Noisy and Subjective Performance Measure in Promotion Tournaments

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This paper considers both incentive and sorting effects of a promotion tournament, and analyzes the optimal degree of uncertainty in the agents' performance measure. In a subjective promotion tournament where the winner is determined by the principal's belief about the agents' ability, this paper shows that a noisy performance measure can have a positive incentive effect and a negative sorting effect. Therefore, it can be optimal for the principal to intentionally choose a noisy performance measure.

Keywords: Noisy performance measure, Subjective tournament, Sorting

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I. Introduction

Tournament theory, à la Lazear and Rosen (1981), has been one of the key theoretical building blocks in the analysis of personnel policies within firms, especially in the analysis of promotions. The literature has extended the standard model in various directions, including multi-stage tournaments, asymmetric tournaments, and repeated tournaments, although the basic focus of the literature has been the *incentive* effects on the agents' efforts. However, promotions in typical organizations also serve as a *sorting* device to select the most able agent and re-assign him/her to the next higher (and often more skill-demanding) job level.

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To incorporate the sorting effect of promotions, the current paper considers the subjective tournament model proposed by Kwon (2011). In a subjective tournament, the winner is determined by the principal's belief about the agents' ability. This means that the principal observes the agents' performance, updates her belief about the agents' ability, and promotes the agent with the highest expected ability. Note that this type of subjective tournament more closely resembles the actual promotion system in typical organizations than a standard tournament model, where the winner is chosen based only on the agents' realized performance.

In a subjective tournament, the principal has two objectives. First, she wants to maximize the agents' effort (incentive effect). Second, the principal wants to maximize the precision of her expectation about the agents' abilities (sorting effect). Therefore, the subjective tournament model allows us to analyze the potential trade-offs between the incentive effect and the sorting effect in promotions.

In particular, the current paper shows that in a subjective tournament, the noise in the agents' performance measure can have positive incentive effects but negative sorting effects. Therefore, it is optimal for the principal to maintain some degree of noise (or uncertainty) in the agents' performance measure. This result contrasts with previous studies, where it is always optimal to minimize the noise in the performance measure due to its negative incentive effect (*e.g.*, Hvide 2002).

These results may explain why some organizations use noisy and subjective performance measures (*e.g.*, supervisors' performance ratings), even when objective performance data (*e.g.*, sales records) are available. Alternatively, this paper suggests that in organizations with potentially heterogeneous agents, relying on objective and precise performance measures may only diminish the workers' incentives.

Intuitively, in a standard tournament model, the noise in the performance measure has negative incentive effects, because the winner is chosen by luck rather than by the agents' efforts. In contrast, in a subjective tournament, the principal can take into account the effect of the noise (or luck) in forming her belief about the agents' ability.

In a subjective tournament, this paper shows that when the noise in the performance measure increases, its incentive effect is ambiguous. When the noise in the performance measure increases, the principal's belief about the agents' ability depends more on her prior, rather than on, the agents' realized performance. Therefore, the agents have less incentive to work hard to increase their performance. This means that the noise in the performance measure has a *negative* incentive effect. Note that the reason for this negative incentive effect differs from that in the standard tournament model as discussed above.

Moreover, the importance of the sorting effect implies potentially large unobserved heterogeneity among the agents' abilities. Recall that agents work harder in a symmetric tournament where the agents' abilities are homogeneous (*e.g.*, O'Keeffe, Viscusi, and Zeckhauser 1984; Kräkel and Sliwka 2004). When the noise in the performance measure increases, the agents are less certain about who has higher (or lower) ability, making the tournament more symmetric. Therefore, the noise in the performance measure can have a *positive* incentive effect.

For the sorting effect, the principal wishes to minimize the noise in the performance measure to form a more precise belief about the agents' ability. That is, the noise in the performance measure has a *negative* sorting effect. Due to these opposing effects, the current shows that it is optimal for the principal to maintain some degree of noise in the agents' performance measure.

The rest of the paper is organized as follows. Section 2 summarizes the related literature. Section 3 introduces the basic model. In Section 4, I consider the agents' choice of effort in a subjective tournament. Section 5 characterizes the principal's optimal choice of uncertainty in the agents' performance measure. Section 6 discusses the potential extensions and conclusion.

