Recent Applications of Generalized Instrumental Variable Models

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This study comprehensively reviews recent developments in the application of the generalized instrument variable (GIV) framework introduced by Chesher and Rosen (2017, *Econometrica*). The GIV framework effectively derives sharp bounds (equivalent to identified sets) in incomplete models. Focusing on limited dependent variable models with endogeneity, this study demonstrates the application of general identification results to obtain the identified set in specific settings. Moreover, practical implementation challenges that may arise are discussed, and potential strategies for overcoming them are highlighted in empirical research.

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JEL Classification: C10, C26, C50

I. Introduction

An instrumental variable method, which focused on estimating demand and supply curves of flaxseed, was first proposed by Wright (1928). The author emphasized that exogenous variations are necessary to identify the structural relationship between the outcome and the explanatory variables. For instance, in the demand and supply system,

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price and demanded quantity have simultaneous causality; thus, regressing quantity on price does not identify either the demand or supply curves. To determine demand, variations in the supply schedule are necessary, maintaining a constant demand curve. These exogenous variations can be provided by an observable variable, referred to as an *instrumental variable* (IV), which is excluded from the structural demand function but is correlated with the price, thereby affecting the demand. IV methods have long been popularized in applied studies, not only in economics but also in many other disciplines for causal identification.

IVs are straightforward to employ in linear regressions when the outcome and endogenous regressors are continuously distributed. Two-stage least squares (TSLS) and generalized method of moments (GMM) estimators are commonly applied in most empirical studies. However, when dealing with limited dependent variables, such as binary or discrete responses, the structural outcome equation naturally becomes nonlinear. In this case, substituting the endogenous variables in the nonlinear regression with their fitted values from the first stage regression, such as TSLS, does not work. Wooldridge (2010) referred to this attempt as the "forbidden regression." If the endogenous explanatory variables are continuous, then the control function (CF) approach (Blundell and Powell 2003) is a commonly used remedy. Instead of replacing the endogenous variables with their predicted values from the first stage auxiliary regression, the CF approach includes the estimated unobserved heterogeneity (regression residuals in the context of linear models) from the first stage into the nonlinear outcome regression.

The CF approach cannot be applied in nonlinear models when endogenous variables are discrete because the regression residual in this case is not point identified. The CF approach relies on the *invertibility* of the first-stage regression, which is not satisfied in a nonlinear first stage.[1] The CF approach also exhibits other undesirable features. For instance, it requires a triangular structure in which full simultaneity between variables is ruled out. The invertibility condition

[1] Recently, Han and Kaido (2024) propose a partial identification strategy using a set-valued control function in the context of treatment effect models. Their framework allows for rich heterogeneity in treatment effects, thereby not nested in the GIV model framework.

restricts the unobserved heterogeneity to be scalar.[2] These limitations have led econometricians to develop more generally applicable methodologies within the IV framework.

Deviating from the triangular structure often fails to yield point identification. Econometricians previously believed that the model parameters were unidentified in this case, implying no useful methods are available to make inferences on the parameters. However, early studies by Charles Manski, such as Manski (1990), Manski and Pepper (2000), and Manski (2003), changed this perspective by investigating informative bounds on the unidentified parameters of interest. These attempts laid the foundation for the development of partial identification.

In the context of nonlinear IV models, Andrew Chesher and his coauthors have made significant contributions to the literature. Initially, separate investigations were undertaken to explore partial identification in each individual model. Specifically, the primary objective of such investigations is to obtain sharp bounds (equivalent to the identified set), which refer to the tightest possible bounds that exhaust all the information and restrictions imposed on the model. Chesher (2010) developed the bounds on structural parameters for threshold-crossing discrete response models and confirmed the sharpness of the bounds when the outcome is binary. Chesher and Smolinski (2012) focused on ordered outcome models and derived bounds on structural functions, which are sharp if either the outcome or the endogenous variable is binary. Subsequently, Chesher and Rosen (2017) introduced a generalized instrumental variable (GIV) model framework that works for a broad class of models using mathematical theory of random sets. Their framework nests the single equation IV models studied in Chesher's earlier work and allows for multidimensional unobserved heterogeneity.

