Post-Malthusian Population Model of the British Industrial Revolution in a Lewis' Unlimited Supply of Labor Model

Shin-Haing Kim and Donghyu Yang *

To explain the "post-Malthusian" population equilibrium of the British Industrial Revolution, this study proposes a two-sector economy that produces both agricultural foods and manufacturing goods. A Lewis model is used to discuss the interrelations among population increase, capital accumulation, and structural change of the British economy in the 19th century. We place capital accumulation at the center of the model to explain the growth in population size. Structural changes in the economy that are induced by capital accumulation favor the employment of labor in the manufacturing sector and trigger population increase. We examine this hypothesis by applying Granger causality tests to such variables as population size, capital accumulation, trade volume, and structural change in the British economy during the Industrial Revolution.

Keywords: Britain's Industrial Revolution, Lewis's model, Capital accumulation, Population increase

JEL Classification: J11, N10

I. Introduction

The growth of population size is essential to the transition from a "post-Malthusian regime" to a "modern economy regime." The post-Malthusian implies the stage after the escape from the Malthusian trap. This argument is rooted in the Boserupian viewpoint (1981) on the positive effects

* Professor Emeritus of Economics, Department of Economics, Seoul National University, Seoul 151-742, Korea, (E-mail) shk@plaza.snu.ac.kr; Professor of Economics, Department of Economics, Seoul National University, Seoul 151-742, Korea, (E-mail) dyang@snu.ac.kr, respectively. We are grateful to two anonymous referees for helpful suggestions.

of population size on technical progress. Large-scale economies, which result from the increased demand caused by the increase in population size, generate technical progress. The unified growth model of Galor and Weil (1980) demonstrates that large-scale economies lead to economic conditions in which investments on human capital are favored. However, a gap remains between a "post-Malthusian population economy" and a "modern economy" with respect to determining population size. How the increase in population is achieved remains unclear, and how such an increase provides economic environments that are conducive for human capital investments is poorly understood.

This study employs capital accumulation to determine the "post-Malthusian population equilibrium," which can fill the gap in the research on the population equilibrium between the "Malthusian stationary state" and the "Modern neo-classical population equilibrium" of family size choice. The function of capital goods in the context of the British Industrial Revolution as a growth model is rarely discussed. The function of capital goods in the transitional growth path of a unified growth model remains uncertain. Voigtländer and Voth (2006) presented a growth model of the British Industrial Revolution. This model considered capital goods. Their study indicated that an Ashton "wave of gadgets" in the Industrial Revolution generated the growth of the British economy. The endogenous growth model of a "variety" kind did not determine the population size.

We employ the "Unlimited Supply of Labor model" of Lewis to discuss the function of capital accumulation during the British Industrial Revolution and relate it to the determination of population size. Yang and Kim (2013) suggested a population equilibrium for the Malthusian economy of a one-sector agricultural economy. Their study emphasized the improvement of the marginal physical productivity of the agricultural worker in escaping the Malthusian trap. The present work retains the biophysical wage rate hypothesis of Fogel. Thus, the post-Malthusian model implies that income significantly affects the choice of family size.

The three factors of economic production are land, labor, and capital. Land is specific to the agricultural sector, whereas labor and capital move between the manufacturing and agricultural sectors. Capital and labor are utilized in fixed proportions for agricultural produce but are interchangeable for the production of goods. Land is specific to agricultural produce.

An economy is Malthusian when the land factor can effectively constrain population increase. A huge amount of land available for agricultural produce provides a huge amount of food to support a huge population size. Similarly, an increase in population size can be caused by the per capita rise in land ratio. The transfer of an agricultural worker into the manufacturing sector raises the per capita land ratio. The marginal physical productivity of the agricultural worker rises, such that this worker can, in turn, support a large number of children. A division of labor between the manufacturing and agricultural sectors relates to the endowment of land and capital stock of the economy. This study focuses on how the total size of capital stock determines the division of labor between the two sectors.

Another factor associated with structural changes in the economy is the relative productivity between the manufacturing and agricultural sectors. A high relative productivity of the manufacturing sector to the agricultural sector causes a huge pulling effect of labor in the manufacturing sector from the agricultural sector. The migration cost of rural workers to urban cities, where the manufacturing sector is located, also relates to a division of labor between the two sectors. The high migration cost of rural workers to urban cities hinders structural changes in the economy and delays the increase in population size. Notably, international trade is a crucial factor influencing structural changes in the economy because it alters the relative prices of agricultural foods relative to those of manufacturing goods. In an open-economy with a comparative advantage in terms of manufacturing goods, we can predict that trade contracts the employment of the agricultural sector.

Section II describes the division of labor between the agricultural and manufacturing sectors in 19th century Britain. This section presents the interrelations among changes in capital stock, total trade volume, and population size in relation to the distribution of employment in the two sectors. Section III presents the model used in this study. Section IV presents the Granger causality test results among the variables of the model. The causality test determines the effects of structural changes in the economy on the population size. Section V reviews the British experience of economic growth and population change during the Industrial Revolution with respect to the increase in capital stock and structural changes in the British economy. Section VI concludes the paper.

II. Distribution of Labor and Capital Stock between the Agricultural and Manufacturing Sectors in the British Economy from 1801 to 1891

"Studies in Capital Formation in the United Kingdom" of Feinstein and Pollard (1988) demonstrated the capital stock trend during the British Industrial Revolution. The study analyzed the allocation of capital stock among sectors. The British capital stock increased more than eight times during the 19^{th} century. The capital stock was £431 million in the beginning of the century and then increased to £3,412 million by the end of the century at 1900 constant prices.[1] Most of the fixed capital was derived from dwellings, railways, transport and communication, distribution and other services, public and social services, as well as gas, water, and electricity. All these factors took sixty-seven percentage points on average, and the remaining thirty-three percentage points were derived from both agriculture (*e.g.*, mining and quarrying) and manufacturing.