II. Related Literature

There are a series of studies that analyze agents' choice of risk in a tournament setting (see, *e.g.*, Hvide 2002; Kräkel and Sliwka 2004). Moreover, in a non-tournament setting, Hellwig (1994), Biais and Casamatta (1999), and Palomino and Prat (2003) have considered an agent's choice of effort and risk. However, in most of these studies, the principal's optimal choice of risk is trivial (that is, zero) and does not receive serious attention.

Many recent studies have explored the idea that a principal may intentionally withhold information on the agents' mid-term performance in a dynamic model. Some of these studies include Ederer (2009), Gershkov and Perry (2006), Goltsman and Mukherjee (2006), Kim (2005), Yildirim (2005), and Aoyagi (2010). However, in these models, the principal privately observes the agents' mid-term performance. Therefore, the decision to release the mid-term performance information does not change the principal's belief about the agents' ability, or the sorting effect of the tournament.

Note that the aforementioned studies are interested in maximizing the agents' effort (*i.e.*, the incentive effect), not in selecting the highest ability agent (*i.e.*, the sorting effect). There exists a related recent literature on sequential auctions and elimination contests that focuses on the efficiency of the sorting effects. See, for example, Moldovanu and Sela (2006), Mezzetti *et al.* (2008), Cai *et al.* (2007), and Wang and Zhang (2009). In this literature, however, the mid-term information revelation structure (*i.e.*, uncertainty in the performance information) is exogenous.

Kwon (2011) also studied incentive and sorting effects in a subjective tournament. However, Kwon (2011) examined the choice of uncertainty and the choice of effort *by the agents*. In contrast, this paper considers a case where *the principal* chooses the uncertainty, while the agents choose their effort. For example, Kwon (2011) applies to a situation where the agents can choose the risk of their investments, while this paper applies to a situation where the principal can choose the accuracy of the agents' performance measure.

Using a contest model, Wang (2010) showed a similar result: there exists an optimal accuracy level, which maximizes the contestants' total effort. However, Wang (2010) takes the power contest success function, à la Tullock (1980), as given. In contrast, the current paper introduces the subjective tournament as a better model of the actual practice of promotions than the standard tournament model à la Lazear and Rosen (1981), and then derives the agents' winning probability. Furthermore, Wang (2010) does not discuss the sorting effect of promotions.

Kräkel (2012) considered competitive career contests, where the losers have different fall-back options depending on their productivity. He has shown that if the winner for a top position is chosen based on the players' performance only, the least productive player may have the highest probability of winning the top position because s/he has worse fall-back positions than the others. In other words, career contests based on the players' performance can only lead to bad sorting for the top position. Thus, Kräkel (2012) is concerned about both the sorting and the incentive effects in career contests. However, in Kräkel (2012), the players' productivities are common knowledge. Thus, he does not analyze the choice of uncertainty about the players' productivity or performance measure.

III. Model

Following the same notation as in Kwon (2011), suppose that there are two risk-neutral agents ($i=A, B$). Neither the agents nor the principal observe the agents' ability η_i. The prior distribution of η_i is normal:

$$\eta_i \sim N(m_0, \sigma_0^2).$$

Note that the agents are ex-ante symmetric, but that their ability can be different ex-post.

There are two periods. In the first period, each agent's performance (y_{i1}) is determined as follows:

$$y_{i1}=\eta_i+\varepsilon_{i1},$$

where ε_{i1} is a random noise.[1]

In the second period, each agent's performance (y_{i2}) is determined as follows:

$$y_{i2}=\eta_i+a_i+\varepsilon_{i2},$$

where a_i is agent i's effort and ε_{i2} is a random noise. Assume that ε_{i1} and ε_{i2} are *iid* normal, then we arrive at:

$$\varepsilon_{it} \sim N(0, \sigma_\varepsilon^2).$$

The cost of effort is $g(a)=e^a-1$.[2]

The principal does not observe the agents' effort a_i. However, she can observe the realized performance of each agent, and update her belief about the agents' ability. At the end of the second period, the principal promotes the agent who has higher expected ability. This type of tournament is called *subjective tournament* (Kwon 2011). Note that in a standard tournament model, the agent who has higher realized performance wins the tournament. The promoted agent (the winner) receives the prize

[1] For simplicity, I assume that the agents do not exert effort in the first period. Relaxing this assumption should not change the qualitative results of this paper.