The GIV framework can effectively obtain the identified sets without constructive proofs, which are often challenging and technically demanding. Chesher and Rosen (2017, CR17 henceforth) conducted an identification analysis of a highly general model under high-level conditions. This approach enables the framework to be applied to a

[2] More detailed discussions on the limitations of the CF approach can be found in Chesher (2009).

broad spectrum of problems. However, for a specific model, meticulously tailoring the framework to carry out identification analysis is crucial. This study provides a comprehensive overview of recent applications of the GIV model framework for limited dependent variable models in a non-technical manner, specifically for the applied audience.[3] Numerous studies have used the CR17's framework for such models. Chesher, Rosen, and Smolinski (2013) investigated multinomial choice models, and Chesher and Rosen (2014) studied random coefficient binary outcome models. Kim (2020) focused on endogenous count data models. Kim (2023) leveraged the results from CR17 to derive the sharp bounds on marginal distributions of latent durations in competing risks models with discretely measured durations. Chesher, Kim, and Rosen (2023) applied the framework to censored outcome models.

II. GIV model framework

Y and Z denote the observed endogenous and exogenous variables, respectively. U corresponds to a vector of unobserved factors representing unobserved heterogeneity. In the GIV models, the structural relationships among Y, Z, and U are determined by the structural function h, such that $h(Y, Z, U) = 0$ with probability 1. U is assumed to be scalar and $Y = (Y_1, Y_2)$, where Y_1 indicates a scalar binary outcome, and Y_2 refers to a vector of endogenous regressors. Then, the structural function is defined as $h(Y, Z, U) = Y_1 - 1[\alpha + Y_2'\beta + Z'\gamma \geq U]$. When $h(Y, Z, U) = 0$, the model can be rewritten as $Y_1 = 1[\alpha + Y_2'\beta + Z'\gamma \geq U]$. When U is standard normally distributed and is independent of Z, the model becomes a standard probit model with endogeneity. One crucial feature of such a model is that it is incomplete in the sense that, given the values of Z and U, the Y values are not uniquely determined.

Incompleteness is the consequence of imposing less restrictive assumptions in the model. However, they often fail to identify the structural function h unless the unobserved variable U is uniquely determined by Y and Z.[4] In this endogenous probit model, a range of values of U can be compatible with specific values of Y and Z.

[3] For more technical, extensive reviews, see Chesher and Rosen (2020).

[4] If U is a single-valued function of Y and Z, then the CF approach can be used to identify the structural function assuming a triangular structure.

Suppose Y_2 and Z are scalar and $\gamma = 0$. Then, Z satisfies the exclusion restriction. Given $Y_2 = 1$, the value of Y_1 only depends on U and the model parameters. Any value of U larger than $\alpha + \beta$ delivers $Y_1 = 0$. When $U \leq \alpha + \beta$, Y_1 becomes 1. In this case, U is a set-valued function of the observed variables. Thus, the set of values of U that are compatible with (Y, Z) can be defined as $\mathfrak{U}(Y, Z; h)$, which is a random set given that Y and Z are random vectors. CR17 derives the identified set of the structural function h and the conditional distribution of U, given Z, using properties of random sets.

