Wealth Effects When the Cost of Effort is Money

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We study the effects of the agent's wealth on the agency cost and the principal's profit in the principal-agent model in which the agent's effort entails a monetary cost. We show that if the inverse of the marginal utility function is concave in the utility function, then an increase in the agent's wealth lowers the agency cost for any effort level, directly implying that the principal clearly benefits from such a decrease in the agency cost. However, even if the convexity of the marginal utility function with respect to the utility function is assumed, as in most of the previous results, the effects of the agent's wealth on the agency cost remain unclear in our model. The main reason is because a rise in wealth inevitably makes the incentive problem easier by lowering the marginal cost of effort, reducing the agency cost whereas that convexity raises the agency cost.

Keywords: Wealth effects, Moral hazard, Pecuniary effort, Agency costs

JEL Classification: D86

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This Research was Supported by Kangnam University Research Grants(2022).

[**Seoul Journal of Economics** 2025, Vol.38, No.1]
DOI: 10.22904/sje.2025.38.1.006

I. Introduction

In the principal-agent model that deals with the moral hazard issue, changes in the agent's preferences for income and effort can have an impact on the principal's compensation cost. Since changes in the agent's wealth may alter his preferences, research on the relationship between the agent's wealth and the principal's compensation cost has been carried out over the past few decades. Prior research has demonstrated that if the agent's degree of risk aversion is not low, an increase in the agent's wealth has a negative impact on the principal's compensation cost, thereby resulting in lowering the principal's surplus. The underlying premise of this result is that the agent's utility for income and effort is an additively separable form. Beyond such additive separability, we consider the principal-agent model in which the agent's effort involves a monetary cost and then theoretically analyze how the agent's wealth affects the principal's compensation cost. This topic has not yet been covered in the literature.

Most of the research findings are derived from analyzing the standard principal-agent model, where a risk-neutral principal and a risk-averse agent with additively separable utility for income and effort confront a moral hazard problem. Thiele and Wambach (1999) (henceforth TW) show that if the agent's degree of absolute prudence is not greater than three times his degree of absolute risk aversion (or equivalently, the inverse of the marginal utility function is *convex* in the utility function), then an increase in the agent's wealth raises the compensation cost that the principal must bear, or the agency cost. In addition, they demonstrate that if the agent's participation constraint does not bind at the optimum, an increase in the agent's wealth immediately lowers the agency cost.[1] Later, Chade and Serio (2014) prove that the converse of the first proposition presented by TW also holds true by verifying that if an increase in the agent's wealth raises the agency cost for any principal-agent setting where arbitrary choice is possible, then the agent's utility must satisfy the convexity suggested by TW.

Kadan and Swinkels (2013), even without depending on the validity of the first order approach, provide wealth effect results that are more widely accepted than those of TW. They provide the theoretical

[1] See Propositions 1 and 3 in Thiele and Wambach (1999).

conclusion that if the sum of the incentive wage paid to the agent and his wealth is not less than the income level from outside options in every state in the world, then an increase in the agent's wealth has a negative effect on the agency cost, even in situations where the convexity suggested by TW is not met. They also offer the additional result that an increase in the agent's wealth always leads to an increase in the agency cost if there is a minimum payment constraint that the agent's incentive payment cannot go below a certain amount. Furthermore, Jung (2022) offers a more general result than the second finding provided by Kadan and Swinkels (2013). He shows that, regardless of whether the participation or minimum payment constraints bind at the optimum, an increase in the agent's wealth has a negative impact on the agency cost if the agent's utility satisfies the convexity proposed by TW when the income level from outside options is not less than the minimum payment level.

According to the findings of those earlier studies, an increase in the agent's wealth raises the agency cost and, as a result, reduces the principal's profit if the requirement that the inverse of the marginal utility function be convex in the utility function is met. It should be noted that these theoretical findings were obtained when an additively separable utility function was used to represent the agent's preferences for income and effort. In this case, money and effort are independent commodities to the agent since the marginal utility of income and the marginal cost of effort are unaffected by each other.

Some research findings have also focused on the wealth effect when the agent's utility exceeds additive separability. Thiele and Wambach (1999) demonstrate that an increase in the agent's wealth raises the agency cost, which ultimately has a negative impact on the principal's profit if the agent's utility function satisfies such convexity and the cross derivative of the agent's utility functions for income and effort are negative (*i.e.,* income and effort are supplements).[2] According to Jung (2017), however, if the agent has a modified constant absolute risk aversion (CARA) utility function such that its degree decreases as his wealth increases, then under certain circumstances, an increase in the agent's wealth lowers the agency cost at an increasing rate, which raises the principal's profit at a decreasing rate.

