On the Theory of Labor Supply with Wage Rate Uncertainty

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Kim (1994) uses the uncertainty counterpart of price compensations to decompose the effect of increased wage uncertainty on labor supply into income and substitution terms. This paper complements Kim's work by formulating the uncertainty counterpart of income compensations commonly used in economic theory. Substitution and income effects based on income compensations are derived and compared to those derived by Kim. Necessary and sufficient conditions on preferences to sign these effects are also provided. (*JEL* Classification: D81)

I. Introduction

In the literature on labor supply under wage uncertainty, the effect of increased wage uncertainty is frequently separated into "Slutsky-like" income and substitution effects.[1] While intuitively appealing, this decomposition is *ad hoc* since it is not based on any identifiable compensation method. Recently, Kim (1994) provided a decomposition based on a sound theoretical foundation. Specifically, Kim adjusts the expected wage rate to compensate for the increase in wage rate uncertainty prior to the resolution of uncertainty so as to keep expected utility constant.

Kim's compensation method is in the spirit of "price compen-

[1] "Slutsky-like" income and substitution effects were introduced by Block and Heineke (1973).

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sations".[2] Since the standard decomposition under certainty is based on the Hicks-Slutsky income compensation, and given the widespread interest in separating the effect of increased wage uncertainty into income and substitution effects, it would be valuable to provide a decomposition that is the uncertainty counterpart of Hicks-Slutsky decompositions. In this paper we derive the income and substitution effects of an increase in wage rate uncertainty on labor supply using income compensation. We adjust the non-labor (non-random) income so as to keep expected utility constant. Like Kim, the income compensation occurs prior to the resolution of uncertainty.

The two compensation methods decompose the same total effect,[3] and both keep expected utility at the level prior to the increase in wage uncertainty to obtain the substitution effect. We show that the wage-compensated substitution effect is composed of the income-compensated substitution effect and the substitution effect induced by the change in the expected wage rate. Kim has shown that wage-compensated income and substitution effects are controlled respectively by the magnitude and the behavior of endogenous partial risk aversion. We show that income-compensated income and substitution effects are controlled respectively by the behavior of expected marginal utility of income and the behavior of a measure of aversion to incremental-risk (called the incremental-risk premium) along a budget line.

The next two sections briefly review Kim's wage-compensated effects and derive the income-compensated effects. Section IV introduces the incremental-risk premium, relates it to the compensating risk premium, and shows how its behavior is related to the behavior of absolute risk aversion and of endogenous partial risk aversion. Section V and VI provide algebraic and geometric comparisons of income and substitution effects under the two compensation methods.

[2]See Hadar (1967) and Allen and Mishan (1967) for price compensations and their relationship to income compensations under certainty.

[3]Necessary and sufficient conditions to sign the total effect of increased wage uncertainty on labor supply have been derived in the literature, e.g. Tressler and Menezes (1980).

II. Kim's Wage-Compensated Income and Substitution Effects

Following Kim (1994), $U(c, L)$ is a thrice continuously differentiable von Neumann-Morgenstern utility function that is increasing in consumption (c), decreasing in labor (L) and strictly concave in (c, L). Consumption is $c=m+wL$, where m is exogenous, non-random, non-labor income and wage rate w is a positive random variable. The random variable w can be written as $w=\overline{w}+\gamma\varepsilon$, where $\overline{w}=Ew$ is the expected wage rate, ε is an actuarially neutral random variable, and γ is a positive scalar that serves as the spread parameter. An increase in γ induces a multiplicative mean-preserving spread of w. The individual chooses L to maximize expected utility, i.e.

$$\text{Max } V(L, \overline{w}, m, \gamma)=EU(m+wL, L). \tag{1}$$

The first and second order conditions for an interior solution are

$$V_L=E[wU_c+U_L]=0, \tag{2}$$

and $V_{LL}=E[w^2U_{cc}+2wU_{cL}+U_{LL}]<0$, where, and hereafter, subscripts denote partial derivatives.