The key property used to derive the moment inequalities that characterize the sharp bounds is called Artstein's inequality (Artstein, 1983). S represents a closed subset of the support of U. In the probit example, the support of U is the real line $(-\infty, +\infty)$, and S is a closed interval $[a, b]$. For any S, the following inequality holds:

$$P[U \in S \mid Z = z] \geq P[\mathfrak{U}(Y, Z; h) \subseteq S \mid Z = z], \ \textit{for all } z.$$

In the probit example, the left-hand side (LHS) of the inequality is computed using an integral $\int_a^b \phi(u)\, du$, where $\phi(\cdot)$ denotes the standard normal density function. The right-hand side (RHS) is calculated by determining the Y values producing $\mathfrak{U}(Y, Z; h) \subseteq [a, b]$, given that Z is irrelevant for determining $\mathfrak{U}$. This inequality must hold for any closed subset S. Thus, in general, an infinite number of inequalities are required to define the identified set. An innovation proposed by CR17 enables us to work on a smaller (and possibly finite) collection of S, referred to as *core determining* sets first proposed by Galichon and Henry (2011), to obtain sharp bounds.

The collection of core determining sets can be obtained by considering the support of the random set $\mathfrak{U}(Y, Z; h)$. In the probit example with binary variables, Y can take four possible values, (0, 0), (0, 1), (1, 0), and (1, 1). Hence, the support of $\mathfrak{U}$ comprises four intervals, as shown in Table 1. Then, any connected unions of these intervals are core determining sets, except $(-\infty, +\infty)$ for which Artstein's inequality trivially holds with equality. The core determining sets must be determined by the researcher in each model at hand, often a challenging task. The following section illustrates the identification results of a few specific models using Artstein's inequality and core determining sets.

TABLE 1
SUPPORT OF THE RANDOM SET $\mathcal{U}$ IN THE PROBIT EXAMPLE

Y	$\mathcal{U}$
(0,0)	$(\alpha, +\infty)$
(0,1)	$(\alpha+\beta, +\infty)$
(1,0)	$(-\infty, \alpha]$
(1,1)	$(-\infty, \alpha+\beta]$

III. Recent applications

This section provides comprehensive illustrations of identification analysis using the GIV model framework for a binary response model, a count data model, and censored outcome model with endogeneity.

A. Binary response IV model

A generalized version of the example model used in the previous section is considered as follows:

$$Y_1 = 1\,[g(Y_2) \geq U], \quad Z \perp\!\!\!\perp U$$

Chesher (2010) explored this model, in which the proof of the bound sharpness is constructive. Sharp bounds are derived using the GIV framework without a constructive proof. The threshold function g is non-parametrically specified; hence, U can be normalized and uniformly distributed in the unit interval [0, 1] without loss of generality.[5] To simplify the exposition, suppose Y_2 is binary and $g(0) \leq g(1)$. Table 2 shows the four intervals of the U random set $\mathcal{U}(Y\,;\,g)$, namely, $[0, g(0)]$, $[0, g(1)]$, $[g(0), 1]$, and $[g(1), 1]$, in its support. These intervals are core determining sets that characterize the sharp bounds on $g(0)$ and $g(1)$. $[0, g(1)]$ contains $[0, g(0)]$, and their union is expressed as $[0, g(1)]$. Similarly, $[g(0), 1]$ includes $[g(1), 1]$, and their union is $[g(0), 1]$. The union of $[0, g(0)]$ and $[g(0), 1]$ is [0, 1], which is trivial. Similarly, all other combinations of the four intervals are either trivial or unconnected

[5] Let the true model be $Y_1 = 1[g^*(Y_2) \geq \varepsilon]$ and let $F(\cdot)$ be the cumulative distribution function (CDF) ε. Then $Y_1 = 1[F(g^*(Y_2)) \geq F(\varepsilon)] = 1[g(Y_2) \geq U]$, which is observationally equivalent to the true model.

TABLE 2

VALUES OF Y THAT DELIVER THE U RANDOM SET

$\mathfrak{U}$	Y
$[0, g(0)]$	$(0, 0)$
$[0, g(1)]$	$(0, 0), (0, 1)$
$[g(0), 1]$	$(1, 0), (1, 1)$
$[g(1), 1]$	$(1, 1)$

(*e.g.*, the union of $[0, g(0)]$ and $[g(1), 0]$). Therefore, applying Artstein's inequality to the four intervals deliver the identified set of the threshold function g.