[2] See Proposition 2 in Thiele and Wambach (1999).

This study focuses on the wealth effects when the agent's effort entails a monetary cost. Our model specifically involves a situation that has not been covered in previous studies: income and effort are complements, leading to a positive cross derivative. A situation where the cost of effort is money is frequently observed in the insurance or financial markets. Consider an insured person (the agent), for example, who is likely to lose money as a result of an accident or illness. The insured can attempt to prevent the loss of wealth by purchasing an accident prevention device. In this situation, the insured's self-prevention effort results in money costs to reduce the likelihood of the accident or the resulting wealth loss.[3] Of course, if insurance companies can observe the monetary costs of the insured, this information could be used to design insurance contracts. However, there are many cases where this is not the case. For instance, car-insured individuals who use a black-box usually get discounts from auto insurance companies. However, they do not provide discounts even if the insured personally installs other safety devices in their own old car (not a new car in which they are already installed) because they find it difficult to observe such costs. So, insurance firms must practically design insurance contracts to provide incentives for the insured to prevent car accidents by investing in such hard-to-observe safety devices.

We show that a rise in the agent's wealth results in a fall in the agency cost, which eventually boosts the principal's profit if the inverse of the marginal utility function is *concave* in the utility function. This implies that the effect of the agent's wealth on the agency cost is not entirely established, if as in most previous ones, the agent's utility satisfies such convexity proposed by TW. Here's the reason. In our model, when the principal induces the agent to select a target effort level, an increase in the agent's wealth lowers his marginal cost of effort. This reduces the agency cost by relaxing the incentive compatibility constraint. Thus, it is difficult to determine if the agency cost-reducing effect associated with this incentive provision is outweighed by the agency cost-increasing effect that arises under such convexity as previously proposed. We instead derive the result that an increase in the agent's wealth actually lowers the agency cost if the agent's utility

[3] Refer to Jullien *et. al.* (1999), Eeckhoudt and Gollier (2012) and Peter (2021) for the models in which the cost of effort is money.

satisfies our concavity condition rather than the convexity condition.

The key assumption of our paper is that the cost of effort is money. So, the agent's utility function has a positive cross derivative between income and effort, implying that income and effort are complementary. Thus, our results derived in the paper can apply even to the more general case that income and effort are complementary. However, our setting for the agent's utility differs from previous studies which made the assumption that income and effort are supplements or independent goods (additively separable utility case). As a result, we address the effects of the agent's wealth in the case that was not covered in the earlier research. Nevertheless, our main results of the paper would hold even with the additively separable utility function where income and effort are independent goods. Because this fact is immediately implied by Chade and Serio (2014), we will not discuss this issue in the paper.

As explained earlier, TW also discuss the case of non-additively separable utility functions. They consider the case that income and effort are supplements and suggest their result that an increase in the agent's wealth *raises* the agency cost if the inverse of the marginal utility function is convex in the utility function. However, we deal with the case that income and effort are complementary and then provides our result that an increase in the agent's wealth *reduces* the agency cost if the inverse of the marginal utility function is concave in the utility function. As a result, we derive the exact opposite result from Thiele and Wambach (1999). Furthermore, we show that in our model where the agent's effort incurs a pecuniary cost, the TW's result cannot be always guaranteed even if the agent's utility satisfies such a convexity.

This paper is organized as follows: Section II introduces the basic model in which the agent has pecuniary effort; Section III provides our results on wealth effects; and Section IV concludes this paper.

II. Basic Model

We consider a one-period principal-agent model with moral hazard problems. An agent chooses effort level $a \in A \equiv [0, \bar{a}]$ on behalf of a risk-neutral principal, where the upper bound $\bar{a}$ may be infinity. The agent's effort a is hidden to the principal but, the output $x \in X \equiv (\underline{x}, \bar{x})$ which stochastically depends on the agent's effort a, is observable to the principal without costs. Thus, $F(x \mid a)$ denotes the cumulative distribution function of x conditional on a, and $f(x \mid a) \in \mathbb{C}^2$ is the corresponding probability density function. The expected output is denoted by $\mu(a) \equiv \int x f(x \mid a)\, dx$.

