Technological Asymmetry, Externality, and Merger: The Case of a Three-Firm Industry

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We construct a model of three firms oligopoly with homogeneous goods and portray situations where firms fail to merge into monopoly, although such a merger maximizes aggregate profits. The degree of technological asymmetry and the effects of externalities determine the outcome via their effects on the profitability of a bilateral merger. There are situations when an inefficient firm, that cannot survive in a Cournot competition, obtains a positive payoff in the grand coalition. There are also cases when the efficient firm has a disadvantage to bargain.

Keywords: Externality, Technological asymmetry, Bilateral merger, Grand coalition, Bargaining.

JEL Classification: C71, D43, L13.

I. Introduction

Merger and acquisition are common in business place. If we turn the business pages of the daily newspapers and business magazines, we can notice that everyday some firms are merging with some other firms, and some firms are negotiating for merger. Still sometimes firms fail to merge. Different characteristics and

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business strategies may prevent them from merging. Of course, antitrust laws might prohibit horizontal merger. Then, if one assumes that antitrust laws are not applicable, at least to some oligopoly industries of our interest, one may then presume that oligopolists, producing homogenous goods, should always merge to avoid competition and prevent dissipation of profits. The nature of product market competition determines non-cooperative payoffs of the players. Since the monopoly payoff strictly dominates the oligopoly industry payoff, one may tempt to conclude that a grand merger (*i.e.*, a single coalition of all firms) should always take place, because firms can now share a larger profit. This paper examines this hypothesis and proves the invalidity of the statement. Even in a homogeneous good industry, firms may not successfully come up with forming a grand coalition. Therefore, the absence of any antitrust rules does not necessarily mean perfect or complete monopolization of the industry. The reason is that firms may fail to agree on a division of pie. In our paper this occurs when firms have asymmetric technologies and merger of a sub-group of firms creates externalities. In such a situation firms might find it even more profitable to stand outside the merger. In particular, externalities under merger occur when the product market is characterized by Cournot type competition. We show that when the effects of externalities are large enough because of some technological asymmetry, firms under quantity competition may fail to agree on a grand merger.

We construct a model of three firms. Hence, to the question of all firms merger, it is necessary that we discuss and understand the possibility of a bilateral merger in this context. Following the analysis of Salant, Switzer, and Reynolds (SSR) (1983), we know that when the firms hold identical and constant returns to scale technology, a bilateral merger is never feasible.[1] This is due to the pecuniary externality, as identified in the SSR paper. So, one purpose of the present paper is also to examine under what conditions cost asymmetries may overcome this pecuniary externality of Cournot competition.[2]

[1]See also Levin (1990) and Fauli-Oller (1997).

[2]Perry and Porter (1985) have shown that if merger is associated with some efficiency gain in the form of cost reduction, then merger of any size can be profitable. Farrell and Shapiro (1990) have extended the model to welfare analysis. Kabiraj and Chaudhuri (1999) have discussed the choice

We consider a cooperative model of coalition formation, and assume that firms have complete information about their own and rivals' strategies and payoffs under all possible contingencies. So, in the negotiation table, players bargain together with perfect communication. When negotiation starts, each one bargains for a larger share, each tries to convince the proposed partners that he can get more by going alone, or he has other better alternatives than to sign the contract on the proposed division. If they come to an agreement, it must be essentially a stable outcome in the sense that the constituent members will have no further incentives to deviate, unilaterally or as a subgroup. In game theory language, the proposed allocation must be in the 'core.'[3] An efficient allocation means that the sum of payoffs under coalition equals the maximum attainable industry payoff. One should also insist on a "fair" division in the sense that equally efficient firms should get equal payoffs, and relatively more efficient or strong firms should get larger payoffs. In the bargaining process, the disagreement payoff of a player is determined by its default payoff that it is expecting to get by going alone. We call this the disagreement or bargaining payoff. Quite naturally, the disagreement payoffs depend on the technological positions of the firms. Hence the technological asymmetry plays a crucial role in our analysis. We look at the core, giving each player its minimum amount guaranteed by the core. Then we apply the Nash bargaining solution to the residual, that is, we assume that the surplus that comes from coalition, is divided by Nash bargaining. Thus, under this division rule, each coalition partner derives its reservation payoff plus an equal share of the surplus generated by such a coalition. The net payoffs of the players differ to the extent they have different reservation payoffs. Given this rule of division, we portray situations when firms fail to form a grand merger.[4]

between cross-border mergers and inside-border mergers from the viewpoint of welfare of the local country.

[3]The core of a game consists of those utility (payoff) vectors which are feasible for the entire group of players and which cannot be blocked by any coalition.

[4]It should be mentioned that for parameter values for which the core does not exist, the Nash scheme will obviously not be possible. However, a merger might be impossible under the Nash scheme, and still it might be in the core.

existence of a feasible set of solutions for different possible coalitions. Our concern is to study how the values of different coalitions may be affected by the differences in technologies of the firms, and by the nature of product market competition. We define a rule for "fair division." Then we examine whether, under the given division rule, firms agree on a grand coalition. In that sense we have a positivist's approach to the problem. Because of the externalities, a firm's bargaining payoffs become larger than its non-cooperative payoffs when a subgroup forms a coalition. This gives some firms an extra edge while bargaining. Then there are situations when the group as a whole finds it difficult to satisfy the demand of all firms. This gives some other interesting points. Sometimes some firms might have more incentives to be outsiders rather than insiders. As a result there is a possibility that no merger at all will occur. There are also situations when, under non-cooperative competition, some firms cannot operate because of their technological inefficiency, and if these firms would not exist, the other firms could merge to share profits among themselves, but the existence of such inefficient firms might prevent the other firms to merge. We portray situations under which an inefficient firm that cannot survive in a Cournot competition, obtains a positive payoff in a grand coalition; this means the efficient firms are to pay to the inefficient firm if to form a grand merger.