[2] This functional form of the cost function allows a closed form solution for a simple analysis. However, the qualitative results of the paper should not change as long as the cost function is increasing and convex.

M, while the loser receives the payoff m, where $M > m > 0$.

Note that this model is motivated by typical promotion tournaments in a hierarchical organization, where performance in a higher level job is more sensitive to the agent's ability than performance in a lower level job (Gibbons and Waldman 1999). Therefore, while the principal wants the agents to work hard in their current job levels, she also wants to promote the agent with the highest ability to a higher job level.

However, the agent with the highest performance is not necessarily the agent with the highest ability. For example, the agents may intentionally choose a risky strategy in a winner-take-all contest (Hvide 2002). Then, in this case, the agent with the highest realized performance is not necessarily the one with the highest ability. Furthermore, the agent with the highest ability may not work hard to win the tournament because s/he has a better fall-back option from losing (Kräkel 2012). Therefore, to determine the winner in a promotion tournament, it can be better for the principal to rely on her belief about the agents' ability taking into account the agents' incentives for effort.

In this model, by Bayes' rule, the principal's posterior belief about player i's ability η_i conditional on the first and the second period performance (y_{i1} and y_{i2}) can be characterized by the following normal distribution:

$$\eta_i \mid y_{A1}, y_{B1}, y_{A2}, y_{B2} \sim N\left(\frac{(\sigma_\varepsilon^2 m_0 + \sigma_0^2 y_{i1}) + \sigma_0^2(y_{i2} - \hat{a}_i)}{\sigma_\varepsilon^2 + 2\sigma_0^2}, \frac{\sigma_0^2\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + 2\sigma_0^2}\right), \quad (1)$$

where $\hat{a}_i$ is the principal's expectation of agent i's effort level.

An important departure from the previous literature is that I allow the principal to choose the noise in the performance measure, σ_ε^2. As discussed above, some previous studies have allowed the agents, not the principal, to choose the noise or risk of their performance in a tournament (see, *e.g.*, Hvide 2002; Kräkel and Sliwka 2004; Kwon 2011).

The timing of the game is as follows. First, the principal chooses the noise in the performance measure, σ_ε^2. Then, the first period performance is realized. In the second period, each agent simultaneously chooses his effort level and the second period performance is realized. Finally, the principal promotes the agent with the higher expected ability.

Note that the model does not specify the payoff function of the principal. For the purpose of this paper, it is sufficient to assume that the principal's expected payoff increases with the agents' efforts and with the precision of the expected agent's ability. In other words, the results of this paper are more general than those obtained with a specific payoff function of the principal.

IV. Agents' Choice of Effort

To solve the model backwards, I first consider the agents' choice of effort in the second period given σ_ε^2, y_{A1}, and y_{B1}. From (1), without loss of generality, agent A would win the promotion if

$$E[\eta_A | y_{A1}, y_{B1}, y_{A2}, y_{B2}] \geq E[\eta_B | y_{A1}, y_{B1}, y_{A2}, y_{B2}]$$
$$\Updownarrow$$
$$(\eta_A + \varepsilon_{A2} - \eta_B - \varepsilon_{B2}) \geq (\hat{a}_A - \hat{a}_B) - (a_A - a_B) - (y_{A1} - y_{B1}).$$

Note that conditional on the first period performance y_{A1} and y_{B1},

$$(\eta_A + \varepsilon_{A2} - \eta_B - \varepsilon_{B2})|_{y_{A1}, y_{B1}} \sim N\left(\frac{\sigma_0^2(y_{A1} - y_{B1})}{\sigma_\varepsilon^2 + \sigma_0^2}, \frac{2\sigma_\varepsilon^4 + 4\sigma_0^2\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_0^2}\right). \qquad (2)$$