In this section, we categorize the fixed capital formation among the agricultural, mining, quarrying, and manufacturing sectors, as well as the remainder of the total fixed capital formation in 19^{th} century Britain. We apply the distributive share of labor data by Deane and Cole (1962) on the capital stock data of Feinstein to yield the per capita capital available for the manufacturing sector. The capital stock of the manufacturing sector should consist of mining and quarrying to be comparable with the labor share of Dean and Cole.

Table 1 summarizes the per capita capital by sector and the population change during the British Industrial Revolution.

In column (1) of Table 1, K is the capital stock in million £ at 1900 constant prices of the sum of the agricultural and manufacturing sectors. K is sourced from the capital stock data of Feinstein (1988, Table XIII). The labor share of the manufacturing sector to the agricultural sector is presented as *magl* in column (2) *agk* in column (3) is the amount of capital stock in million £ at 1900 constant prices of the agricultural sector, whereas *nk* in column (4) is that of the manufacturing sector. We determine the net amount of capital available for the manufacturing sector, *nk*, by determining the per capita capital stock, which is the

[1] These data are obtained from Feinstein and Pollard's (1988) "Capital Formation" Table XIII in its Appendix. The value of the capital stock on the first half of the century is adjusted for the 1900 constant prices.

TABLE 1
AMOUNT OF CAPITAL AND PER CAPITA CAPITAL STOCK OF BRITAIN BY SECTOR FROM 1801 TO 1891

year	(1) K (£ million)	(2) *magl* (l_M/l_A)	(3) *agk* (£ million)	(4) *nk* (£)	(5) *pop* (million)
1801	168.95	0.82	135.17	209.97	8.66
1831	261.73	1.67	172.55	315.24	13.28
1861	489	2.35	229	599.64	18.94
1891	706	4.06	271	813.08	27.59

Notes: $nk = (1/l_M)K - (l_A/l_M)(agk);\ l_A + l_M \equiv 1$.

agk: amount of capital goods employed for the agricultural sector in million £.

l_A: share of labor for the agricultural sector; $l_A \equiv \dfrac{L_A}{L_A + L_M}$.

l_M: share of labor for the manufacturing sector; $l_M \equiv \dfrac{L_M}{L_A + L_M}$.

L_A: amount of labor employed for the agricultural sector.
L_M: amount of labor employed for the manufacturing sector.

weighted average of the per capita capital of the two sectors. Weight is the employment share of the two sectors. A large amount of *nk* results in a large amount of K for a given amount of *agk* and a given share of employment for the two sectors, (l_A/l_M). A decrease in employment in the agricultural sector attributed to an increase in productivity results in an increase in *nk*. Column (5) presents the population size of British *pop* (in million) from Table A9.2 of Wrigley and Schofields (1993).

Figure 1 shows the major macro-indicators of Table 1. The capital stock, which is represented by K and comprises the capital stock of the agricultural and manufacturing sectors, increased from £169 million to £706 million. Concurrent with the increase in capital stock are the structural changes in the British economy. The ratio of the employment share of labor of the manufacturing sector to that of the agricultural sector, *magl*, is presented in column (2) of Table 1.[2] The ratio increased more than four times. The amount of labor employed for the manufacturing sector, represented by L_M, increased almost five times, from 1.40

[2] This ratio was obtained from Deane and Cole's Table 30 (1962). We note that 1814 is the year in which the employment share of labor of the agricultural sector was equal to that of the manufacturing sector.

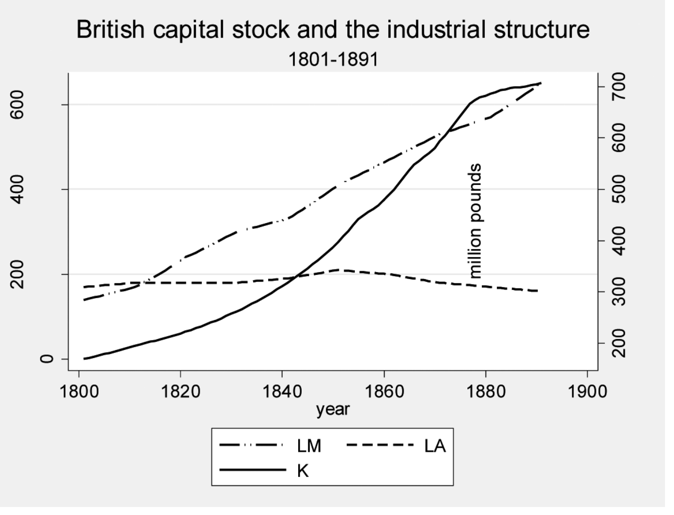

FIGURE 1

Notes: L_A: amount of labor in ten thousands employed for the agricultural sector.
L_M: amount of labor in ten thousands employed for the manufacturing sector, as sourced from Deane and Cole (1962, p. 143, Table 31); annual data are obtained from geometric interpolation.
K: amount of capital stock (*e.g.*, agricultural, manufacturing, mining, and quarrying) in £million, as sourced from the capital stock data of Feinstein (1988, Table XIII).

million in 1801 to 6.50 million in 1891. By contrast, a decrease in employment size is observed for the agricultural sector, represented by L_A. In 1801, employment size for the agricultural sector was 1.7 million. The value reached its peak of 2.1 million in the middle of the century, thereafter decreasing gradually to 1.6 million by 1891. The size of the British population more than tripled during this period; the value was approximately 8.7 million in 1801 and increased to 27.6 million by 1891.

The macroeconomic description (Table 1) and graph (Figure 1) of the British economy during the Industrial Revolution demonstrate the inter-relations among structural change, capital accumulation, and population growth. We take structural change as the focal point for explaining population change, as inspired by the "unlimited labor supply hypothesis" of Lewis (1954). The thesis emphasized capital accumulation of the manufacturing sector for economic development with the agricultural sector as the source of labor.