Under independence restriction between Z and U, the LHS probability of Artstein's inequality $P[U \in S \mid Z = z] = P[U \in S]$ is simply the length of the interval S given the uniform distribution of U. The RHS probability $P[\mathfrak{U}(Y ; g) \subseteq S \mid Z = z]$ is computed by summing up the probabilities of the values of Y that deliver the U random set in the interval S. For instance, the interval $[0, g(1)]$ contains $[0, g(0)]$ delivered by $Y = (0, 0)$ and $[0, g(1)]$ provided by $Y = (0, 1)$. Therefore, $P[\mathfrak{U}(Y ; g) \subseteq [0, g(1)] \mid Z = z] = P[Y = (0, 0) \mid Z = z] + P[Y = (0, 1) \mid Z = z]$. The values of Y that deliver the U random set contained in each interval are displayed in Table 2.

Given these results, deriving the sharp bounds on $g(0)$ and $g(1)$ is straightforward. For $g(0)$, we only need Artstein's inequalities with two intervals, $[0, g(0)]$ and $[g(0), 1]$, as follows:

$$P[U \in [0, g(0)]] \geq P[\mathfrak{U}(Y; g) \subseteq [0, g(0)] \mid Z = z] \rightarrow$$
$$g(0) \geq P[Y = (0, 0) \mid Z = z],$$

$$P[U \in [g(0), 1]] \geq P[\mathfrak{U}(Y; g) \subseteq [g(0), 1] \mid Z = z] \rightarrow$$
$$1 - g(0) \geq P[Y = (1, 0) \text{ or } (1, 1) \mid Z = z].$$

These inequalities hold for all z so the sharp bounds on $g(0)$ is expressed as follows:

$$\sup_z P[Y = (0, 0) \mid Z = z] \leq g(0) \leq 1 - \sup_z P[Y = (1, 0) \text{ or } (1, 1) \mid Z = z].$$

Likewise, the sharp bounds on $g(1)$ is constructed as follows:

$$P[U \in [0,\ g(1)]] \geq P[\mathfrak{U}(Y; g) \subseteq [0,\ g(1)] \mid Z = z] \rightarrow$$
$$g(1) \geq P[Y = (0,\ 0)\ or\ (0,\ 1) \mid Z = z],$$

$$P[U \in [g(1),\ 1]] \geq P[\mathfrak{U}(Y\ ;\ g) \subseteq [g(1),\ 1] \mid Z = z] \rightarrow$$
$$1 - g(1) \geq P[Y = (1,\ 1) \mid Z = z].$$

Collecting the inequalities for all z, the sharp bounds on $g(1)$ is derived as follows:

$$\sup_z P[Y = (0,\ 0)\ or\ (0,\ 1) \mid Z = z] \leq g(1) \leq 1 - \sup_z P[Y = (1,\ 1) \mid Z = z].$$

Given the identification results, estimation and inference are relatively straightforward. The upper and lower bounds are easily estimated by taking maximums of empirical probability masses. The sharp bounds are constructed as an intersection of multiple bounds from each value of z. Therefore, the inference can be made using the method in Chernozhukov, Rosen, and Lee (2013), readily available in statistical packages such as Stata.

B. Count data IV model

The binary choice model investigated in the previous example can be extended to accommodate other discrete outcomes such as ordered outcomes and count data. Kim (2020) derived the identified set in a nonparametric, nonseparable count data IV model. His results can be applied to ordered choice models because he formulated the count data model as a threshold-crossing ordered outcome model. For any non-negative integer k, the model is specified as follows:

$$Y_1 = k \quad if \quad p_k(Y_2) \leq U < p_{k+1}(Y_2), \quad Z \perp\!\!\!\perp U,$$

where U is normalized and uniformly distributed in the unit interval and $p_0(Y_2) = 0$. For modelling count data, generalized method of moments (GMM)-based approaches such as Mullahy (1997) and Windmeijer and Santos Silva (1997) are widely used. However, their approaches do not work under this threshold crossing model structure because the moment conditions they impose under strong separability are not satisfied.