Kim decomposes the total effect of an increase in γ on labor supply into income and substitution effects by adjusting the expected wage rate so as to keep expected utility constant. His wage compensation is given by

$$\left.\frac{d\overline{w}}{d\gamma}\right|_{V=\text{constant}}=-\frac{E(\varepsilon U_c)}{EU_c}. \tag{3}$$

Kim shows that his wage-compensated income and substitution effects depend on the magnitude and the behavior of the index of endogenous partial risk aversion, given by

$$P^*(Y)=-\frac{LU_{cL}(m+Y, L)+YU_{cc}(m+Y, L)}{U_c(m+Y, L)},$$

where $Y=wL$ is labor income. Specifically, the wage-compensated income effect (WIE) is given by

$$WIE=\frac{E(\varepsilon U_c)E[P^*(Y)-1]}{V_{LL}}, \tag{4}$$

and the wage-compensated substitution effect (WSE) is given by

$$WSE=\frac{E[P^*(Y)\phi U_c]}{V_{LL}}, \tag{5}$$

where $\phi = \varepsilon - \{E(\varepsilon U_c)/EU_c\}$. Kim's Propositions 1 and 2 establish that the sign of *WIE* is controlled by the magnitude of $P^*(Y)$ and the sign of *WSE* is controlled by the behavior of $P^*(Y)$.

III. Income-Compensated Income and Substitution Effects

The total effect of an increase in wage rate uncertainty on labor supply can alternatively be decomposed by adjusting non-random income to compensate for the increased wage uncertainty so as to keep expected utility constant. Specifically, the income compensation is given by

$$\left.\frac{dm}{d\gamma}\right|_{V=\text{constant}} = -\frac{LE(\varepsilon U_c)}{EU_c}. \tag{6}$$

Differentiating the first-order condition (2) with respect to m gives

$$\frac{\partial L}{\partial m} = \frac{E(U_{cL}+wU_{cc})}{V_{LL}}. \tag{7}$$

The income-compensated income effect (*IIE*) is the product of (6) and (7), i.e.

$$IIE = \frac{E(U_{cL}+wU_{cc})}{V_{LL}} \times \left\{-\frac{LE(\varepsilon U_c)}{EU_c}\right\}. \tag{8}$$

Since $V_{LL}<0$ and $-LE(\varepsilon U_c)/EU_c>0$, a necessary and sufficient condition for labor to be a normal good under wage uncertainty (i.e. $IIE>0$) is that $E(U_{cL}+wU_{cc})<0$, and this happens if and only if the expected marginal utility of income is decreasing in labor along the budget line $c=m+wL$. A sufficient condition for labor to be a normal good under wage uncertainty is that it is a normal good under certainty (i.e. $U_{cL}+wU_{cc}>0$). Or alternatively, labor is a normal good under wage uncertainty if the individual is both risk averse (i.e. $U_{cc}<0$) and multivariate risk averse (i.e. $U_{cL}<0$).[4]

Totally differentiating (2) with respect to γ and using (6) gives the income-compensated substitution effect (*ISE*)

[4]Let $u(x, y)$ be a von Neumann-Morgenstern utility function. An individual is multivariate risk averse if for $x_0<x_1$ and $y_0<y_1$, he prefers the lottery which gives outcomes (x_0, y_1) and (x_1, y_0) each with probability 1/2 to the lottery which gives outcomes (x_0, y_0) and (x_1, y_1) each with probability 1/2. A necessary and sufficient condition for this is that $u_{xy}<0$. See Theorem 1 in Richard (1975).

$$ISE = -\frac{1}{V_{LL}}\{E(\varepsilon U_c) + LE[(U_{cL} + wU_{cc})(\varepsilon - \frac{E(\varepsilon U_c)}{EU_c})]\}$$
$$= \frac{-E(\varepsilon U_c) + E[P^*(Y)\phi U_c]}{V_{LL}}. \tag{9}$$

IV. Incremental-Risk Premium and the Sign of the Income-Compensated Substitution Effect

In this section, we first introduce the incremental-risk premium (IRP) and relate it to compensating premium. We then show how the behavior of IRP is related to the behavior of absolute risk aversion and of endogenous partial risk aversion. Finally, we sign the income-compensated substitution effect in terms of the behavior of IRP.