We discuss the interrelations among population, capital accumulation, and structural change in the context of a post-Malthusian economy. A post-Malthusian economy in this study is characterized as follows:

(1) Land is a factor of production that influences the wage rate of a family unit and the determination of the size of the family unit.
(2) The marginal productivity of manufacturing workers is greater than the wage. Hence, the demand for labor from the manufacturing sector is infinitely elastic at wage rate.
(3) Income affects the wage rate and is inclusive of the decline in death rate.
(4) The number of agricultural workers is greater than that of manufacturing workers.

The mechanism by which population increase influences structural changes in the economy is as follows: The increased amount of labor attributed to population increase causes employment in the manufacturing sector to meet the infinitely elastic demand for labor at a given subsistence wage rate in the Lewis model.[3] By condition (2) of the post-Malthusian economy, profits increase at the manufacturing sector, which accrue to the owner of the capital good as rentals. Reinvestments of rentals from capital goods in the manufacturing sector draw agricultural workers to the manufacturing sector. The reduction of the number of agricultural farmers increases marginal productivity by condition (1). The number of a family unit increases by condition (3), which results in an increase in population size. Consequently, a feedback effect of the population increase occurs. Section III demonstrates this self-generating population equilibrium.

Section III presents a model to capture the stylized facts between the population and the change in the share of labor between the agricultural and manufacturing sectors associated with capital accumulation, as shown in Table 1.

III. The Model

We assume that using a farm machine or vehicle for agricultural pro-

[3] Instead of the surplus from the agricultural sector, the increase of the population is an alternative source for supplying labor to the manufacturing sector.

duction is complementary to farm labor. A fixed bundle of a unit of farm capital and of farm labor is applied to a given plot of land for agricultural production. Given that this unit is a fixed bundle, the number of farm workers, denoted by L_A, also measures the amount of agricultural capital stock.

A productive farm capital results in efficient farm labor because of the increase in efficiency. For example, soil cultivation is more efficient when using a tractor than a horse or a cow-harness cart. The efficiency level of agricultural capital is denoted by $1<B<\infty$. Hence, BL_A represents a measure of agricultural capital goods in efficiency units. A decrease in agricultural capital goods from the total amount of capital stock, denoted by K, leaves the amount of capital stock available for the manufacturing sector in our two-sector economy.

Equation (1) is the production function for agricultural foods in Cobb-Douglas form:

$$\begin{aligned} Y_A(t) &= T^{\alpha} K_A(t)^{1-\alpha} \\ &= T^{\alpha} \left(BL_A(t)\right)^{1-\alpha} . \end{aligned} \tag{1}$$

where T and $K_A(t)$ are the input of land and capital for agricultural produce, respectively, for time period t. The amount of agricultural workers substitutes for agricultural capital $K_A(t)$ in the second line of Equation (1). The exponent $0<\alpha<1$ on factor input T of land represents a distributive share on the latter.

We then consider a production function for the manufacturing sector. Unlike the case of production for agricultural produce, capital and labor are interchangeable in the production of manufacturing goods. We also express the production function for manufacturing goods in Cobb-Douglas form:[4]

$$Y_M(t) = \varphi K_M(t)^{\alpha} L_M(t)^{1-\alpha}, \tag{2}$$

where $0<\alpha<1$ is a distributive parameter for the capital good owner. The parameter φ represents the technological efficiency level of manufacturing goods relative to that of agricultural produce. The variables

[4] To avoid complexity in notation, we simply use the same distributive parameter α for capital and land for agricultural produce. A variation of this parameter between the two sectors does not change the substance of the model.

$L_M(t)$ and $K_M(t)$ are the input of labor and capital for manufacturing good production, respectively, for time period t. The movement and adjustment costs are added to the subsistence wage rate of the agricultural worker, which is a fraction of the wage rate in terms of manufacturing goods.

The wage rate of the agricultural worker for the given price of agricultural produce p_A is:

$$w_A(t) = p_A(1-\alpha)B^{\alpha-1}\left(\frac{T}{L_A(t)}\right)^{\alpha},$$

Meanwhile, the wage rate of the manufacturing worker for the given price of manufacturing good p_M is:

$$w_M(t) = p_M\xi\varphi(1-\alpha)\left(\frac{K_M(t)}{L_M(t)}\right)^{\alpha},$$

where $0<(1-\xi)<1$ represents the mobility cost of an agricultural worker moving from a farm to an urban manufacturing company, which is a fraction of the wage rate in terms of manufacturing goods. We obtain the wage rate of the manufacturing worker by deducting the mobility cost from the marginal productivity of the worker.

Equality of wage rates across the two sectors provides equation (3) of the price of manufacture goods $p_M(t)$ in terms of agricultural foods prices $p_A(t)$ at time period t as denoted by $p(t)$:

$$p(t) \equiv \frac{p_M(t)}{p_A(t)} = \frac{1}{\xi\varphi}\left[\frac{B^{(1-\alpha)/\alpha}T}{K_M(t)\left(L_A(t)/L_M(t)\right)}\right]^{\alpha}$$
$$;\, K_M(t) \equiv \left(K(t)-K_A(t)\right) \equiv \left(K(t)-L_A(t)\right) > 0. \quad (3)$$

We assume that $(K(t)-L_A(t))$ is positive, suggesting that a certain amount of capital stock is available for investments in the manufacturing sector after equipping the agricultural workers. This amount is the net amount of capital stock available for the manufacturing sector *nk* in column (4) of Table 1. The per capita capital stock available for the manufacturing sector in 1801 was approximately £210, which increased to £813.

The wage-parity condition of (3) yields a distribution of labor between the agricultural and manufacturing sectors, $L_A(t)/L_M(t)$, for a given relative price, $p(t)$, for a given amount of capital stock, $K(t)$, and for that of the amount of labor available for time period t, $L(t)$.