A binary Y_2 is considered for exposition purposes. Then, the support of the U random set exhibits a collection of many intervals as follows:

$$Supp\,(\mathfrak{U}) = \{[p_k\,(y_2),\ p_{k+1}\,(y_2)] : k \in \{0, 1, 2, \cdots\},\ y_2 \in \{0, 1\}\}.$$

All the connected unions of elements in $Supp\,(\mathfrak{U})$, denoted as $\mathfrak{U}^*$, are core determining sets. $Supp\,(\mathfrak{U})$ already provides an infinite number of moment inequalities; hence, it is computationally infeasible to deal with. Even with a bounded count outcome, the number of elements in $\mathfrak{U}^*$ explosively increases as the upper bound of Y_1 and the support of Y_2 increase. To realize a computationally tractable model, Kim (2020) suggested the use of a subset of $\mathfrak{U}^*$, which delivers an outer region (a set that nests the identified set).

$$Q = \{[0,\ p_k\,(y_2)],\ [p_k\,(y_2),\ 1] : k \in \{0, 1, 2, \cdots\},\ y_2 \in \{0, 1\}\}.$$

The number of elements in Q increases more gradually than $\mathfrak{U}^*$, identifying computationally feasible structural functions. He further showed that the bounds obtained using the suggested subset Q are sharp under some shape restrictions.[6]

Analogous to the binary outcome case, every threshold value is set-identified by moment inequalities. However, for elements in Q, computing the RHS of Artstein's inequality, $P\,[\mathfrak{U}\,(Y\ ;\ h) \subseteq S \mid Z = z]$, is not as straightforward. Consider deriving bounds on $p_5(1)$, given $y_2 = 1$, any $y_1 \leq 5$ provides a U random set that is contained in $[0,\ p_5(1)]$. A key difficulty arises when determining what values of y_1 provide a U random set that lies within $[0,\ p_5(1)]$ when $y_2 = 0$ because the ordering among the threshold values, given different values of y_2, must be known. A particular ordering is considered, as follows: $p_7(0) \leq p_5(1) \leq p_8(0)$. Then, given $y_2 = 0$, any $y_1 \leq 7$ provides a U set contained in the interval. The application of Artstein's inequality yields the following:

[6] Kim (2020) confirmed that under the conditions of i) monotonicity: $p_k(0) \leq p_k\,(1)$ or *vice versa* and ii) complete separability: max $\{p_k\,(0),\ p_k(1)\} \leq$ min $\{p_{k+1}(0),\ p_{k+1}(1)\}$, the set Q is core-determining. Many parametric count data models satisfy monotonicity. Complete separability indicates that the impact of Y_2 on the threshold values is close to zero. We parameterize the threshold functions as $p_k\,(y_2) = F\,(k,\ \exp(\alpha + \beta y_2))$, where F is a CDF that belongs to a parametric family satisfying monotonicity. In applied studies, whether the identified set of β includes zero is often a primary concern. The set Q offers the sharp criterion to evaluate the values of β around zero.

$$p_5(1) \geq P[\mathfrak{U}(Y;h) \subseteq [0,\ p_5(1)] \mid Z = z] = P[y_1 \leq 5 \cap y_2 = 1 \mid z] + P[y_1 \leq 7 \cap y_2 = 0].$$

Similarly, the upper bound is obtained as follows:

$$1 - p_5(1) \geq P[\mathfrak{U}(Y\ ;\ h) \subseteq [p_5(1),\ 1] \mid Z = z] = P[y_1 \geq 6 \cap y_2 = 1 \mid z] + P[y_1 \geq 8 \cap y_2 = 0].$$