If I denote the cumulative distribution function of $(\eta_A + \varepsilon_{A2} - \eta_B - \varepsilon_{B2})$ by F, then at the beginning of the second period, agent A's probability of winning given the first period performance is given by:

$$P_A = 1 - F((\hat{a}_A - \hat{a}_B) - (a_A - a_B) - (y_{A1} - y_{B1})).$$

Likewise, agent B's probability of winning given the first period performance is given by:

$$P_B = F((\hat{a}_A - \hat{a}_B) - (a_A - a_B) - (y_{A1} - y_{B1})).$$

Then, given the first period performance, agent A's maximization problem in the second period is as follows:

$$\max_{a_A \geq 0} U_A = m + P_A(M - m) - g(a_A).$$

If I denote the pdf of F by f, then the first order condition for agent A's optimal effort a_A^* is given by:

$$\begin{aligned}\frac{\partial U_A}{\partial a_A} &= f((\hat{a}_A - \hat{a}_B) - (a_A^* - a_B^*) - (y_{A1} - y_{B1}))(M-m) - g'(a_A^*) \\ &= f(-(y_{A1} - y_{B1}))(M-m) - g'(a_A^*) \leq 0,\end{aligned} \tag{3}$$

where equality holds if $a_A^* > 0$. The second equality follows from the rational expectations assumption, that is, $\hat{a}_A = a_A^*$ and $\hat{a}_B = a_B^*$ in equilibrium.

Likewise, the first order condition for agent B's optimal effort a_B^* is given by:

$$\frac{\partial U_B}{\partial a_B} = f(-(y_{A1} - y_{B1}))(M-m) - g'(a_B^*) \leq 0, \tag{4}$$

where equality holds if $a_B^* > 0$.

Then, the agents' optimal choice of effort can be characterized as follows:

***Proposition* 1.** (i) Regardless of the first period performance (y_{A1}, y_{B1}), both agents choose the same level of effort $(a_A^* = a_B^* = a^*)$.

(ii) If $|y_{A1} - y_{B1}|$ increases, then the effort level (a^*) decreases.

Proof. See Appendix. ■

Intuitively, in an asymmetric tournament, if the difference in ability between the agents increases, the agent with higher (expected) ability exerts less effort because he is likely to win anyway even with little effort. Furthermore, the agent with lower (expected) ability exerts less effort, because he is likely to lose even if he works hard. This means that the agents' effort levels largely depend on the difference in expected ability. Therefore, given that the agents are ex-ante symmetric in this model, their efforts in the second period are the same, because the perceived difference in their expected ability after the first period must be the same for both agents.

Furthermore, if $y_{A1} - y_{B1}$ increases, the posterior expected difference in the agents' ability increases. Therefore, both agents would work less (O'Keeffe, Viscusi, and Zeckhauser 1984; Kräkel and Sliwka 2004).

V. Principal's Optimal Choice of Uncertainty

Next, I analyze the principal's optimal choice of uncertainty in the performance measure. For precise sorting (*i.e.*, to reduce the type I error of promotion), the principal must minimize the variance of her expected posterior belief. From (1), it is straightforward to show that the variance of her posterior belief about the agents' ability is increasing in σ_ε^2. Therefore, for precise sorting, the principal should minimize σ_ε^2.

If reducing σ_ε^2 increases the agents' effort at the same time, then there would be no trade-off between the incentive effect and the sorting effect, and the principal chooses the smallest σ_ε^2 possible. However, if reducing σ_ε^2 decreases the agents' effort, there would be a potential trade-off between the incentive effect and the sorting effect, and the optimal σ_ε^2 can be strictly positive. Then, the principal may intentionally choose a noisy performance measure.

Recall that in a standard tournament model, reducing σ_ε^2 increases the agents' effort (*e.g.*, Hvide 2002). Therefore, the previous studies have not considered the potential trade-offs between incentive and sorting effects. However, in a subjective tournament, the following proposition shows that there can be a trade-off.