The wage-parity condition of (3) suggests that a higher share of labor for the manufacturing sector relative to that of the agricultural sector yields high capital stock. Thus, a high relative employment share in the manufacturing sector facilitates the easy movement of agricultural workers to the manufacturing sector, and the technological level of the manufacturing sector is higher than that of the agricultural sector. The functions of ξ and φ relative to the wage-parity condition of (3) will be discussed according to the international volume of trade.[5]

We then consider the capital funds market. We consider the hypothesis that the reinvestment of profits in the manufacturing sector mostly involves the financing of investments (Neal 1994). In this context, we are in line with Allen (2009) in utilizing the capital market for the growth and income distribution model of the British Industrial Revolution. However, we extend his one-sector model to a two-sector model, retaining the "unlimited labor supply" in the hypothesis of Lewis. We assume that wages and rents are consumed.[6] For a simple model, we employ a classical savings assumption.

Capital fund is malleable. We assume that such fund can be transformed into capital goods. A capital owner consumes a constant fraction $0<1-s<1$ of his rental income so that savings are in $srK_M(t)$ and are re-invested. Hence, the capital goods market equilibrium for a given rate of rentals $\bar{r}$ is expressed as:

$$K(t) = K(t-1) + s\bar{r}K_M(t) = K_A(t) + K_M(t).$$

A manufacturing firm employs labor at wage rate $\bar{w}$. A rental rate for utilizing capital good is given at rate $\bar{r}$. The profits of a representative manufacturing firm at time period t is defined as:

$$\pi(t) = p_M(t)Y_M(t) - \bar{r}K_M(t) - \bar{w}(t)L_M(t),$$

[5] The relative productivity of the two sectors is reflective of international trade on the basis of a Ricardian comparative advantage doctrine.

[6] The land-owner's investment for agriculture sector infrastructure, such as irrigation, and for the reformation of the arable land is not included in our discussion of capital stock.

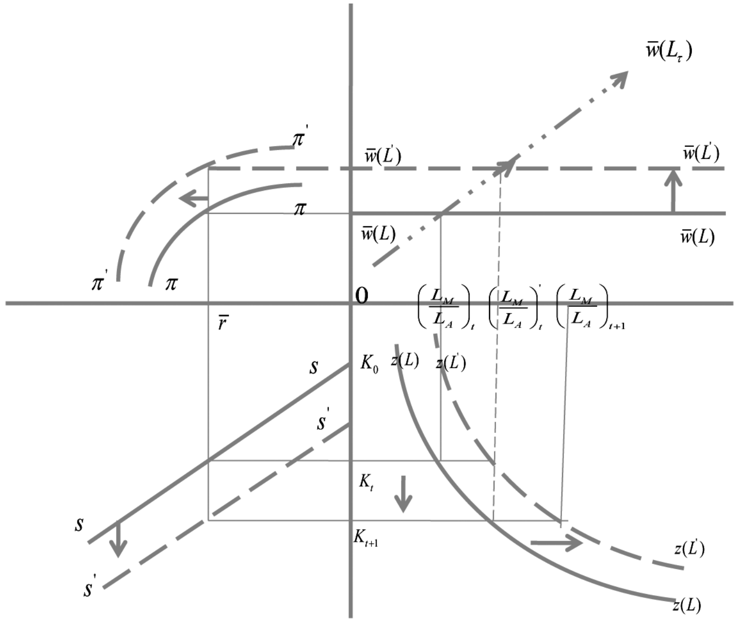

FIGURE 2
POPULATION INCREASE AND CAPITAL ACCUMULATION

where $\bar{w}(t)$ is the wage rate of a family unit, required to support the number of children at time period t, which should increase when family size increases. Thus, wage increases with population size. The total amount of labor supply is obtained by dividing the population size by the average number of children per unit in a family.

This hypothesis is consistent with the trend of wage rate and population size in the British economy in the 19^{th} century. Wrigley-Scholfield (1993) data demonstrated that the real wage index of Britain increased 2.8 times, from 497 in 1801 to 1,402 by the beginning of the 20^{th} century. This increase is no more than the increase in the population size of 8.66 million at the beginning of the 19^{th} century to 31.09 million by the 20^{th} century for a millennium, which far exceeds the threefold increase of the 19^{th} century. This trend is consistent with the post-Malthusian economy.

Figure 2 illustrates the endogenous determination of population size in relation to capital accumulation in the manufacturing sector.

The horizontal line of the north-east (NE) quadrant in Figure 2 indicates the relative share of manufacturing labor to agricultural labor, L_M/L_A. The vertical line is the wage rate of the family unit $\bar{w}$. The population size at given time period t, L_t, is supported at the wage rate. The marginal productivity of a manufacturing worker is greater than the family-supporting wage rate in our post-Malthusian economy. Hence, the relative demand of a manufacturing firm for labor is perfectly elastic at the wage rate of family unit $\bar{w}$., which is represented by $\bar{w}\bar{w}$. The horizontal line of the north-west (NW) quadrant is the rental rate r, on the capital goods employed for the manufacturing firm. The downward sloping curve $\pi\pi$ concave to the origin on the quadrant represents a factor-price frontier for a given price of the manufacturing goods relative to that of the agricultural produce. The profit for a manufacturing firm along the curve is zero.

Multiplying the rental on the manufacturing capital goods, rK_M, by the propensity to save, s, yields savings to the economy. The vertical line below zero represents the amount of capital stock. The upward-sloping curve ss on the south-west (SW) quadrant displays the amount of savings supplied to the economy. Adding the savings, which are re-invested in the manufacturing sector, to the previously given amount of capital stock of the economy, K_t, yields the capital stock of the next period, K_{t+1}.

Finally, the upward-sloping curve on the south-east (SE) quadrant, denoted as curve zz, represents the wage-parity condition (3) between the two sectors. A positive relation is observed between the amount of capital stock and the relative employment of manufacturing labor to that of agricultural labor.

We discuss the equilibrium in this economy. We assume that a given amount of labor, L_t, which is supported by the wage rate, $\bar{w}$, is allocated between the manufacturing and agricultural sectors. The relative price of the manufacturing good, p, is determined on the zero-profit curve $\pi\pi$, which passes through the cross-over point of $\bar{w}$ and $\bar{r}$.[7]

The wage-parity condition (3) on the SE quadrant yields the amount of capital stock, K_t, which is consistent with the relative share of labor employed between the two sectors. Finally, we obtain the amount of capital employed for the manufacturing sector, K_M, from the savings schedule of the ss curve on the SW quadrant. Thus, the equilibrium of the population size, L_t, is determined.