Consider an individual with income $y = m + \beta z$, where m is sure income and z is an actuarially neutral random variable.[5] Let $u(y)$ be the individual's von Neumann-Morgenstern utility function. Suppose the risk βz cannot be insured but that increments in risk can be insured. What is the maximum amount that the individual is willing to pay to avoid an increase in risk? The conventional (equivalent and compensating) risk premiums are not designed to handle this problem since they pertain only to situations where the entire risk is insurable. To formulate a risk measure when only the incremental portion of an existing risk can be insured, consider the ratio $\triangle m/\triangle\beta$ where $\triangle m$ and $\triangle\beta$ are defined by $Eu(m+\beta z) = Eu(m+\triangle m+(\beta+\triangle\beta)z)$. As $\triangle\beta \to 0$, $\triangle m/\triangle\beta$ becomes

$$\rho(m) = -\frac{E[zu'(m+\beta z)]}{Eu'(m+\beta z)}.$$

We call ρ the *incremental-risk premium*. It is the maximum amount that the individual is willing to pay to avoid additional risk when existing risk cannot be insured. Equivalently, it is the minimum amount that the individual requires as compensation to assume the additional risk.

The compensating risk premium π^c is defined by $Eu(m+\beta z+\pi^c) = u(m)$. It is the minimum amount that the individual requires as

[5]The assumption that z is actuarially neutral is for notational convenience only.

compensation to assume the risk βz. Differentiating the preceeding identity with respect to β and evaluating at $\pi^c=0$ gives

$$\left.\frac{\partial \pi^c}{\partial \beta}\right|_{\pi^c=0} = -\frac{E[zu'(m+\beta z)]}{Eu'(m+\beta z)} = \rho\,. \tag{10}$$

That is, the derivative of the compensating risk premium with respect to the risk parameter measures aversion to incremental risk *only* when evaluated at $\pi^c=0$.[6]

To sign the income-compensated substitution effect, we require the multivariate counterpart of the incremental-risk premium. It is defined by

$$\rho(c,\ L) = \left.\frac{\partial m}{\partial \gamma}\right|_{EU=\text{constant}} = -\frac{LE[\varepsilon U_c(c,\ L)]}{EU_c(c,\ L)}. \tag{11}$$

We now show that the behavior of incremental-risk premium with respect to income is controlled by the behavior of absolute risk aversion ($A(c,\ L) = -U_{cc}/U_c$) with respect to income, while the behavior of incremental-risk premium per unit of labor with respect to labor is controlled by the behavior of endogenous partial risk aversion with respect to labor income.

***Proposition* 1**

(i) The incremental-risk premium $\rho(c,\ L)$ is decreasing (increasing) in c according as the absolute risk aversion $A(c,\ L)$ is decreasing (increasing) in c.

(ii) The incremental-risk premium per unit of labor $\rho(c,\ L)/L$ is decreasing (increasing) in L according as the endogenous partial risk aversion $P^*(Y)$ is decreasing (increasing) in Y.[7]

***Proof*:** See the Appendix.

It follows from this proposition that decreasing absolute risk aversion implies the amount of income required to compensate for a small increment in risk decreases as income increases and that increasing endogenous partial risk aversion implies a one percent increase in labor will result in a more than one percent increase in the amount of income required to compensate for the increase in

[6] ρ and the equivalent risk premium π^e are related by $\partial\pi^e/\partial\beta = [1-(\partial\pi^e/\partial m)]\rho$, which follows from (10) and the identity $\pi^c(m) = \pi^e(m+\pi^c(m))$, derived by Kimball (1990, p. 57).