The agent's monetary utility function from final income I is denoted by $u(I)$, where $u : (\underline{I}, \infty) \to \mathbb{R}$ is three times continuously differentiable, where the lower bound $\underline{I}$ can be negative infinity. We assume that $u'(I) > 0$ and $u''(I) < 0$ for all I. Since the cost of effort is pecuniary, the agent's utility when income s and effort a are given can be represented by

$$U(s, a) = u(s - c(a)),$$

where $c(a)$ denote the monetary cost from taking effort a. For the convenience of analysis, it is assumed that $c(a) \equiv a$. Thus, we have

$$U_s(s, a) = u'(s - a) > 0 \text{ and } U_{ss}(s, a) = u''(s - a) < 0 \text{ for all } (s, a),$$

implying that the agent is risk averse with respect to income regardless of his effort choice, and

$$U_a(s, a) = -\, u'(s - a) < 0 \text{ and } U_{aa}(s, a) = u''(s - a) < 0 \text{ for all } (s, a),$$

indicating that the agent dislikes working hard regardless of his income level.[4]

In order to induce the agent to voluntarily take $a > 0$, the principal should offer the agent an incentive contract $s(x)$ which depends on the output x. For the agent to accept the incentive contract $s(x)$, the expected utility level when $s(x)$ is given and the agent selects effort a is not less than the utility level from outside options when he takes $a = 0$. Thus, the participation constraint of the agent with initial wealth w is represented by

$$\int u\left(s(x) + w - a\right) f(x \mid a)\, dx \geq u(\alpha + w)$$

where α is the income level from the outside options and so $u(\alpha + w)$ means the agent's reservation utility level. And, assuming that the first

[4] Hereafter, subscripts denote partial derivatives.

order approach is valid,[5] the (doubly) relaxed incentive compatibility constraint is given by

$$\int [-\ u'\left(s\,(x)+w-a\right)f\ (x\,|\,a)+u\left(s\,(x)+w-a\right)f_a(x\,|\,a)]\ dx \geq 0.$$

Let $\hat{s}\,(x)=s\,(x)+w$. Then, the principal's cost minimization problem given ($a > 0$, w) is given by

$$C\,(a,\ w) \equiv \min_{\hat{s}\,(x)} \int \hat{s}\,(x) f\ (x\,|\,a)\ dx - w$$

$$s.t.\ \ i)\ \int u\,(\hat{s}\,(x)-a) f\ (x\,|\,a)\ dx - u\,(\alpha + w) \geq 0 \qquad \text{(PC)}$$

$$ii)\ \int \left[-u'\left(\hat{s}\,(x)-a\right)+u\left(s\,(x)-a\right)\frac{f_a\,(x\mid a)}{f\,(x\mid a)} \right] f(x\,|\,a)\ dx \geq 0 \qquad \text{(IC)}$$

Assume that there exists the optimal contract to solve the above cost minimization problem and denote it by $\hat{s}(x;\ a,\ w)$. Hence, the optimal contract should satisfy the following equation:

$$\frac{1\ +\ \mu u''\left(\hat{s}\,(x;\ a,\ w)\ -\ a\right)}{u'\left(\hat{s}\,(x;\ a,\ w)\ -\ a\right)} = \lambda\ +\ \mu\,\frac{f_a\,(x\mid a)}{f\,(x\mid a)},\qquad \forall x \in X, \qquad (1)$$

where λ and μ are the Lagrange multipliers of (PC) and (IC), respectively. The value function $C(a,\ w)$ means the agency cost that occurs when the principal induces the agent with wealth w to take a given effort a.

Note that if there exists the optimal contract to solve the cost minimization problem, then it must satisfy equation (1) at the optimum. For guaranteeing the existence of the optimal contract $\hat{s}(x;\ a,\ w)$ which is increasing in the likelihood ratio $\frac{f_a\,(x\mid a)}{f\,(x\mid a)}$ in equation (1), therefore, we need the assumption that the degree of absolute risk aversion $R\,(I) \equiv -\,\frac{u''\,(I)}{u'\,(I)}$ is decreasing in I. Furthermore, we could require an additional assumption that $\lim_{I\,\to\,\underline{I}} R\,(I) = \infty$ for the existence of the

[5] For the sufficient conditions for the validity of the first-order approach in our model, see Fagart and Fluet (2013).

optimal contract even in cases where the likelihood ratio $\frac{f_a(x \mid a)}{f(x \mid a)}$ is unbounded below *i.e.,* $\lim_{x \to \underline{x}} \frac{f_a(x \mid a)}{f(x \mid a)} = -\infty$.

