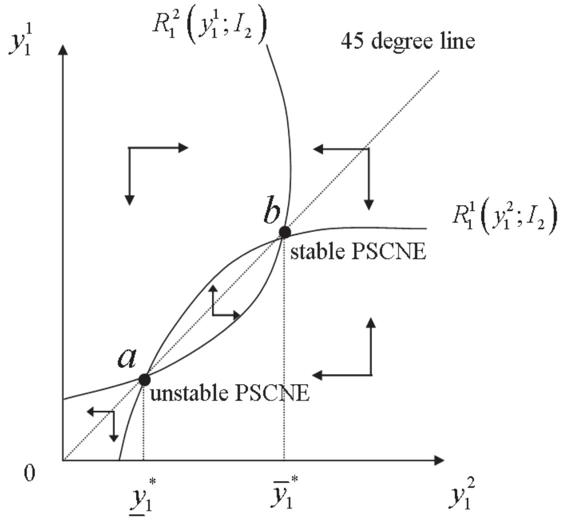

FIGURE 3
PARTIAL SYMMETRIC C-N EQUILIBRIA IN SECTOR 1

$$\frac{dR_1^i(y_1^j;\, I_2)}{dy_1^j} > 1, \text{ and } 0 < \frac{dR_1^i(\bar{y}_1^*;\, I_2)}{dy_1^j} < 1,$$

from which the assertion of the proposition holds for sector 1. As for sector 2, a similar argument can be applied (see Figure 3). *Q.E.D.*

III. General Cournot-Nash Equilibrium

Now we are going to analyze the general equilibrium. Since any partial equilibrium is symmetric between firms, the outputs of two firms are equal at any partial equilibrium. That is, $y_1^1 = y_1^2 = y_1^*$, and it is assumed as $n = 1$. The feasibility condition for good i market becomes $c_i = 2y_i$, for $i = 1, 2$. On the other hand, the feasibility condition for type i labor market can be written as $L_i^s = 2L_i^d$, for $i = 1, 2$.

From the production function and the labor constraint that $\bar{L} = L_i^s + l_i$, the feasibility condition of market i can be derived as

$$l_i = \bar{L} - \frac{c_i}{\alpha}, \text{ for } i = 1,\ 2. \tag{14}$$

Consider the symmetric Nash equilibrium (SNE) in each sector. Suppose the labor as a numeraire; $w_1 = w_2 = 1$. Substituting the feasibility condition (14) into (4), we obtain,

$$p_1(c_1,\ c_2) = \frac{\beta_c(\bar{L} - \frac{c_2}{\alpha})}{\beta_l(c_1 - \underline{c})}. \tag{15}$$

Since we focus on the SNE in each sector, it should be satisfied that; $y_j^i = y_j = Y_j/2$ for $j = 1,\ 2$. From (10), firm's f.o.c. in sector 1 industry can be written as:

$$\frac{Ep_1}{Ec_1} = 2\left[1 - \frac{1}{p_1\alpha}\right]. \tag{16}$$

Substituting (15) and (7) into RHS and LHS respectively in (16), we obtain,

$$\frac{\beta c_1}{\beta c_1 - \beta_l \underline{c}} = 2\left[1 - \frac{\beta_1(c_1 - \underline{c})}{\beta_c(\alpha\bar{L} - c_2)}\right]. \tag{17}$$

This equation exhibits the output relations between sectors. We can calculate an industrial output of sector 1 given that of sector 2.

From (17) and the symmetric structure between sectors, the symmetric general equilibrium can be obtained by substituting $c_1 = c_2 = c$,

$$\frac{\beta c}{\beta c - \beta_l \underline{c}} = 2\left[1 - \frac{\beta_1(c - \underline{c})}{\beta_c(\alpha\bar{L} - c)}\right]. \tag{18}$$

From (18), the following proposition is established.