[7] Note that $\partial(\rho/L)/\partial L \leq (\geq) 0$ if and only if the elasticity of ρ with respect to L is less (greater) than 1.

risk induced by the increase in labor.

From (9), a sufficient condition for the income-compensated substitution effect to be negative is that endogenous partial risk aversion $P^*(Y)$ is increasing in Y, assuming risk aversion. Proposition 2 provides a necessary and sufficient condition to sign the income-compensated substitution effect of wage uncertainty in terms of the behavior of incremental-risk premium.

***Proposition* 2**

The income-compensated substitution effect is negative (positive) if and only if the incremental-risk premium is increasing (decreasing) in L along the budget line $c=m+wL$, i.e.

$$ISE \begin{cases} \leq 0 \\ \text{or} \\ \geq 0 \end{cases} \text{if and only if } \left.\frac{\partial \rho}{\partial L}\right|_{c=m+wL} \begin{cases} \geq 0 \\ \text{or} \\ \leq 0. \end{cases}$$

Proof: See the Appendix.

We now consider how ρ changes with L along the budget line $c=m+wL$. From (11), $\rho=0$ at $L=0$ and $\rho>0$ for $L>0$. Hence, ρ must be increasing in L immediately to the right of $L=0$ and must be either everywhere increasing or non-monotone in L. Consequently, the income-compensated substitution effect cannot be uniformly positive, i.e. it is either uniformly negative or changes in sign.

V. Comparison of Wage- and Income-Compensated Income and Substitution Effects

A comparison of (4) with (8) and (5) with (9) identifies the difference between income and substitution effects under wage and income compensations. Specifically,

$$WIE-IIE=-\frac{E(\varepsilon U_c)}{V_{LL}}, \qquad (12)$$

$$WSE-ISE=\frac{E(\varepsilon U_c)}{V_{LL}}. \qquad (13)$$

From (3),

$$\frac{E(\varepsilon U_c)}{V_{LL}} = -\frac{E(\varepsilon U_c)}{EU_c} \times \left(-\frac{EU_c}{V_{LL}}\right) = \left(\frac{d\bar{w}}{d\gamma}\bigg|_{V=\text{constant}}\right) \times \left(\frac{dL}{d\bar{w}}\bigg|_{V=\text{constant}}\right), \quad (14)$$

where

$$\frac{dL}{d\bar{w}}\bigg|_{V=\text{constant}} = -\frac{1}{V_{LL}}\left(V_{L\bar{w}} + V_{Lm}\frac{dm}{d\bar{w}}\bigg|_{V=\text{constant}}\right) = -\frac{EU_c}{V_{LL}}$$

is the substitution effect of an increase in expected wage rate on labor. Thus, the key difference between the two kinds of compensation is reflected in the term $E(\varepsilon U_c)/V_{LL}$, which represents the change in labor induced by the change in the expected wage rate required to keep expected utility constant when wage uncertainty increases.

As $E(\varepsilon U_c)/V_{LL}$ is positive, (12) indicates that the wage-compensated income effect is smaller than the income-compensated income effect. Hence, if endogenous partial risk aversion $P^*(Y)$ is uniformly greater than unity (implying that labor is normal under wage compensation), then labor has to be normal under income compensation. Equation (13) implies that the wage-compensated substitution effect is larger than the income-compensated substitution effect. Hence, if $P^*(Y)$ is increasing in Y (implying that compensated labor supply decreases under wage compensation), then compensated labor supply must also decrease under income compensation.