It is true that $\mu > 0$ at the optimum if there exists the optimal contract $\hat{s}(x;\, a,\, w)$ to solve the cost minimization problem. To verify this, note that $\mu \geq 0$ by the Kuhn-Tucker necessary conditions for (IC). Suppose that it is not true that $\mu > 0$ at the optimum *i.e.,* $\mu = 0$. In this case, equation (1) reduces to $\frac{1}{u'(\hat{s}(x;\, a,\, w) - a)} = \lambda$.[6] Let $s = s_\lambda$ solve $\frac{1}{u'(s - a)} = \lambda$, where s_λ is constant in x. Thus, the optimal contract that should satisfy equation (1) is $\hat{s}(x;\, a,\, w) = s_\lambda$, indicating that if $\mu = 0$, the optimal contract is a fixed wage contract. From this, (IC) becomes

$$- u'(s_\lambda - a) + u(s_\lambda - a) \int f_a(x \mid a)\, dx = - u'(s_\lambda - a) < 0,$$

where the equality comes from the fact that $\int f_a(x \mid a)\, dx = 0$. This is a contradiction. Thus, we have the following lemma.

Lemma 1. $\mu > 0$ at the optimum.

On the other hand, (PC) may not be binding at the optimum in our model with pecuniary effort in contrast to the cases that the agent has an additively or multiplicatively separable utility function with respect to income and effort. In general, in the settings with additive or multiplicative separability, it is well known that the participation constraint may not be binding at the optimum when there exists a constraint that an incentive contract should be bounded below by a certain level and it is binding for some x. It is because the principal is unable to penalize the agent for poor performance and so it is possible for the agent to enjoy the rent from such a constraint. However, note that there is no minimum payment constraint in our model. Nevertheless, Thiele and Wambach (1999) have pointed out that in models including pecuniary effort as in our model, the participation constraint may not be binding at the optimum.[7] Therefore, it is possible that λ is equal to zero at the optimum in our model.

[6] Note that since $u'(I) > 0$ for all I, λ must be positive in this case.

[7] Refer to section 3 in Thiele and Wambach (1999) about this issue.

III. Results

We will first deal with how the agent's wealth w affects the agency cost $C(a, w)$. Using the Lagrange multiplier method, the agency cost is represented by

$$
\begin{aligned}
C(a, w) = \mathbb{E}[\hat{s}(x; a, w)] - w - \lambda\{\mathbb{E}[u(\hat{s}(x; a, w) - a)] \\
- u(\alpha + w)\} - \mu\mathbb{E}\Big[- u'(\hat{s}(x; a, w) - a) \\
+ u(\hat{s}(x; a, w) - a)\frac{f_a(x \mid a)}{f(x \mid a)}\Big],
\end{aligned} \tag{2}
$$

where $\mathbb{E}$ denotes the conditional expectation given a. Differentiating equation (2) with respect to w and applying the envelope theorem yield

$$C_w(a, w) = -1 + \lambda u'(\alpha + w) \tag{3}$$

Note that as explained earlier, (PC) can be non-binding at the optimum. Thus, we will provide our results dividing the case that it is binding and the one that it is not. They may be distinguished according to the value of effort a. For this, define $A_b(w) \equiv \{a \mid EU(a, w) = u(a + w)\}$ and $A_{nb}(w) \equiv \{a \mid EU(a, w) > u(a + w)\}$ for a given w, where $EU(a, w) \equiv \int u(\hat{s}(x; a, w) - a) f(x \mid a)\, dx$ is the agent's expected utility given the optimal contract. Note that since there exist no other cases except for the ones that (PC) binds or does not at the optimum, it is true that $A_b(w) \cup A_{nb}(w) = A$.

Based on equation (3), we can obtain the following proposition.

Proposition 1. Suppose that (PC) does not bind at the optimum. Then, we have $C_w(a, w) < 0$ for any $a \in A_{nb}(w)$.

Proof. Since (PC) does not bind at the optimum, it is true that $\lambda = 0$ and $a \in A_{nb}$. Therefore, from equation (3) we have $C_w(a, w) = -1 < 0$ for any $a \in A_{nb}(w)$. Q.E.D.