From (3) and (4), I can characterize the effect of σ_ε^2 on the agents' expected optimal effort $E[a^*]$ as follows:

***Proposition* 2.** There exists $s(\sigma_0^2)>0$ such that
(i) if $\sigma_\varepsilon^2>s(\sigma_0^2)$, $E[a^*]$ decreases in σ_ε^2,
(ii) if $\sigma_\varepsilon^2<s(\sigma_0^2)$, $E[a^*]$ increases in σ_ε^2,
(iii) if $\sigma_\varepsilon^2=0$, then $E[a^*]=0$,
(iv) $s(\sigma_0^2)$ increases in σ_0^2.
Proof. See Appendix. ■

Intuitively, the noise in the performance measure, σ_ε^2, has two opposing incentive effects. First, if σ_ε^2 increases, the agents' performance would reflect luck more than the agents' ability or effort. Then, from (1), the agents' performance would have little marginal effect on the principal's expectation of the agents ability. Therefore, if the noise in the performance measure (σ_ε^2) increases, the agents would have *less* incentive for effort.

Second, if σ_ε^2 increases, the difference in the agents' first period performance does not necessarily reflect the difference in the agents' ability. This means that if σ_ε^2 increases, the tournament in the second period becomes more symmetric in terms of the expected ability of the agents. Then, as discussed above, the agents' would have *more* incentive to exert effort.

Proposition 2 shows that if σ_ε^2 is large enough relative to σ_0^2, the first negative incentive effect dominates the second positive effect. Thus, the agents' expected effort decreases in σ_ε^2. However, if σ_ε^2 is small enough relative to σ_0^2, the second positive incentive effect dominates the first negative effect. Thus, the agents' expected effort increases in σ_ε^2.

Now, let us suppose that the principal's payoffs increase in the agents' effort (given σ_ε^2), and decrease in σ_ε^2 (given effort). Note that I do not need to assume any specific functional form for the principal's payoff function.[3]

If $\sigma_\varepsilon^2=0$, the difference in the first period reveals the difference in ability for sure. Then, there would be no incentive for any effort in the second period. Therefore, assuming that the principal never wants the agents to choose zero effort, the principal would not choose $\sigma_\varepsilon^2=0$.

Then, I can state the following proposition for the optimal σ_ε^2.

***Proposition* 3.** The optimal σ_ε^2 for the principal is $0<\sigma_\varepsilon^2<s(\sigma_0^2)$.

Proof. From Proposition 2, decreasing σ_ε^2 leads to more effort if and only if $\sigma_\varepsilon^2>s(\sigma_0^2)$. Given that the principal's payoffs increase in the agents' effort and decrease in σ_ε^2, if $\sigma_\varepsilon^2\geq s(\sigma_0^2)$, reducing σ_ε^2 must lead to higher payoffs for the principal. Therefore, the optimal σ_ε^2 must be less than $s(\sigma_0^2)$. ■

Note that even when the principal can choose a precise performance measure with $\sigma_\varepsilon^2=0$ for perfect sorting, the principal would still intentionally choose a noisy performance measure in order to increase the incentives for second period effort. This result contrasts with the conventional wisdom in agency models that the more precise the performance measure is, the better it would be. Therefore, Proposition 3 can provide a potential explanation for why some organizations use seemingly subjective and noisy performance measures (*e.g.*, performance rating) instead of objective and precise performance measures (*e.g.*, sales records).

Meanwhile, if sorting is not important, either because the agents are symmetric or because performance at higher job levels is not more sensitive to an agent's ability than that at lower job levels, then it would be optimal for the principal to minimize the noise in the agents' performance measure. These results may explain why, in promotion tournaments, some organizations seem to rely on objective performance measures while

[3] I do assume that the second order conditions for the principal's optimization problem are satisfied.

others rely on the subjective opinion of the supervisors.[4]

Proposition 3 is also related to the recent literature on mid-term performance revelation discussed at the beginning. For example, Ederer (2009) shows that the principal may intentionally hide the information on agents' mid-term performance to increase the agents' incentives in the second period. It is worth emphasizing, however, that in Ederer (2009), the trade-off is between first period incentives and second period incentives. Proposition 2 shows that there is another mechanism, which drives the principal to choose a noisy performance measure, *i.e.*, the trade-off between the sorting effect and the (second period) incentives effect.