***Proposition* 3.** If $\alpha > B$ and $\beta_l > \beta_c$, where

$$B = \max\left[\frac{((2\beta_l + 4\beta_c - 1)\beta_l - \beta_c)\underline{c}}{(\beta_l - \beta_c)\beta_c \bar{L}}, \frac{2\beta_l \underline{c}}{\beta\beta_c \bar{L}}\left(2\beta_l + \sqrt{4\beta_l^2 - 2\beta\beta_c}\right)\right],$$

then there exist two general symmetric Cournot-Nash equilibria (GSCNE).

Proof: First, solving (18) for c, we obtain,

$$c = \frac{\beta(\alpha\bar{L}\beta_c + 4\beta_l \underline{c}) \pm \sqrt{\beta\beta_c((\alpha\bar{L})^2\beta\beta_c - 8\beta_l^2(\alpha\bar{L} - \underline{c}))}}{2\beta(\beta + \beta_l)}. \qquad (19)$$

If there are two solutions in (19), the determinant must be positive, *i.e.*,

$$D = \beta\beta_c((\alpha\bar{L})^2\beta\beta_c - 8\beta_l^2(\alpha\bar{L} - \underline{c})) > 0. \qquad (20)$$

The sufficient condition to satisfy (20) is that

$$\alpha > \frac{2\beta_l \underline{c}}{\beta\beta_c \bar{L}}\left(2\beta_l + \sqrt{4\beta_l^2 - 2\beta\beta_c}\right), \text{ and } \frac{\beta_l}{\beta_c} > \frac{1 + 2\sqrt{2}}{4}. \qquad (21)$$

Next, from the condition of the lower solution in (19) is more than $\underline{c}$, the following equation must be satisfied,

$$\beta_c(\beta_l - \beta_c)\alpha\bar{L} + (2\beta_l - 4\beta)\beta_l \underline{c} + \beta\underline{c} > 0. \qquad (22)$$

The sufficient condition of (22) is that

$$\alpha > \frac{((2\beta_l + 4\beta_c - 1)\beta_l - \beta_c)\underline{c}}{(\beta_l - \beta_c)\beta_c \bar{L}}, \text{ and } \beta_l > \beta_c. \qquad (23)$$

Combining (21) and (23), we obtain the sufficient conditions in the statement of proposition. *Q.E.D.*

The condition that $\alpha > B$ and $\beta_l > \beta_c$ in Proposition 3 plays a similar role to those of Theorem 2 and Theorem 4 of Heller (1986). Especially, the latter inequality is deduced from the condition in Heller's Theorem

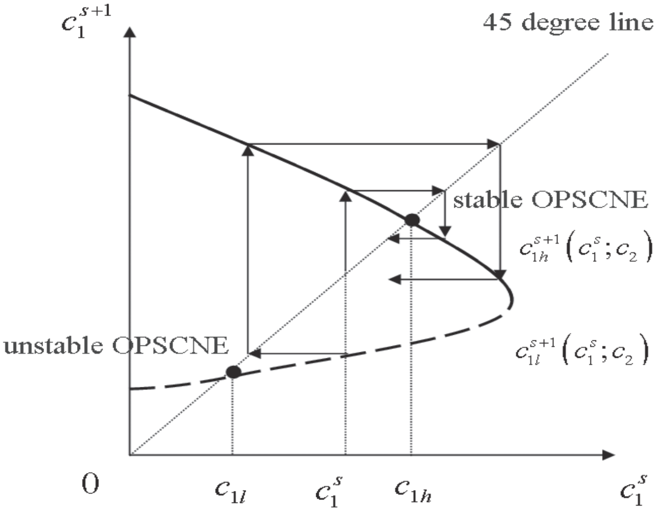

FIGURE 4
OBJECTIVE PARTIAL SYMMETRIC C-N EQUILIBRIA IN SECTOR 1

4. In fact, from (7), Heller's condition in Theorem 4 becomes

$$\lim_{c \to \underline{c}^+} \frac{\beta c}{\beta c - \beta_l \underline{c}} > 2,$$

This condition implies that the price elasticity of demand is highly inelastic near the subsistence level, which corresponds to the key source of multiple equilibria.