Let us briefly explain the source of externality. To illustrate, consider linear cost functions with no fixed costs. Then, if merger of a subset of firms takes place, it does not matter whether firms operate on different plants (but using the same production technology) or on a single plant, we can treat the merged firm as a single competitor. So, if there are n firms in the industry, and out of them s $(s<n)$ firms form a merger, then effectively there are now $(n-s+1)$ firms in the industry. Therefore, under Cournot competition each of $(n-s)$ outsiders will now derive more than its initial non-cooperative payoff. This has negative effect to the profit of the merged firm. Because of this (negative) externality, a horizontal merger of any subset may not be privately profitable. To the extent merger of asymmetric firms means that some inefficient firms are dropped from operation, one efficient firm's payoff also goes up. If the degree of asymmetry is large enough, merger of a subgroup of firms becomes profitable.[5]

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The organization of the paper is the following. The second section describes the structure of the model. The third section provides the formation of mergers under all possible assumptions regarding technology asymmetry. The last section is a conclusion.

II. Model

Consider an industry for a homogeneous good, with three firms in the industry, 1, 2, and 3. Their production technologies are represented by constant marginal costs of production, c_1, c_2 and c_3, respectively. Without loss of generality, assume $0 \leq c_1 \leq c_2 \leq c_3$. Product market is characterised by Cournot competition.

Let the non-cooperative payoff under Cournot competition of the ith firm be π_i^N. If firms i and j merge together, the merged firm's payoff is denoted by π_{ij}^0, and the outsider k's payoff is π_k^0. It is always assumed that the merged firm uses the technology which corresponds to the lowest marginal cost available to its constituents.[6]

We assume that a bilateral merger will arise only if such a merger is privately profitable to its members. A bilateral merger between i and j is profitable if and only if

$$\pi_{ij}^0 > \pi_i^N + \pi_j^N \quad \forall i, j = 1,2,3; \; i \neq j. \tag{1}$$

Thus π_k^0 is defined only when condition (1) holds.[7]

Industry profit is maximized when all firms merge together. We call this a grand merger or grand coalition (a Pareto optimal situation). We denote this by G. The industry profit under G is the

[5] In case of price competition under homogeneous goods, only the efficient firm survives, and it does not depend on how many inefficient firms are in the industry. Hence under price competition, there is no such externality of merger as in the case of Cournot competition. This means that when the product market competition is given by Bertrand competition, firms can always form a grand merger. The possibility of merger under price competition with differentiated products is examined in Deneckere and Davidson (1985).

[6] If firm i and firm j merge together, then merged firm's technology (marginal cost of production) is $c_m = \min(c_i, c_j)$. If all firms merge together, then the merged firm has access to the technology, c_1.

[7] Suppose $c_i < c_j$. Then $\pi_k^0 > \pi_k^N$ if and only if $a - 3c_i + c_j + c_k > 0$ for $i \neq j \neq k$.

monopoly profit for c_1 technology. Let $\pi(c_1) = \pi^m$ be the payoff to the grand coalition. Our question is: Can we divide π^m among the players in such a way that the allocation is acceptable, and no firm will have any further incentive to leave the coalition? Let v_i be any allocation to the ith player under G. Then for G to be stable, following conditions must hold:

$$v_1 + v_2 + v_3 = \pi^m \tag{S1}$$

$$v_i + v_j \geq \max[\pi^0_{ij}, \pi_i^N + \pi_j^N];\; i, j = 1,2,3;\; i \neq j \tag{S2}$$

$$v_k \geq \begin{cases} \pi^0_k & \text{if (1) holds} \\ \pi_k^N & \text{otherwise} \end{cases} \tag{S3}$$

The conditions stated above have easy interpretation. The first condition is the division of profits under grand coalition. Any allocation satisfying (S1) is Pareto efficient in the sense that any reallocation implies that at least one firm is worse off. We call this the 'Pareto optimality' condition. The left hand side of (S2) is the sum of profits of any two firms, i and j, under G. If (1) is satisfied, by forming a bilateral merger among themselves i and j can together get π^0_{ij}, and if bilateral merger is not available, their profits will be just the noncooperative profits. Hence the second inequality ensures that, given an allocation in G, any two firms have no incentives to go for a bilateral merger or to compete non-cooperatively. The third condition similarly ensures that individually no firm has any incentive to leave the grand coalition because under grand coalition firm k gets v_k, whereas if it leaves G and (1) is satisfied for firms i and j (so that i and j form a bilateral merger), k will get π^0_k as an outsider, and if (1) is not satisfied for i and j (so that the market structure is oligopoly of all three firms), k can get just its noncooperative profit, π_k^N. The second and third conditions may be called, respectively, 'group rationality' and 'individual rationality' conditions. Thus the conditions (S1) through (S3) are similar to core, with the exception that we have modified (S2) and (S3) to accommodate externalities, and a bilateral merger will occur only when it is profitable.

Then our problem in the paper is to study whether meaningfully we can allocate pie among the players, given their technologies. We show that there are situations where firms cannot agree on a division, and hence they fail to form a grand merger.

Regarding the division of payoffs, we assume following two rules which appear to us innocuous:

- Identical players will get identical payoffs (symmetry property).
- The surplus payoff, over and above the sum of reservation payoffs, which comes from their cooperation, will be divided equally among the insiders.