VI. Conclusion

Theoretically, this paper uncovers two opposing incentive effects in a subjective tournament. When the noise in the performance measure increases, it has a negative incentive effect, because the principal does not trust the agents' performance. However, the noise in the performance measure also has a positive incentive effect because the agents do not learn much about the difference in their abilities and compete more vigorously. This paper provides a tractable tournament model, which combines both incentive effects and sorting effects.

With the growing importance of performance pay in many organizations, it is crucial that we find ways by which to measure accurately the agents' performance. It is well-known that an imprecise performance measure can reduce incentives for risk-averse agents and distort the agents' task allocation. Consequently, much research and resources are devoted to develop more objective and precise performance measures. In contrast, this paper shows that it can be optimal to use a subjective and noisy performance measure, especially in promotion tournaments with heterogenous agents. An interesting empirical implication for future research is that in organizations with homogeneous agents, the performance measure is likely to be more objective and precise, while in organizations with potentially heterogeneous agents, the performance measure

[4] In this case, there is a concern that skilled agents would not join an organization with subjective tournaments because their ability is likely to be discounted by the principal's belief. However, it appears that the jobs using objective tournaments and those using subjective tournaments are in different occupations or industries. As such, it would be difficult for skilled agents to avoid subjective tournaments and join objective tournaments in different occupations or industries.

is likely to be relatively more subjective and imprecise. Perhaps more importantly, this paper shows that in the choice of performance measure for promotion tournaments, one should consider both incentive and sorting effects of promotions.

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Appendix

***Proof of Proposition* 1.** (i) From (3) and (4), the first order conditions for players A and B are symmetric. Therefore, $a^*_A = a^*_B$.

(ii) Without loss of generality, consider the posterior belief about player A's ability. The posterior mean and variance of η_A conditional on the first period performance are given as:

$$m_{A1} = \frac{\frac{1}{\sigma_0^2} m_0 + \frac{1}{\sigma_\varepsilon^2} y_{A1}}{\frac{1}{\sigma_0^2} + \frac{1}{\sigma_\varepsilon^2}} = \frac{\sigma_\varepsilon^2 m_0 + \sigma_0^2 y_{A1}}{\sigma_\varepsilon^2 + \sigma_0^2}, \tag{A.1}$$

$$\sigma_1^2 = \left(\frac{1}{\sigma_0^2} + \frac{1}{\sigma_\varepsilon^2} \right)^{-1} = \frac{\sigma_0^2 \sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_0^2}. \tag{A.2}$$

The posterior mean and variance of η_A conditional on the first and second period performance are given as:

$$\begin{aligned} m_{A2} &= \frac{\frac{1}{\sigma_1^2} m_{A1} + \frac{1}{\sigma_{\varepsilon A}^2} (y_{A2} - \hat{a}_A)}{\frac{1}{\sigma_1^2} + \frac{1}{\sigma_{\varepsilon A}^2}} = \frac{\sigma_{\varepsilon A}^2 m_{A1} + \sigma_1^2 (y_{A2} - \hat{a}_A)}{\sigma_{\varepsilon A}^2 + \sigma_1^2} \\ &= \frac{\sigma_{\varepsilon A}^2 \frac{\sigma_\varepsilon^2 m_0 + \sigma_0^2 y_{A1}}{\sigma_\varepsilon^2 + \sigma_0^2} + \frac{\sigma_0^2 \sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_0^2} (y_{A2} - \hat{a}_A)}{\sigma_{\varepsilon A}^2 + \frac{\sigma_0^2 \sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_0^2}} \\ &= \frac{\sigma_{\varepsilon A}^2 (\sigma_\varepsilon^2 m_0 + \sigma_0^2 y_{A1}) + \sigma_0^2 \sigma_\varepsilon^2 (y_{A2} - \hat{a}_A)}{\sigma_{\varepsilon A}^2 (\sigma_\varepsilon^2 + \sigma_0^2) + \sigma_0^2 \sigma_\varepsilon^2}, \end{aligned} \tag{A.3}$$

$$\sigma_{A2}^2 = \left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_{\varepsilon A}^2}\right)^{-1} = \left(\frac{\sigma_{\varepsilon A}^2 + \dfrac{\sigma_0^2\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_0^2}}{\dfrac{\sigma_0^2\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_0^2}\sigma_{\varepsilon A}^2}\right)^{-1} = \frac{\sigma_0^2\sigma_\varepsilon^2\sigma_{\varepsilon A}^2}{\sigma_{\varepsilon A}^2(\sigma_\varepsilon^2 + \sigma_0^2) + \sigma_0^2\sigma_\varepsilon^2}. \tag{A.4}$$