Result 1 summarizes the discussion on the equilibrium share of

[7] We assume that the profits of a manufacturing firm accrue to the owners of the capital goods in the form of rentals.

manufacturing labor to agricultural labor.

***Result* 1:** In our post-Malthusian economy, an equilibrium distribution of labor exists between the agricultural and manufacturing sectors at the given relative price of the manufacturing good, p rate of interest, $\bar{r}$, and wage rate $\bar{w}$ for a given amount of labor, $L(0)$, and for capital stock $K(0) > \hat{K}$, such that $\hat{K} = K_A$.

We discuss the effects of the increase in labor supply on capital accumulation and on the labor distribution between the manufacturing and agricultural sectors.

The increase in wage rate from $\bar{w}$ to $\bar{w}'$ and the population increase from L to L' has two effects. The first effect is on the demand side. The relative price of the manufacturing good increases because of the high income elasticity of such good, which shifts the zero-profit curve upward from schedule $\pi\pi$ to schedule $\pi'\pi'$ on the NW quadrant. The second effect is on the supply side. The increased amount of labor could work either for the agricultural or manufacturing sectors. The wage-parity condition (3) suggests that the manufacturing sector is favored by the increased amount of labor provided that the employment share of the agricultural sector is greater than that of the manufacturing sector.[8] Consequently, the wage-parity schedule zz on the SE quadrant shift to the right of schedule $z'z'$. Condition (4) of our post-Malthusian economy satisfies this condition. Column (2) in Table 1 indicates that the share of the agricultural sector is 20% greater than that of the manufacturing sector in the beginning of the 19^{th} century. The increase in the labor supply absorbed by the manufacturing sector, by condition (2) of our post-Malthusian economy, increases the employment share of the manufacturing sector from (L_M/L_A) to $(L_M/L_A)'$.

The profits of a manufacturing firm increases when the employment share of the manufacturing sector increases. Investments in the manufacturing sector increase the capital for the manufacturing sector K_M at a given rental-wage ratio, which shifts the savings schedule ss on the SW quadrant downward to the savings schedule $s's'$. The supply of capital stock increases from K to K'. Finally, the increase in capital stock increases the relative share of manufacturing workers from $(L_M/L_A)'$ to $(L_M/L_A)_{t+1}$ on the wage-parity schedule $z'z'$ of the SE quadrant. Agricultural workers are drawn toward the manufacturing sector, and the mar-

[8] We can verify this observation in the parity condition of (3).

ginal productivity of the agricultural worker increases. The population increases by condition (3) in our post-Malthusian economy. In conclusion, a feedback effect of the population increase occurs.

The amount of capital stock is crucial in discussing self-generating population equilibrium. An economy endowed with the amount of capital stock at the initial period above critical level $\hat{K}$ is bound to experience a feedback of population increase. Population increase leads to the population increase of the next generation through capital stock accumulation. Population increases concurrently with family supporting wage rate $\bar{w}$. Line $\bar{w}(L_r)$ on the NE quadrant indicates the population size of period τ, which increases concurrently with the family supporting wage rate $\bar{w}$.

However, the endogenous increase in population size is determined when a negative substitution effect of the quality-choice of a child instead of the number of children comes into effect with respect to the increase in wage rate. Considering the critical wage rate $\hat{w}$ at which the negative substitution effect matches the positive income effect, the self-generating effect of the population concludes, and a population equilibrium is reached.

***Result* 2:** In our post-Malthusian economy, a population equilibrium exists at critical wage rate $\hat{w}$.

Structural change is influenced by the level of relative productivity of the manufacturing sector to that of the agricultural sector and by the opening of international trade instead of capital accumulation. These factors could also determine the population equilibrium.

For example, the opening of trade increased the profits of the manufacturing sector, yielded a high capital accumulation, and reinforced structural change in favor of the manufacturing sector in the British economy during the Industrial Revolution.[9] The increase in relative productivity in the manufacturing sector relative to that of the agricultural sector not only reinforced the movement of the labor force from the latter to the former, but also contributed to the increase in trade volume.

[9] In terms of Figure 2 the curve $\pi\pi$ in the NW quadrant shifts upward by the opening of trade, whereas the savings curve ss in the SW quadrant shifts downward, resulting in the accumulation of capital goods. A draw of the agricultural workers to the manufacturing sector contributes to the increase in the marginal productivity of agricultural workers. As a result, the opening of trade in our model contributes to the increase in population.

IV. Granger Causality Wald Test Results in Vector Auto Regressions (VAR)

VAR is performed on four variables from 1801 to 1891, listed on Appendix Table A1. The four variables are the natural log of population (*lnpop*), ratio of the share of the manufacturing labor to that of the agricultural labor *(magl)*,[10] natural log of capital stock *(lnk)*, and natural log of the total trade volume (*lntv*).[11] The trade volume reflects the relative technological level of the manufacturing sector to that of the agricultural sector.

Table 2 presents the Granger causality test results on the structural changes in the British economy during the 19^{th} century.[12] The results on causality vary with the time lags allowed. Table 2 reports the results based on time lags of 3, 5, and 11.

The first row reports the test results for time lag 3. Both *magl* and *lntv* are Granger-caused by the other remaining variables. The Granger-causality for *magl* by *lnpop* is significant at a 5% level. The other remaining variables (*i.e.*, *lnk* and *lntv*) cause *magl* at a significance level of 1%. *lntv* is Granger-caused by *lnk* at a significance level of 1%. Both *lnpop* and *magl* are Granger-caused by *lntv* at a 5% significance level. However, no significant Granger causalities are observed for *lnpop* and *lnk*.

The second row extends to the time lag 5 and shows that *lnk*, *magl*, and *lntv* are Granger-caused by the remaining variables. *lnpop* is Granger-caused by the remaining three variables as the time lag extends to 11 on the third row.