The first rule calls for a "fair" division of payoff in the sense that identical firms should get identical payoffs. The second rule specifies that the surplus created due to coalition will be equally divided among the coalition partners. So we are assuming Nash bargaining. Given the technological position, a player's bargaining power in the process of negotiation is determined by its outside option. Thereafter firms have equal bargaining power. Let r_i be the disagreement or reservation payoff of the ith player, and S be the surplus under all firms merger. Then following the above rules, the allocation to the ith player will be

$$v_i = r_i + S/3, \quad i = 1,2,3. \tag{2}$$

where

$$S = \pi^m - \sum_i r_i$$

We have to define the reservation payoff or outside option of a player very carefully; not necessarily these are the non-cooperative payoffs. Given that the firms have Cournot conjectures, we define r_i as

$$r_i = \begin{cases} \pi_i^0 & \text{if } \pi_{jk}^0 > \pi_j^N + \pi_k^N \\ \pi_i^N & \text{otherwise} \end{cases} \tag{3}$$

The reason is the following. Consider any allocation under G. Now, if any player, i, wants to leave G, how much it can expect to get depends on the behavior of the other two players. If the other two players, j and k, form a bilateral merger, i gets π_i^0, and if j and k decide to compete independently, i gets just its non-cooperative payoff, π_i^N. Hence the definition of r_i is like that given above.

Then, (2) and (3) together imply that (S1) and (S3) are satisfied. For stable grand coalition we have to check (S2) separately. Depending on the technologies of the firms, any of the following scenarios is possible: (1) $c_1=c_2=c_3$, (2) $c_1=c_2<c_3$, (3) $c_1<c_2=c_3$, and (4) $c_1<c_2<c_3$. In our analysis we assume that the market demand is linear.[8] The demand function in inverse form is given by:

$$P=a-\sum_i q_i \tag{4}$$

where P is the product price and q_i is the demand for the ith firm's output.

III. The Structure of Merger

In this section we discuss the possibility of formation of a grand merger and the corresponding allocations for the players under all possible assumptions regarding technology asymmetry, given the allocation rules stated in the previous section. Quite naturally, it depends on the incentives of firms to form subgroup coalitions. As we shall show, the structure of bilateral mergers depends on the technological asymmetries of the players. It is possible to have no bilateral merger, merger between only the efficient firms, merger between one efficient firm and one inefficient firm, or merger between the inefficient firms. The possibility of subgroup mergers creates externalities under quantity competition, and the outsider's bargaining payoff goes up. Then there are situations when the grand coalition cannot meet the demand of all firms, and hence a stable grand coalition may not be formed.

A. Assumption 1: $c_1=c_2=c_3$

This is the bench mark case. We have all firms identical. Let $c_i=c\ \forall i$. Then, given (4), it is easy to get $\pi^m=(a-c)^2/4$, $\pi_i^N=(a-c)^2/16$, $\pi_{jk}^0=(a-c)^2/9$.

[8] In section III.D (scenario 4) we have in fact considered general demand function.

***Lemma* 1**: Under assumption 1, bilateral merger is never profitable.

Proof: Condition (1) is never satisfied, given Cournot conjectures. *Q.E.D.*

Here with Cournot competition, the outsider necessarily gains. Since there is no efficiency gain under merger, the concentration effect is dominated by the negative externality, making the bilateral merger privately unprofitable. Hence, by going independently, a player cannot expect a payoff (r_i) more than its non-cooperative payoff. However, the industry monopoly payoff exceeds the sum of non-cooperative payoffs. Since all firms are identical, following our rules of allocation, each firm will get identical payoffs under grand coalition. This is given by $v_i = \pi^m/3\ \forall i$. Hence we have the following proposition.

***Proposition* 1**: When all firms have symmetric technologies, the grand coalition can always be formed, with each firm's payoff being one-third of the industry monopoly payoff.

B. Assumption 2: $c_1 < c_2 = c_3 \equiv c$

This is the case where we have one efficient firm and two identical inefficient firms. Let P_m be the (unrestricted) monopoly price for c_1 technology. Given the demand function (4), we have $P_m = (a + c_1)/2$. Now given c, if $P_m \leq c$, *i.e.*, $c_1 \leq \underline{c} \equiv (2c - a)$, firm 1 is monopoly. There will be no further merger in this case. So assume $c_1 > \underline{c}$. When all firms survive (*i.e.*, if $c_1 > \underline{c}$), the non-cooperative payoffs are $\pi_1^N = (a - 3c_1 + 2c)^2/16$ and $\pi_j^N = (a - 2c + c_1)^2/9$ for $j = 2,3$.

***Lemma* 2**: Given assumption 2, (*i*) the bilateral merger between the inefficient firms (*i.e.*, between 2 and 3) is never profitable, and (ii) a bilateral merger between the efficient firm (firm 1) and one inefficient firm (*i.e.*, either 2 or 3) is profitable if and only if $c_1 \in (\underline{c}, c^0)$, where $c^0 = (14c - a)/13$ and $\underline{c} < c^0 < c$.

Proof: See[9] Appendix A. *Q.E.D.*

[9]Lemma 2 is drawn from Kabiraj and Mukherjee (2000) which analyzes, in a three-firm framework, an interaction between cooperation decisions at the R&D stage and merger decisions at the production stage.

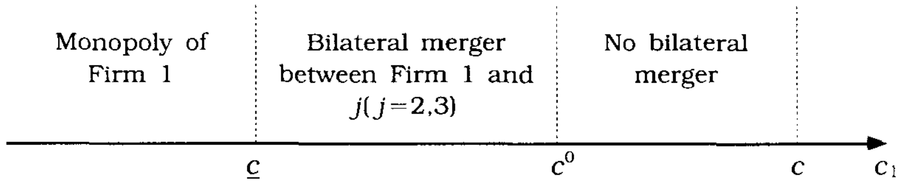

FIGURE 1
POSSIBILITY OF BILATERAL MERGERS UNDER ASSUMPTION 2

The results are shown in Figure 1. Given Lemma 2, we have a number of subcases depending on the extent of technology asymmetry.