Therefore the condition of Proposition 3 implies that the higher productivity and the lower elasticity of consumption demand near the subsistence level may create the lower Cournot-Nash equilibrium.

We can calculate and check that the profit at the lower output equilibrium is higher than that at the higher output equilibrium. Hence, there is incentive for firms to coordinate, not cooperate, to the lower output equilibrium. However, this kind of coordination among firms might be failed since the lower output equilibrium is unstable. This is the kind of coordination failure for firms but it is preferable for markets since the higher output equilibrium corresponds to the Pareto-superior equilibrium. Now we investigate the stability of

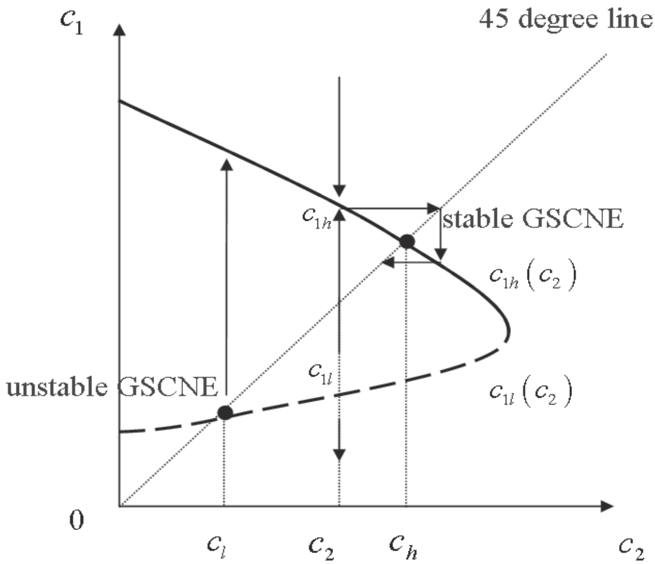

FIGURE 5
GENERAL SYMMETRIC C-N EQUILIBRIA

the general equilibrium in the next section.

A. Stability of General Cournot-Nash Equilibria

From (17), given the output of sector 2, there could be two solutions c_{1h} and c_{1l} for an appropriate range of c_2 as in Figure 5. Notice that these two solutions do not directly coincide with the PSCNE given I_2, since the term r_2 in I_2 does not given in the general equilibrium setting. The term r_2 may changes not only the output of sector 2 but also the output of sector 1. From $w_2=1$, the definition of r_2, and the price condition in (15),

$$\begin{aligned} I_2(c_1, c_2) &= \bar{L} + \left(p_2 - \frac{1}{\alpha}\right)c_2, \\ &= \bar{L} + \left(\frac{\beta_c(\bar{L} - c_1/\alpha)}{\beta_l(c_2 - \underline{c})} - \frac{1}{\alpha}\right)c_2. \end{aligned} \tag{24}$$

***Lemma* 1.** Suppose that the conditions of Proposition 1 and 3 are satisfied, and then there exist two objective PSCNE.

***Proof*:** Substituting (24) into (12) and aggregating them to the market outputs, we obtain the dynamics of partial equilibria given c_2,

$$c_{1h}^{s+1}(c_1^s;\ c_2)=\frac{\beta_l \underline{c}}{\beta}+\frac{A+\sqrt{A(A-8\beta_l \underline{c})}}{4\beta}\ , \tag{25}$$

$$c_{1l}^{s+1}(c_1^s;\ c_2)=\frac{\beta_l \underline{c}}{\beta}+\frac{A-\sqrt{A(A-8\beta_l \underline{c})}}{4\beta}\ , \tag{26}$$

where $A=\alpha\beta_c I_2(c_1^s;\ c_2)$. Since we can check that $\partial c_{1h}^{s+1}/\partial c_1^s<0$, $0<\partial c_{1l}^{s+1}/\partial c_1^s<1$, each function has at most one intersection with the 45 degree line. *Q.E.D.*

The term objective means that it takes into account the indirect effects of c_1 on r_2 objectively, while firms don't take it into account subjectively.