VI. The Geometry

The difference between the two compensation methods becomes more apparent from their geometric representations. For a geometric analysis, we rewrite the labor supply model (1) in an equivalent form. Let $\bar{c} = m + \bar{w}L$ denote expected consumption. The individual's expected utility can be rewritten as $\mu(L, \bar{c}, \gamma) \equiv EU(\bar{c} + \gamma\varepsilon L, L)$. $\mu(\bullet)$ is a *derived utility function* representing preferences over labor supply-expected consumption bundles $(L, \bar{c})$. The assumptions about U imply that μ is decreasing in L and γ, increasing in $\bar{c}$, and strictly concave in $(L, \bar{c})$. The decision problem (1) is equivalent to

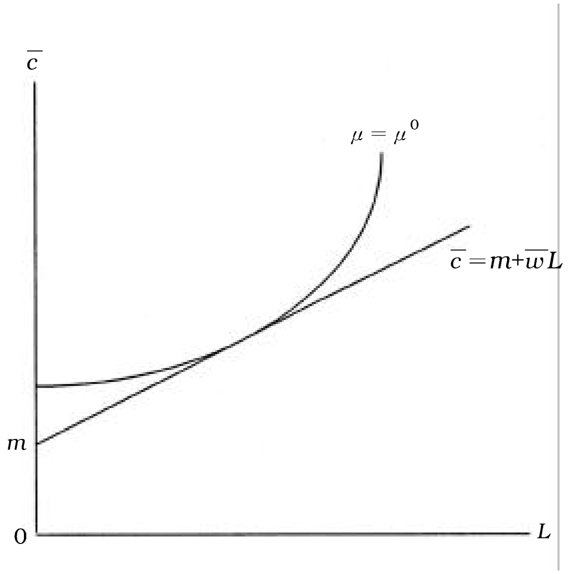

FIGURE 1
OPTIMAL LABOR SUPPLY UNDER WAGE UNCERTAINTY

$$\begin{aligned} &\underset{\{L,\bar{c}\}}{\text{Max}} \ \mu(L, \bar{c}, \gamma) \\ &\text{s.t.} \quad \bar{c} = m + \bar{w}L. \end{aligned} \tag{15}$$

We assume that (15) has an interior solution which satisfies the necessary conditions[8]

$$\frac{-\mu_L(L, \bar{c}, \gamma)}{\mu_{\bar{c}}(L, \bar{c}, \gamma)} = \bar{w} \text{ and } \bar{c} = m + \bar{w}L, \tag{16}$$

where $-\mu_L / \mu_{\bar{c}} = -E[\gamma \varepsilon U_c + U_L]/EU_c$ is the amount of sure consumption $\bar{c}$ required to compensate the individual for a unit increase in labor supply.[9] Geometrically, it is the slope of the derived indifference curve defined by $\mu(L, \bar{c}, \gamma) = \mu^0$. In Figure 1, the optimal (uncompensated) labor supply occurs where the derived indifference curve (i.e. the locus of pairs $(L, \bar{c})$ which give expected

[8]The dual to this formulation is to minimize $\bar{c} - \bar{w}L$ subject to $\mu(L, \bar{c}, \gamma) = \mu^0$. The relationship between the primal and dual problems leads to the Slutsky equation for an increase in wage uncertainty which decomposes the total effect into the income-compensated income effect given in (8) and the income-compensated substitution effect given in (9).

[9]It is easy to verify that the conditions in (16) are equivalent to (2).