***Assumption* 2.1**: $c_1 \leq \underline{c}$.
This is the case of monopoly of the efficient firm, implying that there will be no further merger. Hence $v_1 = \pi^m$ and $v_2 = 0 = v_3$.

***Assumption* 2.2**: $c^0 \leq c_1 < c$.
In this case all firms operate under non-cooperative situation, and there will be no bilateral merger (see Lemma 2). Hence $r_i = \pi_i^N$; $i = 1,2,3$. Under G, therefore, $v_i = r_i + S/3$ where $S = \pi^m - \sum_i \pi_i^N$.

***Assumption* 2.3**: $\underline{c} < c_1 < c^0$.
In this case all firms operate at positive output levels under non-cooperative situation, and the bilateral merger between firm 1 and firm 2 (or 3) is profitable, but not between firm 2 and 3. So if firm 1 leaves G, it cannot get more than its non-cooperative payoff; therefore, $r_1 = \pi_1^N$. But if the jth firm ($j=2,3$) leaves G, the remaining two firms will form a bilateral merger, instead of competing independently. Therefore, as an outsider, the jth firm gets π_j^0; hence, $r_j = \pi_j^0$; $j=2,3$.

Then the question is whether paying all firms at their reservation payoffs is feasible at all. It will be feasible if and only if $S = \pi^m - \pi_1^N - 2\pi_j^0 \geq 0$. Note that S is concave in c_1, with $S=0$ at $c_1 = \underline{c}$ and $c_1 = c^{0'} \equiv (82c - 5a)/77 > c^0$. Also $(dS/dc_1)|_{\underline{c}} > 0$ and $(dS/dc_1)|_{c^{0'}} < 0$. Hence given assumption 2.3, we have $S > 0$. Now, following the rules of allocation we have, $v_1 = \pi_1^N + S/3$ and $v_j = \pi_j^0 + S/3$; $j=2,3$. We can also verify[10] that $v_1 + v_j > \pi_{1j}^0$. Hence, under this assumption, a stable grand merger is possible with the above allocations. Since $\pi_1^N > \pi_j^0$ in this case, the efficient firm is getting the largest payoff under G. In that sense it is a fair division.[11]

***Proposition* 2**: Under assumption 2, grand merger is always formed, and the allocations depend on the asymmetry of technologies.

C. Assumption 3: $\hat{c} \equiv c_1 = c_2 < c_3$

In this case firm 1 and 2 are equally efficient but firm 3 is inefficient. Let us $c_3 = c$, and consider Cournot competition. Then we note that

$$\exists \bar{c} \equiv (3c - a)/2 \mid \hat{c} \leq \bar{c} \Leftrightarrow \pi_3^N = 0;$$

$$\exists \underline{c} \equiv (2c - a) \mid \hat{c} \leq \underline{c} \Leftrightarrow \pi_3^0 = 0.$$

Explanation of the above parameters is quite simple. If firm 1 and 2 each has $mc = \hat{c} \leq \bar{c}$, the inefficient third firm, with $MC = c$, will cease to operate under non-cooperative situation, and the market structure will be reduced to duopoly of the efficient firms. However, the inefficient firm will operate at positive output level if it faces a single efficient firm (the merged firm of the two efficient firms), provided that the MC of the efficient firm is not too small (*i.e.*, $\hat{c} > \underline{c}$).

***Lemma* 3:** Given assumption 3, (i) the bilateral merger between efficient firms (1 and 2) is profitable if and only if $\hat{c} < c^*$, and (ii) a bilateral merger between one efficient firm i ($i = 1,2$) and the inefficient firm 3 is profitable if and only if $\hat{c} \in (\bar{c}, c^{**})$ where $c^* = (c - (\sqrt{2} - 1)a)/(2 - \sqrt{2})$ and $c^{**} = (15c - a)/14$; $\underline{c} < c^* < \bar{c} < c^{**} < c$.

Proof: See[12] Appendix B. *Q.E.D.*

Figure 2 portrays the results of Lemma 3. We have following subcases.

[10]Let $Z = v_1 + v_j - \pi_{1j}^0$. Then we can show that Z is strictly concave in c_1, with $Z = 0$ at $c_1 = \underline{c}$ and at $c_1 = (17a + 38c)/55 > c^0$. Hence $Z > 0$ for $c_1 \in (\underline{c}, c^0)$.

[11]In fact, $\pi_1^N \leq \pi_j^0$ for $c_1 \geq c^0$, but then there will be no merger.

[12]Lemma 3 is again drawn from Kabiraj and Mukherjee (2000).

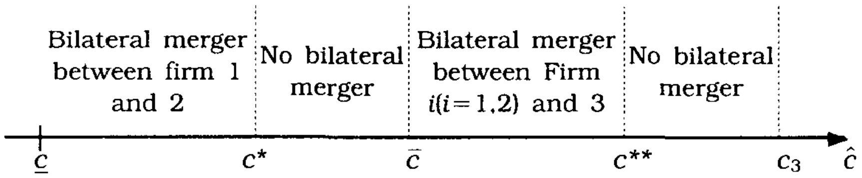

Figure 2
Possibility of Bilateral Mergers under Assumption 3

***Assumption* 3.1**: $\hat{c} \leq \underline{c}$. This is the case when firm 3 cannot enter under non-cooperative competition, and under this situation firm 1 and 2 merge. So firm 3 has no contribution in merger any way. Hence the optimal merger structure will be the bilateral merger of firms 1 and 2 only. The corresponding payoffs will be: $v_i = \pi_i^N + [\pi^m - 2\pi_i^N]/2$ for $i=1,2$, and firm 3 has zero payoff.