Without loss of generality, suppose that $y_{A1}-y_{B1}>0$. From (A.1), the mean of $(\eta_A+\varepsilon_{A2}-\eta_B-\varepsilon_{B2})$ conditional on the first period performance is $\{\sigma_0^2(y_{A1}-y_{B1})\}/(\sigma_\varepsilon^2+\sigma_0^2)$. Given that $(y_{A1}-y_{B1})>\{\sigma_0^2(y_{A1}-y_{B1})\}/(\sigma_\varepsilon^2+\sigma_0^2)>0>-(y_{A1}-y_{B1})$, from the symmetry of f, if $(y_{A1}-y_{B1})$ increases, $f(-(y_{A1}-y_{B1}))$ decreases. Therefore, the second period effort decreases with $(y_{A1}-y_{B1})$. ■

***Proof of Proposition* 2.** (i) and (ii) From (A.1) and (A.2),

$$f(-(y_{A1}-y_{B1})) = \frac{1}{\sqrt{2\pi\dfrac{2\sigma_\varepsilon^4 + 4\sigma_0^2\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_0^2}}}\exp\left(-\frac{\left(-(y_{A1}-y_{B1}) - \dfrac{\sigma_0^2(y_{A1}-y_{B1})}{\sigma_\varepsilon^2 + \sigma_0^2}\right)^2}{2\dfrac{2\sigma_\varepsilon^4 + 4\sigma_0^2\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_0^2}}\right).$$

Then, since $g(a)=e^a-1$, from (3) and (4), we can obtain the closed form solution for the agent's optimal choice of effort as follows:

$$a_A^* = a_B^* = \log(M-m) - \log\left(\sqrt{2\pi\frac{2\sigma_\varepsilon^4 + 4\sigma_0^2\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_0^2}}\right) - \frac{\left(\dfrac{\sigma_\varepsilon^2 + 2\sigma_0^2}{\sigma_\varepsilon^2 + \sigma_0^2}\right)^2}{2\dfrac{2\sigma_\varepsilon^4 + 4\sigma_0^2\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_0^2}}(y_{A1}-y_{B1})^2.$$

Recall that $E[X^2]=\mathrm{Var}(X)+(E[X])^2$. Since $(y_{A1}-y_{B1})\sim N(0, 2(\sigma_0^2+\sigma_\varepsilon^2))$,

$$E[a_i^*] = \log(M-m) - \log\left(\sqrt{2\pi\frac{2\sigma_\varepsilon^4 + 4\sigma_0^2\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_0^2}}\right) - \frac{\left(\dfrac{\sigma_\varepsilon^2 + 2\sigma_0^2}{\sigma_\varepsilon^2 + \sigma_0^2}\right)^2}{2\dfrac{2\sigma_\varepsilon^4 + 4\sigma_0^2\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_0^2}}2(\sigma_0^2 + \sigma_\varepsilon^2)$$

$$= \log(M-m) - \log\left(\sqrt{4\pi\frac{\sigma_\varepsilon^2(\sigma_\varepsilon^2 + 2\sigma_0^2)}{\sigma_\varepsilon^2 + \sigma_0^2}}\right) - \frac{1}{2\sigma_\varepsilon^2}(2\sigma_0^2 + \sigma_\varepsilon^2).$$

It is straightforward to check that $E[a_i^*]$ is maximized at

$$\sigma_\varepsilon^2 = s(\sigma_0^2) \equiv \frac{4}{3}\frac{\sigma_0^4}{\sqrt[3]{\sqrt{\frac{44}{27}\sigma_0^{12}} + 2\sigma_0^6}} + \sqrt[3]{\sqrt{\frac{44}{27}\sigma_0^{12}} + 2\sigma_0^6}.$$

Therefore, if $\sigma_\varepsilon^2 > s(\sigma_0^2)$, $E[a^*]$ decreases in σ_ε^2. And if $\sigma_\varepsilon^2 < s(\sigma_0^2)$, $E[a^*]$ increases in σ_ε^2.