The causality effects took more time for the population increase and capital accumulation than for the structural and trade volume changes.

V. Implication of Population Increase for the Growth of the British Economy during the 19^{th} Century

The function of structural change in the British Industrial Revolution is rarely explored, except by Crafts (1985). We address this issue by employing the endogenous determination of the population equilibrium of a post-Malthusian economy. Given that our focal point is on aug-

[10] Here, $magl \equiv L_M/L_A$.
[11] Natural logs of the variables are prefixed by ln on them.
[12] We use a software-package, STATA11, for running the VAR.

TABLE 2
GRANGER CAUSALITY WALD TESTS

equation		excluded	χ^2	d.f.	Prob > χ^2
lag3	magl	lnpop	8.305	3	0.040
		lnk	10.718	3	0.013
		lntv	17.703	3	0.001
		all	32,213	9	0.000
	lntv	lnpop	10.168	3	0.017
		lnk	22.057	3	0.000
		all	41.36	9	0.000
lag5	lnk	magl	10.963	5	0.052
		lntv	16.26	5	0.006
		all	33.512	15	0.004
	magl	lntv	15.4	5	0.009
		all	35.669	15	0.002
	lntv	lnpop	17.351	5	0.004
		lnk	26.611	5	0.000
		all	57.406	15	0.000
lag11	lnpop	lnk	17.437	11	0.096
		magl	81.244	11	0.000
		lntv	59.184	11	0.000
		all	213	33	0.000
	lnk	lnpop	1550.7	7	0.000
		magl	39.431	11	0.000
		lntv	31.925	11	0.001
		all	57385	29	0.000
	magl	lnpop	3618.3	7	0.000
		lnk	68.054	11	0.000
		lntv	65.629	11	0.000
		all	2.4e+5	29	0.000
	lntv	lnpop	1215.4	7	0.000
		lnk	66	11	0.000
		magl	58.791	11	0.000
		all	4607.5	29	0.000

menting the population associated with capital accumulation during the British Industrial Revolution, the present model can be categorized as a factor-driven growth model. The estimates of Crafts on the contribution of production factors during the Industrial Revolution (1760 to 1831) indicates that approximately 70% to 80% of growth is attributed to the inputs of capital and labor (Crafts 1981, Table 3.4). The present model conflicts with the estimates of Crafts. The contribution of technical progress for growth, measured by the residuals, only takes the share of the remaining 20%.

However, our argument does not undermine the importance of technical progress for growth. The ultimate cause of growth in an economy is technical progress or productivity improvement, which relieves the diminishing returns effect that rises from factor-driven growth. Therefore, direct or indirect effects are associated with factor-driven growth.

Population increase results in large-scale economies, which yields growth in an economy as in the argument of Boserup (1982) on population. Tsoulouhas (1992) proved the correlation of the number of print-patents with population size during the British Industrial Revolution, which supports the hypothesis of Boserup. Besides the mere size of population per se, we examine the growth effect of population from the aspect of supply, which varies with the way in which population growth is brought about. On the basis of a Lewis model, we determine that population increase is caused by capital accumulation. Capital accumulation transitions from manufacturing on the cottage level to that on the factory level. The factory system brings technical progress.

"Learning effects" as presented by Usher (1920) under the mills factory system illustrate the increase in productivity. Although the share of fixed capital investments for the manufacturing sector is not as high as its secondary effects (Field 1985), forthcoming growth is anticipated for the factory system (Berg 1994). Thus, a momentum in the accumulation of knowledge is generated when population increases.

However, the increase in population size associated with capital accumulation in the Lewis model carries "micro-inventions" of Mokyr accumulated over generations. Population increase implies an increase in apprenticeship, craftsmanship, and accumulated knowledge from past cohorts and passed over to forthcoming ones. Capital goods are the vehicles that transmit this accumulated knowledge over to the next generations and provide "learning by doing" effects. The accumulated knowledge embodied on capital goods allows the next generation to invest in human capital, which would sustain the growth of a modern economy. A factor-driven growth model of the present model in this respect is consistent with a Mokyr's statement: Regardless of its source, the Industrial Revolution was above all an age of rapidly changing production technology propelled by technological creativity (Mokyr 2009).

Consequently, population increase associated with capital accumulation provides an economic environment favorable for technical progress and human capital investment. The increase in the amount of labor employed for the manufacturing sector instead of the agricultural sector results in a shift in the economic structure. The function of land input for production decreases, whereas the function of capital input increases. This observation is consistent with the hypothesis of Hansen and Prescott (2002) in the transition of technology from *Malthus technology to Solow technology* during the transitional growth path of the economy.

VI. Conclusion

Economic historians view the technologically innovative economic environments of Britain as one of the causes of the Industrial Revolution. In the rapidly changing British economy during the Industrial Revolution, interactions among population change, capital accumulation, economic structure, and international trade associated with technical changes are observed. Among these economic factors, determining which ones are primary, secondary, and tertiary or a derivative remains controversial. A two-sector model in the Lewis version abstracts itself from the complexities in the interactions among variables. A change in the economic associated with capital accumulation can explain the population size of the economy.

The Granger causality tests among these variables during the British Industrial Revolution are divided into three periods: (1) three years, (2) five years, and (3) 11 years.

A change in the economic structure is Granger-caused by population size, total trade volume, and total capital stock. In the five-year period, capital stock is Granger-caused by the total trade volume and by the rest of the variables. A Granger causality test on the population size occurs only after a lapse of eleven years. The test is crucial to structural change and total trade volume, which can be explained by population change taking more time than the rest of the variables.

Although technical change is presumed to propel the revolution, population increase drives economic growth in a post-Malthusian model. Productivity growth can effectively contribute to population growth. Our two-sector model demonstrates that capital accumulation shifts population upward. Population grows endogenously on this track of capital accumulation. Another important institutional factor is international trade, associated with a relative high technological level of the manufacturing sector to that of the agricultural sector, which contributes to population growth and the growth of the British economy by diversifying labor toward the manufacturing sector.