Proposition 1 shows that when (PC) does not bind at the optimum, an increase in the agent's wealth decreases the agency cost. Since (PC) does not bind, when the agent's wealth increases by one unit, the principal can lower the incentive wage paid to the agent in each state by one unit. Under this new wage contract, both (PC) and (IC) are still satisfied. Consequently, the agency cost decreases by the amount of the wealth increase.

Now, equation (1) must be used in order to reach another result. Since, $\mathbb{E}\left[\frac{f_a(x \mid a)}{f(x \mid a)}\right] = \int \frac{f_a(x \mid a)}{f(x \mid a)} f(x \mid a)\, dx = \int f_a(x \mid a)\, dx = 0$ we have

$$\lambda = \mathbb{E}\left[\lambda + \mu \frac{f_a(x \mid a)}{f(x \mid a)}\right] = \mathbb{E}\left[\frac{1 + \mu u''(\hat{s}(x;\, a,\, w) - a)}{u'(\hat{s}(x;\, a,\, w) - a)}\right] \tag{4}$$

where the second equality holds by equation (1). Therefore, equation (3) becomes

$$\begin{aligned} C_w(a,\, w) &= -1 + u'(\alpha + w)\mathbb{E}\left[\frac{1 + \mu u''(\hat{s}(x;\, a,\, w) - a)}{u'(\hat{s}(x;\, a,\, w) - a)}\right] \\ &= u'(\alpha + w)\left\{\mathbb{E}\left[\frac{1}{u'(\hat{s}(x;\, a,\, w) - a)}\right] - \frac{1}{u'(\alpha + w)}\right\} \\ &\quad + \mu\mathbb{E}\left[-R(\hat{s}(x;\, a,\, w) - a)\right]. \end{aligned} \tag{5}$$

Recall that $R(I) \equiv -\frac{u''(I)}{u'(I)}$ denotes the degree of absolute risk aversion. Using equation (5), we will derive the sufficient conditions for $C_w(a, w)$ to be negative. To this end, we need the following lemma which shows what condition is required in order that the first term of equation (5) is non-positive.

Lemma 2. Assume that (PC) binds at the optimum. If $\frac{1}{u'(I)}$ is concave(convex) in $u(I)$, then $\mathbb{E}\left[\frac{1}{u'(\hat{s}(x;\, a,\, w) - a)}\right] \leq (\geq) \frac{1}{u'(\alpha + w)}$.[8]

Proof. Since $\frac{1}{u'(I)}$ is concave in $u(I)$, there exists a concave(convex) function $h(u)$ such that $\frac{1}{u'(I)} = h\left(u(I)\right)$ for all I. Note that since $u(I)$ is increasing concave in I, $h(u)$ is trivially an increasing function.

[8] It is worth noting that the convexity of $\frac{1}{u'(I)}$ with respect to $u(I)$ implies $\mathbb{E}\left[\frac{1}{u'(\hat{s}(x;\, a,\, w) - a)}\right] \geq \frac{1}{u'(\alpha + w)}$, without the assumption that (PC) binds at the optimum.

By the *binding* participation constraint, we have $\mathbb{E}\left[u\left(\hat{s}\left(x;\ a,\ w\right)-a\right)\right] = u\left(\alpha + w\right)$. Thus, since $h(u)$ is a concave function, using Jensen's inequality yields

$$\begin{aligned}\mathbb{E}\left[\frac{1}{u'\left(\hat{s}\left(x;\ a,\ w\right)-a\right)}\right] &= \mathbb{E}\left[h\left(u\left(\hat{s}\left(x;\ a,\ w\right)-a\right)\right)\right] \\ &\leq (\geq)\ h\left(\mathbb{E}\left[u\left(\hat{s}\left(x;\ a,\ w\right)-a\right)\right]\right) \\ &= h\left(u\left(\alpha + w\right)\right) = \frac{1}{u'\left(\alpha + w\right)},\end{aligned}$$

where the last equality is satisfied by the definition of function h. The proof for the case that $h(u)$ is a convex function is similar to the above. Q.E.D.

We discuss only the concavity case. The condition that $\frac{1}{u'(I)}$ is concave in $u(I)$ in Lemma 2 means that function $\frac{1}{u'(I)}$ is more concave than function $u(I)$ in the sense of absolute risk aversion. Thus, it can be interpreted as that the agent with utility $\frac{1}{u'(I)}$ is more risk averse than the agent with utility $u(I)$.[9] Note that if (PC) is binding at the optimum, we have $\mathbb{E}\left[u\left(\hat{s}\left(x;\ a,\ w\right)-a\right)\right] = u\left(\alpha + w\right)$. Therefore, the concavity of $\frac{1}{u'(I)}$ with respect to $u(I)$ guarantees $\mathbb{E}\left[\frac{1}{u'\left(\hat{s}\left(x;\ a,\ w\right)-a\right)}\right] \leq \frac{1}{u'\left(\alpha + w\right)}$, which is what the Lemma 2 shows.