***Assumption* 3.2**: $\hat{c} \geq c^{**}$.

Since $\hat{c} > \bar{c}$, firm 3 operates at positive output level under Cournot-Nash equilibrium. But no bilateral merger is profitable. Hence non-cooperative payoffs remain to be their reservation payoffs. This gives a payoff to i under G as $v_i = \pi_i^N + [\pi^m - \sum_i \pi_i^N]/3$, $i=1,2,3$.

***Assumption* 3.3**: $c^* \leq \hat{c} \leq \bar{c}$.

This case has an interesting feature. Here firm 3 cannot operate under non-cooperative competition (*i.e.*, $\pi_3^N = 0$). So it is a duopoly of firm 1 and 2. However, if firm 1 and 2 would merge, firm 3 could operate profitably. But given $\hat{c}$ in that interval, no bilateral merger is profitable. Hence again non-cooperative payoffs are their reservation payoffs. Then if the grand merger is formed, firm i ($i=1,2$) will get, $v_i = \pi_i^N + [\pi^m - 2\pi_i^N]/3$ and $v_3 = [\pi^m - 2\pi_i^N]/3$, where π_i^N is the non-cooperative duopoly payoff of one efficient firm.

Note the *interesting* point. Since firm 3's non-cooperative payoff is zero, apparently it seems that firm 3 has no bargaining power, but here the technological asymmetry is such that firm 3 can prevent the efficient firms to merge together. This gives firm 3 some bargaining power and it derives a positive payoff under the grand merger. The greater the efficiency of the efficient firms relative to the inefficient firm (*i.e.*, as $\hat{c}$ is closer to c^*), the larger the benefits the efficient firm derives. Let us highlight the result. *There are situations where an inefficient firm that cannot survive in a non-cooperative competition can secure a positive payoff in a grand coalition.*

***Assumption* 3.4**: $\underline{c}<\hat{c}<c^*$.

Here firm 3's output is zero under non-cooperative competition, but whenever firm 1 and 2 form a merger, firm 3 operates at positive profit (since $\hat{c}>\underline{c}$). Contrary to the previous case, in this interval merger between efficient firms (only) is profitable. This means, firm 3's reservation payoff goes up to $r_3=\pi_3^0>0$. Since bilateral merger between 1 and 2 is profitable, their reservation payoffs are $r_i=\pi_{12}^0/2$; $i=1,2$. Hence, under grand coalition, each of the efficient firms gets a payoff $v_i=\pi_{12}^0/2+S/3$, and firm 3 gets $v_3=\pi_3^0+S/3$ where $S=[\pi^m-\pi_{12}^0-\pi_3^0]$.

Note that, compared to the previous case, now the efficient firms have become more efficient, but a part of the profits due to efficiency goes to the inefficient firm. The greater efficiency of the efficient firms pays partially to the inefficient firm.

***Assumption* 3.5**: $\bar{c}<\hat{c}<c^{**}$.

This is the case when under non-cooperative competition all firms (including the inefficient firm) make positive profits, but, given the interval, a bilateral merger between one efficient firm (*i.e.*, either 1 or 2) and the inefficient firm is profitable, but merger between the efficient firms is not profitable. So if a grand merger is formed, by leaving the coalition firm 3 cannot expect more than its non-cooperative payoff, (*i.e.*, $r_3=\pi_3^N$). But if the ith firm ($i=1,2$) goes out, the jth firm ($j=1,2$) and firm 3 can operate as merged firm. Hence by going out, firm i gets π_i^0, that is, the ith firm's reservation payoff is $r_i=\pi_i^0$, $i=1,2$.

Now given the reservation payoffs as stated above, we have to see whether paying each firm at least its reservation payoff is feasible. As before, define $S=\pi^m-2\pi_i^0-\pi_3^N$. Then $S=0$ at $\hat{c}=c^{**\prime}\equiv(9c-a)/8$ and $\hat{c}=\bar{c}'\equiv(9c-5a)/4$, with $\bar{c}'<\bar{c}<c^{**\prime}<c^{**}$. Also S is concave, with $(dS/d\hat{c})|_{\bar{c}}>0$ and $(dS/d\hat{c})|_{c^{**\prime}}<0$. Therefore, given assumption 3.5, in which case a bilateral merger is profitable between one efficient firm and the inefficient firm, the grand coalition (if formed) can pay at least the reservation payoffs to the players if and only if $\hat{c}\in$

$(\bar{c}, c^{**'}]$. In this interval under grand coalition the optimal allocation will be for $i=1,2$, $v_i = \pi_i^0 + S/3$, and for firm 3, $v_3 = \pi_3^N + S/3$, where S is defined above. But in the other subcase (*i.e.*, $c^{**'} < \hat{c} < c^{**}$), all firms merger under the rules suggested cannot be formed, because with any allocation $\{v_i\}$, $\sum v_i = \pi^m$, at least one firm j ($j=1,2$) finds it profitable to go out of the grand coalition and get a larger payoff than what is allocated. In this case the externalities are strong enough that each firm has much to gain from going out of the grand coalition, but the total profits are not large enough to satisfy the demand of each player. Hence grand coalition cannot be formed. One may think of the following allocation. Since firm 3 under this case has a reservation payoff π_3^N, so suppose $v_3 = \pi_3^N$ and $v_i = [\pi^m - \pi_3^N]/2$ for $i=1,2$. But this is not acceptable for two reasons. First, it violates our second rule of allocation, and secondly, even under this allocation, firm i has an incentive to go out of the coalition, because $[\pi^m - \pi_3^N]/2 < \pi_i^0$. While in this case bilateral merger is Pareto superior, but without further assumptions and different rules of the games we cannot determine the structure of the bilateral merger and also the division of payoffs under bilateral merger.

On the basis of the discussion so far we have made we can write the following proposition.

***Proposition* 3**: Given the rules of allocation and assumption 3, grand merger can be formed if and only if $\hat{c} \notin (c^{**'}, c^{**})$.