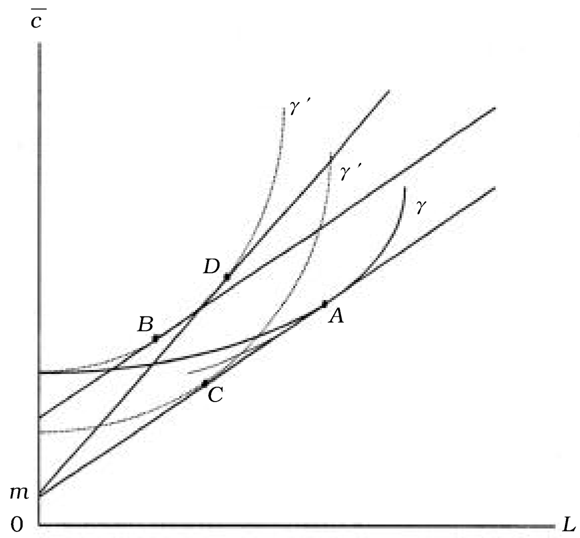

FIGURE 2

WAGE- AND INCOME-COMPENSATED INCOME AND SUBSTITUTION EFFECTS

utility μ^0) is tangent to the expected consumption constraint $\bar{c}=m+\bar{w}L$.

The income and substitution effects under both kinds of compensation are illustrated in Figure 2. The solid γ-indifference curve and the higher dashed γ'-indifference curve have the *same* expected utility but correspond to two different values of the risk parameter (i.e. prior to and after the increase in wage risk, $\gamma'>\gamma$).[10] The individual is initially at point A on the γ-indifference curve. The movement from A to C is the total effect under both wage and income compensations. The movement from A to B is the income-compensated substitution effect. It is obtained by shifting the expected income constraint upward so that it is tangent to the higher γ'-indifference curve (which has the same expected utility as that of the γ-indifference curve). The movement from A to D is the wage-compensated substitution effect. It is obtained by rotating the expected income constraint counter-clockwise through the point m

[10]The indifference map has two important properties: (i) any pair of γ- and γ'-curves having the same expected utility intersect only at $L=0$ where they are tangent; (ii) the vertical distance between such a pair of curves is the incremental-risk premium.

until it is tangent to the higher γ'-indifference curve. The movement from B to D along the higher γ'-indifference curve is the substitution effect induced by the change in the expected wage rate required to keep expected utility constant when wage uncertainty increases, given by (14).[11]

The movement from B to C is the income-compensated income effect. It is obtained by moving from the income-compensated optimal bundle B to the bundle C where the lower γ'-indifference curve is tangent to the initial expected consumption constraint. The movement from D to C is the wage-compensated income effect. It is obtained by moving from the wage-compensated optimal bundle D to the bundle C.

Appendix

***Proof of Proposition* 1:**

Differentiate (11) with respect to c and rearrange to get

$$\frac{\partial \rho}{\partial c} = -\frac{L}{EU_c} E\left[\left\{ \varepsilon - \frac{E(\varepsilon U_c)}{EU_c} \right\} U_{cc} \right] = \frac{L}{EU_c} E[(\phi U_c)A],$$

where ϕ is as defined in (5). Following the argument given by Kim (1994, p. 27), $E[(\phi U_c)A] < (>) 0$ if A is decreasing (increasing) in c. This establishes part (i).

For part (ii), replace c in (11) by $m+wL$ and differentiate to get

$$\frac{\partial}{\partial L}\left(\frac{\rho}{L}\right) = -\frac{1}{EU_c} E\left[\left\{ \varepsilon - \frac{E(\varepsilon U_c)}{EU_c} \right\} (wU_{cc}+U_{cL}) \right] = \frac{1}{EU_c} E[(\phi U_c)P^*(Y)].$$

Again, using the argument given by Kim, $E[(\phi U_c)P^*(Y)] < (>) 0$ if P^* is decreasing (increasing) in Y.

***Proof of Proposition* 2:**

From (11),

$$\left.\frac{\partial \rho}{\partial L}\right|_{c=m+wL} = -\frac{1}{EU_c}\{E(\varepsilon U_c)+LE[(U_{cL}+wU_{cc})(\varepsilon - \frac{E(\varepsilon U_c)}{EU_c})]\} = \frac{V_{LL}}{EU_c} ISE.$$

The proposition follows from this equation and the fact that $V_{LL} < 0$ and $EU_c > 0$.

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[11]Convexity of the induced indifference curve guarantees that the expected wage induced substitution effect is positive.

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