(iii) Moreover, if $\sigma_\varepsilon^2 = 0$, from (3) and (4), $a^* = 0$ for all y_{A1} and y_{B1}. Therefore, $E[a^*] = 0$.

(iv) It is straightforward to check that $s'(\sigma_0^2) > 0$ for all $\sigma_0^2 > 0$. ■

References

Aoyagi, M. "Information Feedback in a Dynamic Tournament." *Games and Economic Behavior* 70 (No. 2 2010): 242-60.

Biais, B., and Casamatta, C. "Optimal Leverage and Aggregate Investment." *Journal of Finance* 54 (No. 4 1999): 1291-323.

Cai, G., Wurman, P.R., and Chao, X. "The Non-existence of Equilibrium in Sequential Auctions When Bids Are Revealed." *Journal of Electronic Commerce Research* 8 (No. 2 2007): 141-56.

Ederer, F. Feedback and Motivation in Dynamic Tournaments. mimeo, UCLA, 2009.

Gershkov, A., and Perry, M. Tournaments with Midterm Reviews. mimeo, University of Bonn, 2006.

Gibbons, R., and Waldman, M. "Careers in Organizations: Theory and Evidence." In O. Ashenfelter and D. Card (eds.) *Handbook of Labor Economics.* 3B, Chapter 36, NY, USA: North Holland, 1999.

Goltsman, M., and Mukherjee, A. Information Disclosure in Multistage Tournaments. mimeo, University of Western Ontario, 2006.

Hellwig, M. A Reconsideration of the Jensen-Meckling Model of Outside Finance. Discussion Paper No. 9422, WWZ University of Basel, 1994.

Hvide, H. K. "Tournament Rewards and Risk Taking." *Journal of Labor Economics* 20 (No. 4 2002): 877-98.

Kim, S. K. "Dual Agency Model, Reporting Frequency." *Seoul Journal of Economics* 18 (No. 1 2005): 21-44.

Kräkel, M. "Competitive Careers as a Way to Mediocracy." *European Journal of Political Economy* 28 (No. 1 2012): 76-87.

Kräkel, M., and Sliwka, D. "Risk Taking in Asymmetric Tournaments." *German Economic Review* 5 (No. 1 2004): 103-16.

Kwon, I. Risk Taking in Subjective Promotion Tournaments. mimeo, Seoul National University, 2011.

Lazear, E., and Rosen, S. "Rank Order Tournaments as Optimal Labor Contracts." *Journal of Political Economy* 89 (No. 5 1981): 841-64.

Mezzetti, C., Pekec, A., and Tsetlin, I. "Sequential vs. Single-Round Uniform-Price Auctions." *Games and Economic Behavior* 62 (No. 2 2008): 591-609.

Moldovanu, B., and Sela, A. "Contests Architecture." *Journal of Economics Theory* 126 (No. 1 2006): 70-96.

O'Keeffe, M., Viscusi, W. K., and Zeckhauser, R. J. "Economic Contests: Comparative Reward Schemes." *Journal of Labor Economics* 2 (No. 1 1984): 27-56.

Palomino, F., and Prat, A. "Risk Taking and Optimal Contracts for Money Managers." *Rand Journal of Economics* 34 (No. 1 2003): 113-37.

Tullock, G. "Efficient Rent Seeking." In J. M. Buchanan, R. D. Tollison, and G. Tullock (eds.), *Toward a Theory of the Rent-Seeking Society.* College Station: Texas A&M University Press, pp. 97-112, 1980.

Wang, R., and Zhang, J. "The Role of Information Revelation in Elimination Contests." *The Economic Journal* 119 (No. 536 2009): 613-41.

Wang, Z. "The Optimal Accuracy Level in Asymmetric Contests." *The B. E. Journal of Theoretical Economics* 10 (No. 1 2010): Article 13.

Yildirim, H. "Contests with Multiple Rounds." *Games Economic Behavior* 51 (No. 1 2005): 213-27.