The intuition of the result is the following. We have already discussed that under assumption 3 a bilateral merger is possible only between one efficient firm and the inefficient firm. Therefore, each efficient firm's reservation payoff is larger than its non-cooperative payoff. Under this situation, a grand coalition can be formed only if it generates sufficient surplus to meet at least the reservation payoff of each firm. Hence, if the efficient firms are efficient to a critical degree, a stable grand coalition can be formed.

D. Assumption 4: $c_1 < c_2 < c_3$

We have already defined P_m to be the monopoly price for c_1 technology. Let P_d be the duopoly price when there are two firms with c_1 and c_2 technologies. For the linear demand function, $P_m = (a$

$+c_1)/2$ and $P_d=(a+c_1+c_2)/3$. Also $P_d<P_m$ when $c_2<P_m$. When $P_m<c_2$ (*i.e.*, $c_1>2c_2-a$), firm 1 emerges as monopoly. We ignore this case.

***Lemma* 4**: Given assumption 4, a bilateral merger between any two firms may be profitable depending on the technological asymmetry of the firms.

Proof: Condition (1) can hold only for one pair or two pairs, or for all pairs, depending on the technological asymmetry of the players.[13] *Q.E.D.*

This is the most general case in the sense that all previous results may be possible under assumption 4. We provide the analysis of the remaining section for a general demand function, while giving examples for linear demand (4). It may be recalled that $P_d<P_m$ when $c_2<P_m$.

***Assumption* 4.1**: $c_1<P_m\leq c_2<c_3$.

It is monopoly of firm 1. Hence $v_1=\pi^m$. All other firms are getting zero payoff.

***Assumption* 4.2**: $c_1<c_2<P_d<P_m\leq c_3$.

It is duopoly of firms 1 and 2 under non-cooperative competition. While merger between 1 and 2 is possible, but firm 3 can never enter. Therefore, $v_i=\pi_i^N+S/2$, $i=1,2$, where π_i^N is the duopoly payoff of firm i and $S=[\pi^m-\sum_i^2\pi_i^N]$. Firm 3 gets nothing. As for example, given the demand function (4), suppose $a=10$, $c_1=2$, $c_2=3$, $c_3\geq 6$, and our result follows.

***Assumption* 4.3**: $c_1<c_2<P_d<c_3<P_m$.

Non-cooperative game is a duopoly of firm 1 and 2. Bilateral merger between 1 and 3 or between 2 and 3 will never occur. But if firm 1 and 2 merge, firm 3 can find entry profitable. Now merger between 1 and 2 is privately profitable if and only if

$$\pi_{12}^0>\pi_1^N+\pi_2^N. \tag{5}$$

When (5) does not hold, there will be no bilateral merger, implying

[13]See examples in the following analysis.

that firms have non-cooperative payoffs as their reservation payoffs under grand coalition. Stable grand coalition is always possible under this situation.[14]

When (5) holds, we must have $r_i = \pi_i^N + (1/2)[\pi_{12}^0 - \pi_1^N - \pi_2^N]$ for $i=1,2$, and $r_3 = \pi_3^0$. Then also note that $S=[\pi^m - (\pi_{12}^0 + \pi_3^0)] > 0$. Hence the firms agree on a division of payoffs under the grand coalition.

***Example* 1**: Suppose $a=10$, $c_1=0$, $c_2=2$, and $4<c_3<5$.

Then we must get

$$\pi^m = 25,\ \pi_1^N = 16,\ \pi_2^N = 4,\ \pi_3^N = 0$$

$$\pi_{12}^0 = \begin{cases} 21.8 & \text{if } c_3 = 4 \\ 25 & \text{if } c_3 = 5 \end{cases} \quad \text{and} \quad \pi_3^0 = \begin{cases} 4/9 & \text{if } c_3 = 4 \\ 0 & \text{if } c_3 = 5 \end{cases}$$

Hence, $\pi_{12}^0 > \pi_1^N + \pi_2^N$ and $\pi^m > \pi_{12}^0 + \pi_3^0$.

***Assumption* 4.4**: $c_1 < c_2 < c_3 < P_d < P_m$.

In this case the non-cooperative profit of each firm is positive. Regarding the structure of the bilateral merger we cannot a priori say anything in general. Given the parameters (c_1, c_2, c_3), let us first consider a case where a bilateral merger between any two firms is profitable, *i.e.*,

$$\pi_{ij}^0 > \pi_i^N + \pi_j^N,\ \forall i \neq j.$$

Therefore, $r_i = \pi_i^0$. Define $S=[\pi^m - \sum_1^3 \pi_i^0]$. Now if $S \geq 0$, the allocations $v_i = r_i + S/3$ will form a stable grand coalition if and only if (S2) is satisfied at the same time ((S1) and (S3) are necessarily satisfied by construction). The following example describes a scenario where grand merger is formed.

***Example* 2**: Suppose, $a=10$, $c_1=1$, $c_2=2$, and $c_3=3$. Then we have, $\pi^m=20.25$, $\pi_{12}^0=13.4$, $\pi_{13}^0=11.1$, $\pi_{23}^0=5.4$, $\pi_1^N=9$, $\pi_2^N=4$, $\pi_3^N=$

[14]To show that (5) may not hold, consider the following example: $a=10$, $c_1=0$, $c_2=0.5$, $c_3=3.6$. Then, $\pi_{12}^0=(a+c_3)^2/9=184.96/9$ and $\pi_1^N + \pi_2^N = (a+c_2)^2/9 + (a-2c_2)^2/9 = 191.25/9$.

1.0, $\pi_1^0 = 11.1$, $\pi_2^0 = 5.4$, $\pi_3^0 = 2.8$, $S = 0.95$. All the relevant conditions of this case are satisfied. Hence $v_1 = 11.42$, $v_2 = 5.72$, $v_3 = 3.11$. Note that the stability conditions are also satisfied.