The condition that $\frac{1}{u'(I)}$ is concave in $u(I)$ is equivalent to the condition of $P(I) \geq 3R(I)$ for all I, where $P(I) \equiv -\frac{u'''(I)}{u''(I)}$ is called the degree of absolute prudence by Kimball (1990). As shown in the proof of Lemma 2, the concavity of $\frac{1}{u'(I)}$ with respect to $u(I)$ implies the existence of a concave function $h(u)$ such that $\frac{1}{u'(I)} \equiv h\left(u\left(I\right)\right)$. Thus, we have $h'\left(u\left(I\right)\right) = -\frac{u''(I)}{\left[u'(I)\right]^3} > 0$, and then

[9] Refer to Thiele and Wambach (1999) for this.

$$\frac{h''(u(I))}{h'(u(I))} = \frac{u'''(I)}{u''(I)} - 3\frac{u''(I)}{u'(I)} = -P(I) + 3R(I) \le 0,$$

where the inequality is satisfied because $h(u)$ is a concave function.

It is known by Kimball (1990) that the degree of absolute prudence, $P(I) \equiv -\dfrac{u'''(I)}{u''(I)}$, is a good measure for the sensitivity of the optimal choice of decision variables to risk in a two-period model with future uncertainty. For example, it affects not only the consumer's strength of the precautionary saving motive under income uncertainty, but also the certainty equivalence for income risk and investment level in risky assets.[10] Moreover, as in Thiele and Wambach (1999), one can derive that the condition of $P(I) \ge 3R(I)$ is equivalent to

$$\frac{I}{R(I)} \cdot \frac{dR(I)}{dI} = I\left[R(I) - P(I)\right] \le -I \cdot 2R(I) = 2 \times \frac{I}{u'(I)} \cdot \frac{u'(I)}{dI},$$

where the left term indicates the income elasticity of absolute risk aversion and the right term is two times the income elasticity of the marginal utility. Based on this reformulation, we can see that when income increases, if the effect of changes in the degree of absolute risk aversion is dominated by two times the effect of decreases in the marginal utility, then the concavity condition is satisfied.

Our concavity condition can be satisfied for the constant relative risk aversion (CRRA) and the hyperbolic absolute risk aversion (HARA) utility functions under some conditions. First, consider the CRRA utility function such as $u(I) = \dfrac{I^{1-r} - 1}{1-r}$ where r, $I > 0$. Since simple algebra yields $R(I) = \dfrac{r}{I}$ and $P(I) = \dfrac{1+r}{I}$, the condition that $P(I) \ge 3R(I)$ (or equivalently, the concavity of $\dfrac{1}{u'(I)}$ in $u(I)$) is satisfied when $r \le \dfrac{1}{2}$. Thus, the CRRA utility case shows that our concavity condition can be satisfied when the agent's degree of relative risk aversion is sufficiently small. Second, consider the HARA utility function such as $u(I) = \dfrac{1-r}{r}\left(\dfrac{\beta}{1-r}I + \eta\right)^r$ where $\beta > 0$ and $\dfrac{\beta}{1-r}I + \eta > 0$. Thus, since

[10] See Kimball (1990) for detailed explanations.

$R(I) = \dfrac{\beta}{\frac{\beta}{1-r}I + \eta}$ and $P(I) = \dfrac{\beta}{\frac{\beta}{1-r}I + \eta} \times \dfrac{r-2}{r-1} = \dfrac{r-2}{r-1}R(I)$, the condition of $P \geq 3R$ (or equivalently the concavity of $\frac{1}{u'(I)}$ in $u(I)$) is satisfied when $\frac{1}{2} \leq r < 1$.[11]

Proposition 2. Suppose that (PC) binds at the optimum. (a) If $\frac{1}{u'(I)}$ is concave in $u(I)$, then we have $C_w(a, w) < 0$ for any $a \in A_b(w)$. However, (b) if $\frac{1}{u'(I)}$ is convex in $u(I)$ for all I, then the sign of $C_w(a, w)$ is ambiguous.