The bounds are derived given an ordering. For any specific structural function h^*, the ordering among the threshold values is determined. If h^* satisfies all the moment inequalities, then it belongs to the identified set (or an outer region). Therefore, under a nonparametric specification of h, all the possible orderings must be considered for identification. The number of orderings rapidly becomes astronomical as the supports of Y_1 and Y_2 increase. Therefore, a parametric restriction, under which the ordering depends on a small number of parameters, is required in practical implementations for computational tractability. Kim (2020) imposed a Poisson and negative binomial restrictions in the application. Under parametric restrictions, the identified set can be computed by grid search.

C. Censored outcome model

Chesher, Kim, and Rosen (2023) considered Tobit-type censored outcome models, in which the latent outcome Y_1^* is only observed if it is greater than a fixed threshold value c as follows:

$$Y_1 = \max\{c,\ Y_1^*\}, \qquad Y_1^* = m(Y_2,\ Z,\ U),$$

where the function m is continuous and strictly increasing in U. The model does not impose independence between U and Y_2. Instead, a type of independence restriction on the conditional distribution of U given Z is imposed to obtain the identified set of the function m. The authors considered full independence, quantile independence, and conditional mean independence. This model is interesting in a theoretical point of view given that the censored outcome exhibits a probability point mass and continuous variations at the same time. Similar to the Gaussian Tobit model (Tobin 1958), which identifies the model parameters without scale normalization (required for parametric binary outcome models such as probit and logit), the IV-censored outcome model can potentially identify the structural function m.

The key identification idea is illustrated using a simple running example where the function m is a standard linear additive function: $Y_1^* = \alpha + \beta Y_2 + U$, the censoring threshold c is 0, and Z is an excluded instrument. An important aspect of this model is that the value of the unobserved variable U is identified by $Y_1 - \alpha - \beta Y_2$ when $Y_1 > 0$. If the outcome is censored ($Y_1 = 0$), then $Y_1^* = \alpha + \beta Y_2 + U \leq 0$ is known and therefore the value of U belongs to $(-\infty, -\alpha - \beta Y_2]$. The identified set of the model parameters is characterized by moment inequalities and equalities because the U random set is either singleton or a semi-infinite interval.

Two types of intervals are considered to derive the identified set. The first type is a finite interval $[t_1, t_2]$, where $t_1 \leq t_2$. This interval provides moment equalities when $t_1 = t_2$. Applying Artstein's inequality to the interval yields the following:

$$P[U \in [t_1, t_2] \mid Z = z] \geq P[t_1 \leq -\alpha - \beta Y_2 \leq t_2 \cap Y_1 > 0 \mid Z = z].$$

The case of $Y_1 = 0$ is excluded in the RHS of the inequality given that the U random set given $Y_1 = 0$ is semi-infinite and cannot be contained within $[t_1, t_2]$. The second type of inequality is a semi-infinite interval $(-\infty, t]$. Then, Artstein's inequality provides the following:

$$P[U \in (-\infty, t] \mid Z = z] \geq P[Y_1 - \alpha - \beta Y_2 \leq t \mid Z = z].$$ [7]

The RHS of the inequalities is computed using data given (α, β). The LHS calculation depends on the conditional distribution restriction of U, conditional on Z. When $U|Z \sim N(0, \sigma^2)$, then $P[U \in [t_1, t_2] \mid Z = z] = \Phi(t_2/\sigma) - \Phi(t_1/\sigma)$ and $P[U \in (-\infty, t] \mid Z = z] = \Phi(t/\sigma)$. In this example, t, t_1, and t_2 can take any real value, resulting in an uncountably infinite number of moment inequalities characterizing the identified set. Therefore, in practical implementations, a finite number of partitions can be used in the real line, *i.e.*, using discrete values of (t, t_1, t_2).