Assuming that pairwise all bilateral mergers are profitable, the grand coalition cannot, however, be formed if $S \leq 0$, although $\pi^m > \sum_i \pi_i^N$. In the above example if we replace the value of c_3 by $2+\varepsilon$ where ε is very small but positive, we shall get $S<0$. Now, given that the grand coalition is not profitable, our question is: What will be the optimal structure of bilateral merger? It is easy to see that merger between i and j will be privately optimal if the following condition holds,[15] that is,

$$\pi_{ij}^0 + \pi_k^N > \max[\pi_{ik}^0 + \pi_j^N, \pi_{jk}^0 + \pi_i^N]. \tag{6}$$

Let us now assume that only one bilateral merger, say between i and j, is profitable, and no other bilateral merger is profitable. In this case, a stable grand coalition can be formed because $S = \pi^m - (\pi_{ij}^0 + \pi_k^N) > 0$. In this case, therefore, $r_i = \pi_i^N + [\pi_{ij}^0 - \pi_i^N - \pi_j^N]/2$, $r_j = \pi_j^N + [\pi_{ij}^0 - \pi_i^N - \pi_j^N]/2$, and $r_k = \pi_k^0$ are the reservation payoffs of firm i, j and k. Under grand merger $v_i = r_i + S/3$ is the payoff for firm i.

Finally, consider that bilateral mergers between i and j and between i and k are profitable, but not between j and k. In this case i cannot get more than its non-cooperative payoff by going out of the grand coalition, but each of j and k can get more than its non-cooperative payoff. So if $\pi^m > \pi_i^N + \pi_j^0 + \pi_k^0$, a stable grand coalition can be formed with an allocation $v_i = r_i + S/3$ if and only if at the same time following two conditions hold: $v_i + v_j \geq \pi_{ij}^0$ and $v_i + v_k \geq \pi_{ik}^0$; otherwise, there will remain some incentives for a bilateral merger. In our scheme, i has relatively disadvantage to bargain in a sense that its bargaining payoff is stuck up at the non-cooperative level.[16] However, if any of the conditions stated in this case fails to hold, the only stable outcome will be the bilateral merger, and then i has the advantage to choose its partner. So i will choose j as its

[15]Player i will prefer player j as its partner iff $\pi_i^N + [\pi_{ij}^0 - \pi_i^N - \pi_j^N]/2 > \pi_i^N + [\pi_{ik}^0 - \pi_i^N - \pi_k^N]/2$, and similarly for j, $\pi_j^N + [\pi_{ij}^0 - \pi_i^N - \pi_j^N]/2 > \pi_j^N + [\pi_{jk}^0 - \pi_i^N - \pi_k^N]/2$. Condition (6) is derived from these inequalities.

[16]Under the scenario described above we cannot rule out the possibility that a relatively efficient firm (say, i) gets, under grand merger, an allocation which is smaller than that of a relatively inefficient firm (say, j). This occurs if $\pi_i^N < \pi_j^0$.

partner if $\pi_{ij}^{0}-\pi_{j}^{0}\geq\pi_{ik}^{0}-\pi_{k}^{0}$; otherwise, partner will be k. In the example below, a stable grand coalition is formed.

Example: Suppose, $a=10$, $c_1=0$, $c_2=0.5$, and $c_3=3.2$. Then we have, $\pi^{m}=25$, $\pi_{12}^{0}=19.36$, $\pi_{13}^{0}=12.25$, $\pi_{23}^{0}=9.0$, $\pi_{1}^{N}=11.73$, $\pi_{2}^{N}=8.56$, $\pi_{3}^{N}=0.05$, $\pi_{1}^{0}=12.25$, $\pi_{2}^{0}=9.0$. This means, bilateral mergers between 1 and 3, and between 2 and 3 are profitable, but not between 1 and 2. In this case, $S=\pi^{m}-\pi_{1}^{0}-\pi_{2}^{0}-\pi_{3}^{N}=3.7$. Then following our rules of allocation, $v_1=\pi_{1}^{0}+S/3=13.48$, $v_2=\pi_{2}^{0}+S/3=10.23$, $v_3=\pi_{3}^{N}+S/3=1.29$. Note that the stability conditions are also satisfied, because $v_1+v_3>\pi_{13}^{0}$ and $v_2+v_3>\pi_{23}^{0}$.

From the discussion of this section we can write the following proposition.

***Proposition* 4**: Given assumption 4 and the rules of payoff allocation, (i) if under noncooperative situation not all firms survive, a stable grand merger is always possible, but (ii) if under non-cooperative competition all firms survive, whether a stable grand merger will occur or not depends on the extent of asymmetry of the players. In particular, given the technological asymmetry, if a bilateral merger is profitable for only one pair, a stable grand merger can always be formed.

Let us summarize the analysis of this section. In a Cournot type set-up the monopoly payoffs strictly dominate oligopoly industrial payoffs. Then it may be considered that a grand merger should always take place if binding agreements are allowed. But we have shown that when the effects of externalities are large enough because of the technological asymmetry, firms fail to agree on a grand merger.

IV. Conclusion

Merger is a business strategy by which firms in an industry consolidate their position to share a larger profit. But the negotiation process for merger is not always smooth enough; it involves lots of threats and counter threats, or objections and counter objections of players. By doing this firms test the bargaining strength of the partners. In this paper we have drawn attention to the role of technological asymmetry in evaluating the bargaining position of each firm across the negotiation table. Although each firm knows that there are larger payoffs to share if a grand merger is formed, but often they fail to determine how to divide the payoff in a mutually agreeable way. Since the size of the bigger cake is fixed, the gain of one player necessarily implies the loss of payoff to that extent to the others. No firm wants to give up the gain to others; hence each player tries to prove how important is its contribution to a particular coalition and what it can otherwise gain without being party to the coalition. This tension may lead to disagreement or formation of a coalition of sub-optimal size.