Proof. It is trivial that the second term in equation (5) is negative because $u(I)$ is increasing and strictly concave in I. (a) Since (PC) binds at the optimum and $\frac{1}{u'(I)}$ is concave in $u(I)$, the first term in equation (5) is non-positive by Lemma 2. Therefore, we have $C_w(a, w) < 0$ for any $a \in A_b(w)$. (b) However, if $\frac{1}{u'(I)}$ convex in $u(I)$ for all I, then the first term in equation (5) is non-negative by Lemma 2. However, because as seen earlier, the second term, $\mu\mathbb{E}\left[-R\left(\hat{s}(x;\, a,\, w) - a\right)\right]$, is definitely negative, there is some confusion regarding the sign of $C_w(a, w)$. Q.E.D.

Proposition 2(a) shows that when (PC) binds at the optimum, an increase in the agent's wealth decreases the agency cost. Our results in Proposition 2 can be partially understood by applying the arguments of Kadan and Swinkels (2013) to our model. Given the optimal contract $\hat{s}(x;\, a,\, w)$, when the principal gives the agent one more util, the agency cost increases by $\mathbb{E}\left[\dfrac{1}{u'\left(\hat{s}(x;\, a,\, w) - a\right)}\right]$. Thus, when the agent's wealth increases by one unit, since the reservation utility level increases by $u'(a+w)$, the agency cost increases by

[11] Consider the condition that $\frac{1}{u'(I)}$ is convex in $u(I)$. This convexity condition is equivalent to that $P(I) \leq 3R(I)$ for all I. Note that the CRRA utility function $u(I) = \frac{I^{1-r} - 1}{1-r}$ qualifies when $r \geq \frac{1}{2}$, and that the HARA utility function $u(I) = \frac{1-r}{r}\left(\frac{\beta}{1-r}I + \eta\right)^r$ qualifies when $r \leq \frac{1}{2}$ or $r > 1$.

$$u'(\alpha + w)\mathbb{E}\left[\frac{1}{u'(\hat{s}(x;\,a,\,w) - a)}\right] - 1$$
$$= u'(\alpha + w)\left\{\mathbb{E}\left[\frac{1}{u'(\hat{s}(x;\,a,\,w) - a)}\right] - \frac{1}{u'(\alpha + w)}\right\},$$

which is identical to the first term in equation (5). Thus, if $\frac{1}{u'(I)}$ is concave in $u(I)$, the first term is non-positive by Lemma 2.

Next, since the agent's utility function is strictly concave in income, an increase in the agent's wealth makes the positive effects on the incentive provision by alleviating (IC). To see this, let $s(x;\,a,\,w) = \hat{s}(x;\,a,\,w) - w$. Note that the expectation $\mathbb{E}\left[u'(s(x;\,a,\,w) + w - a)\right]$ in (IC) plays a role of the marginal cost of effort. Hence, since an increase in w decreases the marginal cost to induce the same effort, it makes the incentive problem easier. By this incentive provision effect, consequently, the agency cost decreases as the agent's wealth increases. This effect is captured by the second term in equation (5). As a result, as the agent's wealth increases, the agency cost decreases by combining the above two effects.

However, Proposition 2(b) shows that as in most literature dealing with the effects of the agent's wealth, even if the convexity of $\frac{1}{u'(I)}$ with respect to $u(I)$ is assumed, the result similar to the previous ones cannot be derived in our model. The crucial reason is that an increase in wealth decreases the agency cost by lowering the marginal cost of effort. Even if the first term in equation (5) is positive by the convexity of $\frac{1}{u'(I)}$ in $u(I)$, since the second term is always negative, the total effect of wealth on the agency cost is ambiguous.

As the agent's wealth w increases, the agency cost decreases when (PC) does not bind by Proposition 1 and also it does when (PC) binds under the condition that $\frac{1}{u'(I)}$ is concave in $u(I)$ by Proposition 2(a). Since (PC) binds or not at the optimum, integrating Propositions 1 and 2(a) directly gives the following proposition.

Proposition 3. If $\frac{1}{u'(I)}$ is concave in $u(I)$ for all I, then we have $C_w(a,\,w) < 0$ for all $a > 0$.

Finally, we will analysis how an increase in the agent's wealth affects the principal's profit. Define the principal's profit maximization problem as

$$\pi(w) \equiv \max_{a \in A} \ \mu(a) - C(a, w).$$

Based on the result in Proposition 3, one can obtain the following proposition about the effects of the agent's wealth w on the principal's profit $\pi(w)$.