(Received 27 March 2014; Revised 13 October 2014; Accepted 14 October 2014)

Appendix

Table A1

British population (*pop*), the structure of the economy in terms of the employment (*magl*), net capital available for the manufacturing (*nk*), and the total volume of trade (*tv*) (1801-1891)

year	*pop*	*magl*	*nk*	ln(*pop*)	ln(*nk*)	ln(*tv*)
1801	866.449	0.823529	209.9692	6.764403	5.346961	10.41577
1802	878.1882	0.834889	212.9569	6.777861	5.36109	10.43877
1803	890.0864	0.846406	215.9969	6.791319	5.375264	10.46177
1804	902.1458	0.858081	219.0905	6.804776	5.389485	10.48477
1805	914.3686	0.869917	222.2389	6.818234	5.403753	10.50777
1806	926.757	0.881917	225.4433	6.831691	5.418069	10.53077
1807	938.8022	0.894082	228.7051	6.844605	5.432433	10.55377
1808	951.0039	0.906415	232.0254	6.857518	5.446847	10.57677
1809	963.3643	0.918918	235.4057	6.870432	5.46131	10.59977
1810	975.8853	0.931594	238.8473	6.883345	5.475825	10.62277
1811	988.569	0.944444	241.8042	6.896258	5.488128	10.64577
1812	1003.434	0.977581	243.8281	6.911183	5.496463	10.66877
1813	1018.522	1.01188	245.8973	6.926108	5.504914	10.69177
1814	1033.838	1.047382	248.0125	6.941033	5.513479	10.71477
1815	1049.383	1.08413	250.1745	6.955958	5.522159	10.73777
1816	1065.163	1.122167	252.3841	6.970883	5.530952	10.76077
1817	1081.461	1.161539	254.6418	6.986068	5.539858	10.78377
1818	1098.008	1.202292	256.9487	7.001253	5.548876	10.80677
1819	1114.809	1.244475	259.3053	7.016438	5.558006	10.82977
1820	1131.866	1.288138	261.7126	7.031623	5.567247	10.85277
1821	1149.185	1.333333	265.3593	7.046808	5.581085	10.87577
1822	1167.007	1.36342	269.6597	7.062197	5.597161	10.89877
1823	1185.104	1.394186	274.1027	7.077586	5.613503	10.92177
1824	1203.483	1.425646	278.6941	7.092975	5.630115	10.94477
1825	1222.147	1.457816	283.4399	7.108364	5.647	10.96777
1826	1241.1	1.490712	288.3466	7.123753	5.664163	10.99077
1827	1258.086	1.52435	293.4207	7.137347	5.681608	11.01377
1828	1275.305	1.558747	298.6693	7.15094	5.699337	11.03677

year	*pop*	*magl*	*nk*	ln(*pop*)	ln(*nk*)	ln(*tv*)
1829	1292.759	1.593921	304.0994	7.164534	5.717355	11.05977
1830	1310.453	1.629888	309.7186	7.178128	5.735664	11.08277
1831	1328.388	1.666667	315.2398	7.191722	5.753334	11.10577
1832	1344.438	1.673555	321.9283	7.203731	5.774329	11.13277
1833	1360.681	1.680471	328.8449	7.215741	5.795586	11.15977
1834	1377.12	1.687416	335.9987	7.22775	5.817107	11.18677
1835	1393.759	1.69439	343.3987	7.239759	5.838892	11.21377
1836	1410.598	1.701393	351.0546	7.251769	5.860942	11.24077
1837	1427.477	1.708424	358.9764	7.263664	5.883257	11.26777
1838	1444.558	1.715485	367.1744	7.275559	5.905837	11.29477
1839	1461.843	1.722575	375.6595	7.287454	5.928683	11.32177
1840	1479.336	1.729694	384.4429	7.299348	5.951795	11.34877
1841	1497.037	1.736842	393.3927	7.311243	5.974808	11.37577
1842	1515.828	1.757279	402.0135	7.323717	5.996486	11.40977
1843	1534.855	1.777957	410.8744	7.336191	6.018288	11.44377
1844	1554.121	1.798878	419.9834	7.348665	6.040215	11.47777
1845	1573.628	1.820045	429.3484	7.361139	6.062269	11.51177
1846	1593.38	1.841461	438.9777	7.373613	6.084449	11.54577
1847	1609.112	1.863129	448.8801	7.383438	6.106756	11.57977
1848	1624.999	1.885052	459.0645	7.393263	6.129191	11.61377
1849	1641.043	1.907233	469.54	7.403088	6.151754	11.64777
1850	1657.246	1.929675	480.3164	7.412912	6.174445	11.68177
1851	1673.608	1.952381	494.3171	7.422737	6.203177	11.71577
1852	1693.678	1.988909	507.5521	7.434658	6.229599	11.75977
1853	1713.988	2.02612	520.1657	7.446578	6.254147	11.80377
1854	1734.542	2.064028	535.1342	7.458499	6.282518	11.84777
1855	1755.342	2.102645	548.4837	7.470419	6.307158	11.89177
1856	1776.392	2.141984	557.7777	7.48234	6.32396	11.93577
1857	1799.268	2.182059	563.613	7.495135	6.334368	11.97977
1858	1822.438	2.222884	568.9678	7.50793	6.343824	12.02377
1859	1845.906	2.264473	576.1929	7.520726	6.356443	12.06777
1860	1869.677	2.30684	588.6738	7.533521	6.377872	12.11177
1861	1893.754	2.35	599.6383	7.546316	6.396327	12.15577
1862	1917.721	2.403596	610.0836	7.558893	6.413596	12.20077
1863	1941.992	2.458414	622.8608	7.571469	6.434323	12.24577
1864	1966.57	2.514482	636.9457	7.584046	6.456684	12.29077
1865	1991.458	2.571829	649.9267	7.596623	6.47686	12.33577