It seems that the lower objective partial equilibrium is stable as in Figure 4. However, it does not mean the Cournot-Nash equilibrium is stable. If a lower partial equilibrium c_{1l}^{s+1} in Figure 4 has been realized, then the total outputs of sector 1 decrease in the next moment. At the same time, the reaction curve of each firm in sector 1 shifts upwards, since the firms do not take it into account subjectively. Hence the point becomes unstable because of its instability of the lower partial equilibrium. Therefore, the higher partial equilibrium is realized (see Figure 3 and 4). In fact, the solutions of (26) at $c_{1l}^{s+1}=c_1^s=c_2$ and $c_{1h}^{s+1}=c_1^s=c_2$ coincide with the lower and higher GSCNE in (19), respectively. Hence there is an external shock at lower GSCNE, the equilibrium deviates from it.

Thus, the following proposition is established.

***Proposition* 4.** The Pareto-inferior general symmetric Cournot-Nash equilibrium (GSCNE) is unstable in the sense of Cournot-adjustment process.

B. Numerical Example

We have discussed the instability of lower PSCNE and GSCNE in the former section. Although it seems clear that both the higher OPSCNE and GSCNE are stable, checking them is highly complex and not tractable. Since the purpose of this paper is to show the condition of Heller's coordination failure is not sufficient. For this purpose, to show a numerical example is an effective way. Hence, we check the local stability of the Heller's example; *i.e.*, $\alpha=20$, $\beta_c=1$, $\beta_l=2$, $\bar{L}=1$, and $\underline{c}=1$. There are two GSCNE, one with $c_h=4.20475$ and the other with $c_l=1.39525$.

In this case, from (25), $\partial c_{1h}^{s+1}(c_h; c_h)/\partial c_1^s=-0.427191$ at $c_1^s=c_2=4.20475$. Hence the GSCNE at $c_2=c_h$ is locally stable when the output of own sector deviates from the equilibrium. Next, from (17), the higher solution function of c_2 becomes

$$c_{1h}(c_2)=\frac{1}{24}\left(80-3c_2+\sqrt{9c_2(c_2-32)+2176}\right). \tag{27}$$

Taking the derivative of (27) with respect to c_2 and evaluate at $c_2=c_h$, we have $\partial c_{1h}(c_h)/\partial c_2=-0.256925$. Therefore, it is seen that the GSCNE is locally stable when the output of the other sector deviates from the equilibrium.

IV. Conclusion

It has shown that the lower general symmetric Cournot-Nash equilibrium is not stable in Heller's multisector Cournot-Nash model. Heller (1986) says that "An external shock pushed the economy toward the low-level state, and adjustment dynamics then placed the economy at E_2 (which corresponds to the lower GSCNE in this paper)." This statement is not true if we consider the standard Cournot adjustment process.

As in Cooper (1999), common knowledge of the multi-sector Cournot-Nash model is the following. Although there is the strategic substitute in each sector, the effect of complementarity of the inter-sector relations is high. This is the main reason of multiple equilibria. However, it is shown that the relations between intra- and inter-sectors are completely different in the case of Stone-Geary type preferences.

That is, there exhibits the strategic complements in each sector, and exhibits the relations of substitutability across the sector if taking into account the stability properties. Hence the source of multiple equilibria stems from the strategic complementarity in each sector. The reason of the relation of the substitution across sector seems the negative correlation of profit and output in the oligopoly. The expansion of market 1 leads to the decreasing of profit of market 1 which in turn reduces the income of market 2, hence the output of market 2 decreased.

Cooper (1999) points out that the presence of imperfect competition creates the possibility of multiple, Pareto-ranked equilibria and thus coordination failures. Cooper (1994) introduces a choice of technologies in a version of this multisector economy. Blanchard and Kiyotaki (1987) introduce the concept of menu costs in the monopolistic competition. In our best knowledge, there are no coordination failure models that stem from the property of demand side except for Heller (1986). Hence, it would be worthwhile to reconsider other sources of coordination failure from demand side like the network externality effects. This is our future research.

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