Proposition 4. An increase in the agent's wealth increases the expected profit of the principal if $\frac{1}{u'(I)}$ is concave in $u(I)$ for all I.

Proof. Consider the agent with wealth level w. We will show that it is true that $\pi'(w) > 0$, where $\pi(w) = \mu(a^*) - C(a^*, w)$ and a^* is the optimal effort level to solve the principal's profit maximization problem. Note that we already know that if $\frac{1}{u'(I)}$ is concave in $u(I)$, $C_w(a, w) < 0$ for any a, implying $C_w(a^*, w) < 0$. Thus, differentiating the profit function $\pi(w)$ and using the envelope theorem yield

$$\pi'(w) = -C_w(a^*, w) > 0.$$

This completes the proof. Q.E.D.

Proposition 4 shows that if $\frac{1}{u'(I)}$ is concave in $u(I)$, then an increase in the agent's wealth results in increasing the principal's profit. This result directly comes from the one of Proposition 3. One can see from Proposition 3 that as the agent's wealth increases, the agency cost decreases for any effort level under the condition that the inverse of the marginal utility function is concave in the utility function. If this conclusion is applied to when the agent takes the optimal effort level to maximize the principal's profit, we can obtain the result of Proposition 4. Therefore, an increase in wealth is beneficial to the principal under that concavity condition.

IV. Conclusion

In the principal-agent problem where the cost of effort is money, we examine how the agent's wealth affects the agency cost and the principal's profit. In particular, we deal with those effects in the case that the cross derivative between income and effort is positive. In contrast to the previous results, an increase in the agent's initial wealth makes the incentive problem easier by decreasing the marginal cost of effort in our model. Therefore, our results are beyond the scope of the existing ones.

We show that either if the participation constraint is non-binding at the optimum or if the inverse of the marginal utility function is concave with respect to the original utility function when it is binding, then an increase in the agent's wealth decreases the agency cost for any effort. This result implies that if the agent's preferences with respect to income satisfies such a concavity, the agency cost decreases for any effort as the agent's wealth increases, implying that the principal prefers rich agent to poor one. Furthermore, we show that as in most literature, even if the inverse of the marginal utility function is convex with respect to the utility function, the effect of the agent's wealth on the agency cost is ambiguous in our model.

We believe that our results can apply to the general case that the agent's utility for income and effort has the positive cross derivative. Nevertheless, our results are somewhat limited in that ours are derived under the assumption that using the first order approach is valid in our model. Actually, the conditions for justifying the first order approach are restrictive. The issue on how an increase in the agent's wealth affects the agency cost without the assumption remains to the future researchers.

(Submitted Jan 13, 2025; revised Feb 13, 2025; accepted Feb 17, 2025)

References

Chade, H., and Vera de Serio, V. N., "Wealth effects and agency costs." *Games and Economic Behavior 86*(2014): 1-11.

Eeckhoudt, L., and Gollier, C., "The impact of prudence on optimal prevention." *Economic Theory 26*(No. 4 2005): 989-994.

Fagart, M., and Fluet, C., "The first-order approach when the cost of effort is money." *Journal of Mathematical Economics 49*(No. 1 2013): 7-16.

Kadan, O., and Swinkels, J. M., "On the moral hazard problem without the first-order approach." *Journal of Economic Theory 148*(No. 6 2013): 2313-2343.

Kimball, M. S., "Precautionary saving in the small and in the large."

Econometrica 58(No. 1 1990): 53-73.

Jullien, B., Salanie, B., and Salanie, F., "Should more risk-averse agents exert more effort?" *The Geneva Papers on Risk and Insurance Theory 24*(No. 1 1999): 19-28.

Jung, J. Y., "Effects of changes in preferences in moral hazard problems." *Journal of Economic Theory 205*(2022): 105527.

Jung, J. Y., "Effects of Wealth and Its Distribution on the Moral Hazard Problem." *Seoul Journal of Economics 30*(No. 4 2017): 487-502.

Peter, R., "Who should exert more effort? Risk aversion, downside risk aversion and optimal prevention." *Economic Theory 71*(No. 4 2021): 1259-1281.

Thiele, H., and Wambach, A., "Wealth Effects in the Principal Agent Model." *ournal of Economic Theory 89*(No. 2 1999): 247-260.