year	*pop*	*magl*	*nk*	ln(*pop*)	ln(*nk*)	ln(*tv*)
1866	2016.662	2.630484	661.4276	7.609199	6.494401	12.38077
1867	2042.665	2.690476	668.3628	7.622011	6.504831	12.42577
1868	2069.003	2.751837	675.2886	7.634822	6.51514	12.47077
1869	2095.681	2.814597	681.206	7.647634	6.523865	12.51577
1870	2122.702	2.878789	689.8105	7.660445	6.536417	12.56077
1871	2150.072	2.944444	705.0755	7.673257	6.558305	12.60577
1872	2179.958	2.982948	716.646	7.687061	6.574582	12.65177
1873	2210.259	3.021954	732.15	7.700865	6.595985	12.69777
1874	2240.982	3.061471	746.2438	7.714669	6.615052	12.74377
1875	2272.132	3.101504	761.5866	7.728474	6.635404	12.78977
1876	2303.714	3.142061	776.8493	7.742278	6.655246	12.83577
1877	2335.736	3.183149	791.0334	7.756082	6.67334	12.88177
1878	2368.203	3.224773	798.5902	7.769887	6.682848	12.92777
1879	2401.121	3.266942	803.1935	7.783691	6.688596	12.97377
1880	2434.496	3.309662	805.484	7.797495	6.691443	13.01977
1881	2468.336	3.352941	810.0702	7.811299	6.697121	13.06577
1882	2495.981	3.417926	811.2964	7.822437	6.698633	13.10677
1883	2523.936	3.484171	812.5452	7.833575	6.700172	13.14777
1884	2552.204	3.551699	812.5348	7.844713	6.700159	13.18877
1885	2580.789	3.620536	813.8335	7.85585	6.701756	13.22977
1886	2609.694	3.690708	812.8831	7.866988	6.700587	13.27077
1887	2638.922	3.762239	810.6988	7.878126	6.697897	13.31177
1888	2668.478	3.835157	811.0776	7.889264	6.698364	13.35277
1889	2698.365	3.909488	811.733	7.900401	6.699172	13.39377
1890	2728.587	3.98526	812.3995	7.911539	6.699992	13.43477
1891	2759.147	4.0625	813.0769	7.922677	6.700826	13.47577

Notes: *pop*: in ten thousands
(In)*pop*: natural log of pop
magl: the amount of labor for the manufacturing sector to that of labor for the agricultural sector
nk: the net amount of capital available for the use of manufacturing sector weight being the employment share of the two sectors
In(*nk*): natural log of *nk*
In(*tv*): natural log of the total volume of trade

References

Allen, Robert C. "Engel's pause: Technical Change, Capital Accumulation and Inequality in British Industrial Revolution," *Explorations in Economic History* 46 (No. 4 2009): 418-35.

Berg, Maxine. "Factories, Workshops and Industrial Organization." In Roderick Floud, and Donald N. McCloskey (eds.), *The Economic History of Britain Since 1700.* 2^{nd} ed. Cambridge: Cambridge University Press, Vol. 1, 1994.

Boserup, Ester. *Poulation and Technological Change.* Chicago: University of Chicago Press, 1982.

Crafts, Nicholas F. R. "The Industrial Revolution." In Floud Roderick and Donald McCloskey (eds.), *The Economic History of Britain since 1700*, Vol. 1, Cambridge: Cambridge University Press, 1981.

Crafts, Nicholas F. R. *British Economic Growth During the Industrial Revolution.* Oxford, New York, Toronto: Oxford University Press, 1985.

Deane, Phyllis, and Cole, W. A. *British Economic Growth, 1688-1959.* 2^{nd} ed. Cambridge: Cambridge University Press, 1962.

Feinstein, Charles H., and Sidney Pollard. *Studies in Capital Formation in the United Kingdom 1750-1920.* Oxford: Clarendon Press, 1988.

Field, A. J. "On the Unimportance of Machinery." *Explorations in Economic History* 22 (No. 4 1985): 402-416.

Galor, Oded, and David N. Weil. "Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond." *American Economic Review* 90 (No. 4 2000): 806-828.

Hansen, Gary D., and Edward C. Prescott. "Malthus to Solow." *American Economic Review* 92 (No. 4 2002): 1205-1217.

Lewis, William Arthur. "Economic Development with Unlimited Supplies of Labor." *Manchester School* 22 (No. 2 1954): 139-191.

McCloskey, Donald. "The Industrial Revolution 1780-1860: A Survey." In Joel Mokyr (ed.), *The Economics of the Industrial Revolution.* Totowa: Roman & Allanheld, 1985.

Mokyr, Joel. *The Enlightened Economy.* New Haven and London: Yale University Press, 2009.

Neal, Larry. "The Finance of Business during the Industrial Revolution." In Roderick Floud and Donald N. McCloskey (eds.), *The Economic History of Britain Since 1700.* 2^{nd} ed. Cambridge: Cambridge University Press, vol. 1, 1994.

Tsoulouhas, Theofanis C. "A New Look at Demographic and Technological Changes, England, 1550 to 1839." *Explorations in Economic History* 29 (No. 2 1992): 169-203.

Usher, Abbott Payson. *An Introduction to the Industrial History of England.* London: Houghton Mifflin Co, 1920.

Voigtländer, Nico, and Hans-Joachim Voth. "Why England? Demographic Factors, Structural Change and Physical Capital Accumulation During the Industrial Revolution." *Journal of Economic Growth* 11 (No. 4 2006): 319-361.

Von Tunzelmann. "Technology in the Early Nineteenth Century." In Roderick Floud and Donald N. McCloskey (eds.), *The Economic History of Britain Since 1700.* 2nd ed. Cambridge: Cambridge University Press, Vol. 1, 1994.

Yang, Donghyu, and Shin-Haing Kim. "An Escape from the "Malthusian Trap": A Case of the Choson Dynasty of Korea from 1701 to 1891 Viewed in Light of the British Industrial Revolution." *Seoul Journal of Economics* 26 (No. 2 2013): 173-201.

Wrigley, E. A., and R. S. Schofield. *The Population History of England, 1541-1871: A Reconstruction.* Cambridge: Cambridge University Press, 1993.