[7] The RHS of the inequality can also be written as follows: $P[Y_1 > 0 \cap Y_1 - \alpha - \beta Y_2 \leq t \mid Z = z] + P[Y_1 = 0 \cap -\alpha - \beta Y_2 \leq t \mid Z = z]$.

If the researcher is not willing to impose full independence, then a weaker alternative is a quantile independence restriction, such as conditional median independence ($med\ (U|Z = z) = 0$). Under median restriction, $P[U \in (-\infty, 0] \mid Z = z] = P[U \in [0, \infty) \mid Z = z] = 0.5$ for all z. Hence, the following upper and lower bounds are obtained for 0.5:

$$P[Y_1 - \alpha - \beta Y_2 \leq 0 \mid Z = z] \leq 0.5 \leq 1 - P[Y_1 > 0 \cap Y_1 - \alpha - \beta Y_2 > 0].$$

This restriction type can be imposed for other quantiles. Another possibility is the conditional mean independence restriction, $E[U|Z = z] = 0$. However, this restriction tends to deliver uninformative bounds.

This model can potentially identify the structural function especially when $m(Y_2, Z, t) > 0$ achieves probability 1 conditional on $Z = z$ for a large value of t. This case exists when Y_2 has a bounded support and m continuously increasing as t increases. However, this condition is difficult to verify in practice. Regardless of the identification status of the considered model, the framework of Chesher, Kim, and Rosen (2023) can robustly apply.

IV. Practical implementations and challenges

The identification results obtained via the GIV framework are highly potent. However, their practical implementation in empirical studies can pose significant challenges. This section discusses important challenges that arise when employing estimation and inference methods for partially identified parameters. In many instances, the computation of the identified set can be computationally intensive. Considering the illustrative binary outcome model presented in the preceding section, even without exogenous variables, the computation of the identified set becomes highly burdensome when Z is continuously distributed. The RHS of the conditional moment inequalities derived in the previous section represents the joint conditional probability that must be estimated from data, conditional on specific values of Z. A nonparametric estimation method, such as kernel and sieve estimation, can be employed. Then, as the number of variables in Z increases, the curse of dimensionality occurs.

A more general linear Tobit model with included exogenous variables is considered as follows:

$$Y_1^* = \alpha + Y_2'\beta + Z'\gamma + U,$$

where Z is continuously distributed. Then, the moment inequalities must be computed on many different combinations of Z values, which can result in a very slow rate of convergence and astronomical computational burden. Chesher, Kim, and Rosen (2023) addressed this problem by discretizing the supports of Z. Alternatively, two indices, $Z_1'\gamma_1$ and $Z_2'\delta$ where Z_1 is a vector of included exogenous variables and Z_2 is a vector of excluded variables, can be considered, following the idea proposed by Lee and Chen (2019). This method does not, in general, deliver the sharp bounds but greatly reduces the dimensionality of the problem when the number of variables in Z is large, because the method relies only on two linear indices regardless of the dimensionality of the problem.

Another important problem is the number of structural functions that need to be tested using moment inequalities. The RHS of moment inequalities are computed given a structural function h. They change their value for different structural functions. For instance, in the context of the exemplary binary choice model, the ordering between $g(y_2)$ matters. As the support of y_2 increases, the number of possible orderings increases rapidly.[8] In ordered choice and count outcome models, the increase becomes astronomical. Appropriate shape or parametric restrictions are necessary to achieve a computationally feasible problem. If parametric restrictions are imposed, then the RHS values of the inequalities depend on a vector of parameters θ. Moreover, the identified set is obtained via a brute-force method (*e.g.*, grid search). The number of grid points grows exponentially as $\dim(\theta)$ increases, requiring a more efficient algorithm. For instance, Kim (2020) and Chesher, Kim, and Rosen (2023) used a complete model that imposes more restrictive assumptions than the partially identifying model they considered to obtain a starting point for grid search. Subsequently, they explored around the starting point using a coarse grid to sketch the identified set, which is refined using a finer grid. The discovery of an algorithm that can rapidly determine the boundary of the identified set (or its convex hull if the identified set is not convex) would be a great

[8] Suppose Y_2 can take M discrete values. Then the number of possible orderings is $M!$.

innovation.