In the process of negotiation, externalities play a very significant role, because firms might gain from the coalition of other firms. In our analysis externalities arise when the product market competition is characterized by Cournot. Hence the nature of product market competition is also important in the process of forming a coalition. Under price competition for homogeneous goods a grand coalition is always formed, because price competition induces no externalities.

It is not just technological asymmetry, but the extent of technology asymmetry that is more important. The extent of technology asymmetry determines whether a merger of a sub-group (in our case, a bilateral merger) is profitable. If merger of any sub-group is not feasible, bargaining payoffs are just their non-cooperative payoffs, and through a Nash bargaining firms can divide the total payoff among themselves. The Nash bargaining allocation seems appealing, and hence it is assumed that firms will agree to that division rule. But when the sub-group mergers are profitable, some firms' bargaining power goes up because of the externalities. When the effects of externalities are large enough, the grand coalition finds it difficult to meet the demand of all partners, and hence firms fail to agree on a grand merger.

In our paper the extent of technological asymmetry also determines the nature of bilateral merger. There are situations where a bilateral merger is possible only between two relatively efficient firms, or between two inefficient firms or between one efficient firm and one inefficient firm. It also determines how many bilateral mergers can be feasible. This gives some other interesting results. There are cases when the inefficient firm cannot just operate because of its inefficient technology, but it can prevent the formation of a (bilateral) merger between other two relatively efficient firms. This gives bargaining power to the inefficient firm and it derives a positive payoff when a grand merger is formed. The relatively efficient firms are, in a sense, to bribe the inefficient firm if a larger payoff is to be shared under grand merger. Even there might be situations when the efficient firm has the disadvantage to bargain in negotiation, and it comes up with a lower payoff compared to a relatively inefficient firm. We have considered a three-firm structure, because it is the simplest structure to capture the role of technological asymmetry and externality in the process of negotiation for merger.

Finally, we note the following. We have discussed situations when the grand merger (of Cournot firms) will not occur because the firms fail to make a binding contract. There are, however, situations when the outcome of the first round of coalition produces duopoly. One may then imagine the second round of coalition between the two operating firms. This might lead to a monopoly of the industry. In our analysis we have not considered the possibility of the second round of coalition. Again, while studying the question of grand coalition we have imposed the assumption that the surplus created by a coalition is divided in a symmetric Nash Bargaining way. Then it is possible that a coalition, which is unstable under this assumption, could be perfectly stable under a different division rule. Hence, an extension of the present work should be to examine the question of a stable grand coalition under alternative division rules.

Appendix

Appendix A: Proof of Lemma 2

(i) For bilateral merger to be profitable, (1) must hold. Consider the possibility of merger between 2 and 3. Given the demand function by (4), we have, $\pi_{23}^0=(a-2c+c_1)^2/9$ and $\pi_2^N+\pi_3^N=(2/16)(a-2c+c_1)^2$. Hence (1) does not hold.

(ii) Now consider the possibility of merger between firm 1 and firm j, $j=2,3$. Let us define

$$S_1(c_1)=\pi_{1j}^0-[\pi_1^N+\pi_j^N].$$

Then, for $c_1 \leq \underline{c}$, $S_1=0$, because $\pi_{1j}^0 = \pi^m$, $\pi_1^N = \pi^m$, and $\pi_j^N = 0$; and $S_1<0$ for $c_1=c$. Also S_1 is continuous and concave for $c_1>\underline{c}$. Hence

$$\exists c^0 | S_1(c_1)>0 \quad \text{iff } c_1 \in (\underline{c}, c^0).$$

For the demand function (4), $c^0=(14c-a)/13$.

Appendix B: Proof of Lemma 3

(i) Bilateral merger between 1 and 2 will be profitable iff

$$S_2(\hat{c}) = \pi_{12}^0 - [\pi_1^N + \pi_2^N] > 0.$$

Now if $\hat{c}>\bar{c}$, firm 3 survives under non-cooperative competition. Given (4), it is easy to show that $S_2<0$ for $\hat{c}>\bar{c}$.

When $\hat{c} \leq \underline{c}$, $\pi_3^0=0 \Rightarrow \pi_{12}^0 = \pi^m$, *i.e.*, the merged firm becomes monopoly, and $\pi_1^N + \pi_2^N = \pi_1^d + \pi_2^d$ (superscript d stands for duopoly). Then gain from merger becomes

$$S_2(\hat{c}) = \pi^m - [\pi_1^d + \pi_2^d] > 0.$$

Thus if $\hat{c}>\bar{c}$, firms 1 and 2 will never merge, but for $\hat{c}<\underline{c}$, they will always merge. Also for $\underline{c}<c<\bar{c}$, S_2 is monotonically decreasing in $\hat{c}$. So there exists $\hat{c}=c^*$ such that $S_2(\hat{c})>0 \Leftrightarrow \hat{c}<c^*$.

(ii) Consider the possibility of merger between firm i ($i=1,2$) and firm 3. Define

$$S_3(\hat{c}) = \pi^m - [\pi_i^N + \pi_3^N].$$

Given (4), S_3 has the following properties. S_3 is inverted U-shaped with $S_3(\hat{c})=0$ for $\hat{c} \leq \bar{c}$, and $S_3(\hat{c})<0$ at $\hat{c}<c$. So there exists $\hat{c}=c^{**}$ at which $S_3(\hat{c})=0$ and $S_3<0$ for $\hat{c}>c^{**}$. Hence $S_3(\hat{c})>0$ for $\bar{c}<\hat{c}<c^{**}$.

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