The final practical concern involves the inference methods. The econometric literature on inference methods for partially identified parameters or structural functions by moment inequalities has advanced significantly over the past two decades. A comprehensive survey on this topic is provided by Canay, Illanes, and Velez (2023). The inference problem tends to be posed as follows:

$$E_p[m\,(W_i;\,\theta)] \leq 0,$$

where $W_i = (Y_i, Z_i)$ represents the i.i.d. observed variables, P indicates the probability measure of W_i, θ refers to a vector of model parameters, and $m(\cdot\ ;\ \cdot)$ denotes the known moment function given θ. The identified set of θ is defined as follows:

$$\vartheta^* = \{\theta \in \Theta : E_p\,[m\,(W_i;\ \theta)] \leq 0\}.$$

An inference problem focuses on finding a confidence set C_n that asymptotically satisfies the following:

$$P\,[\theta \in C_n] \geq 1 - \alpha, \qquad \alpha \in (0,\ 0.5)$$

for a pre-specified significance level α as the sample size n goes to ∞.

Among many available methods, three methods are implemented in Stata: i) Chernozhukov, Lee, and Rosen (2013), implemented by Chernozhukov, Kim, Lee, and Rosen (2015); ii) Andrews and Shi (2013), implemented by Andrews, Kim, and Shi (2017); iii) Cox and Shi (2023), implemented by Gong, Cox, and Shi (2024). The first two methods are widely used and easy to implement. However, they provide a confidence set for the joint identified set of the entire parameter vector, thereby suffering from *projection conservatism*.[9] The third method is developed to overcome this problem, providing non-conservative inference for a subset of θ (so-called subvector inference). This method is relatively easy to use, free from tuning parameters, and computationally tractable.

[9] When the confidence interval of an individual parameter is obtained by projecting the joint confidence set, the resulting confidence interval can be very conservative (meaning that it can be very wide). This problem becomes more severe as the number of parameters in the model increases.

However, this method may not work effectively for models with many moment inequalities.

Other papers propose subvector inference methods, such as Bugni, Canay, and Shi (2017), Belloni, Bugni, and Chernozhukov (2018), and Kaido, Molinari, and Stoye (2019). Among them, Belloni, Bugni, and Chernozhukov (2018) explored many moment inequalities and computationally tractable when the self-normalized critical value is used. However, their method may not be as robust as that of Cox and Shi (2023) because the method relies on the least favorable critical value. These models can be more widely used in empirical research, following innovations in terms of computational tractability of estimation and inference procedures on the identified set.

VI. Conclusions

This study assessed the recent application of the GIV model framework to obtain sharp bounds on model parameters/structural functions. The GIV framework can effectively derive the sharp characterization of the identified set in weakly restrictive IV models. Furthermore, it can be applied beyond the context of IV models discussed in this study. For instance, the framework can deliver the identified set in incomplete auction models proposed by Haile and Tamer (2003), in a game-theoretic entry model with multiple equilibria considered by Kline and Tamer (2016) and Chesher and Rosen (2020), in dynamic empirical IO models (Berry and Compinani 2023), and in nonlinear panel models (Chesher, Rosen, and Zhang 2024).

The GIV model framework can be potentially used in empirical research because it covers a wide range of partially identifying models. However, the technical nature of the original paper by Chesher and Rosen (2017) can be a significant challenge for its potential users. The examples and discussions presented here aim to make the framework more accessible to applied researchers. The framework's ability to unify and extend traditional IV analyses offers significant promise for researchers seeking partial identification tools across diverse empirical contexts.

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