

# The Zero Bound and the Term Structure in a Nonlinear Macroeconomic Model

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The zero bound on nominal interest rates inherently imposes a nonlinearity on models in which money is nonneutral. However, for simplicity, analyses of the zero bound have typically been conducted in models which are otherwise linear. In a nonlinear staggered price-setting model, we examine how the zero bound makes the term structure of interest rates and macroeconomic dynamics sensitive to the economy's average inflation rate. We decompose this sensitivity into two components: (i) A pure expectations component, associated with the fact that the average inflation rate and the zero bound interact to affect the expected future path of short rates; and (ii) a term premium component, associated with the fact that the aforementioned interaction alters the behavior of the term premium. The first component is present in analyses where the zero bound is the only nonlinearity; the second component is absent in those analyses.

*Keywords:* Zero bound, Term structure, Sticky prices

*JEL Classification:* E31, E43, G12

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[*Seoul Journal of Economics* 2006, Vol. 19, No. 1]

## I. Introduction

The fact that nominal interest rates are bounded by zero suggests that the term structure of interest rates may not be invariant to the average level of inflation (the "inflation target"): If the inflation target is very low, short-term nominal rates might be expected to occasionally hit the zero bound, whereas if the inflation target is moderately high, short-term rates would be expected to remain away from zero. We explore the sensitivity of the term structure to the inflation target in a simple staggered price-setting model. Because the model is fundamentally nonlinear, there are two mechanisms through which the inflation target may affect the term structure. First, at low levels of the inflation target the possibility of future short-term nominal interest rates hitting zero means that the expected path of future short rates may be sensitive to the inflation target. This channel is present in linear models embedding the pure expectations hypothesis of the term structure. See for example Ruge-Murcia (*forthcoming*). In addition, the zero bound may make the conditional correlation between future marginal utility and future short rates sensitive to the inflation target. This conditional correlation drives the term premium in the long-term rate.

Our analysis is meant to be illustrative, but extensions of this work may prove to have practical importance. In the United States much attention has been devoted to the "conundrum" of long-term interest rates barely moving even as the Federal Reserve has raised its target for the short-term interest rates by more than two percentage points from 2003 to 2005. To the extent that this behavior is anomalous relative to previous episodes of rising short-term rates, one relevant consideration may be that perceptions about the inflation target have changed. Also, the experience with very low interest rates in 2003 and 2004 may have altered perceptions about the conditional correlation between real outcomes and future short rates.

There has been an explosion of research related to the zero bound on nominal interest rates in recent years. Much of this work is normative monetary (and fiscal) policy analysis. See for example Eggertsson and Woodford (2003), Svensson (2003), and the articles and discussions in the May 2000 *Journal of Money, Credit and*

*Banking.* Research in this vein emphasizes what monetary policy should do if the nominal interest rate is zero, or prescribes policy rules aimed partly at preventing the zero bound from becoming a problem. In contrast, this paper is entirely positive: For a given policy rule, we ask the model how the term structure of interest rates will behave at two different levels of the inflation target.

Interaction between the zero bound and the term structure has not been ignored in the literature. Ruge-Murcia (forthcoming), mentioned above, is perhaps closest to this paper in that his is a positive analysis of how the zero bound can be expected to alter the relationship between long and short-term rates. Skallsjö (2004) studies the implications of optimal monetary policy for the term structure in the presence of the zero bound. His model assumes risk neutrality, so the expectations hypothesis holds. Bernanke, Reinhart, and Sack (2004) among others, examine the extent to which monetary policy actions aimed at influencing the term structure can be effective in achieving policy goals when the short-term interest rate is at or near zero.

Several papers have studied the term structure of interest rates during the ongoing period of near-zero nominal interest rates in Japan, including Fujiki and Shiratsuka (2001), Nagayasu (2004) and Oda and Ueda (2005). One theme of that work has been an assessment of the effect of Japan's policy changes - the zero interest rate policy and then the quantitative easing policy - on the behavior of the term structure. In particular, Oda and Ueda use a small macro model to investigate how these policy changes affected the term structure through both the pure expectations channel and the risk premium channel. The macro model is linear, but they combine it with a no-arbitrage model of the term structure so that there is a risk premium in long-term interest rates. In principle, the approach taken in this paper can be complementary to Oda and Ueda's work. Because we are explicit about the dynamic macroeconomic equilibrium, the macro model directly generates the term structure, including the risk premium. In addition, the fact that the macroeconomic model is itself nonlinear expands the scope for changes in the policy rule to change the nature of the risk premium "process."<sup>1</sup>

<sup>1</sup>The attraction of modeling the term structure and the macroeconomy jointly in an equilibrium framework is clear. However, it should be admitted

The paper proceeds as follows. In section II we provide background on the term structure of interest rates from the perspective of an (unspecified) dynamic general equilibrium model. This material is quite standard, but it will be useful for motivating the rest of the paper. Section III presents the specific macroeconomic model in some detail. We also explain the nonlinear method used to solve the model. Surprisingly, though there is no shortage of research using sticky-price models such as this one, that research has almost exclusively worked with linearized versions of the models. Readers who wish to study the full nonlinear versions of these models may find section III a useful guide. Section IV contains results. For the calibrated model, we first illustrate how the equilibrium behavioral relationships and pricing functions vary depending on whether the inflation target is high enough to keep nominal interest rates away from zero. Then we simulate the model economy, again comparing two inflation targets which differ in their implications for whether nominal interest rates occasionally are zero. Section V concludes.

## II. Background on the Term Structure

The premise of this paper is that the central bank's inflation target may affect the behavior of the term structure of interest rates. Because the term structure is at the heart of the paper, in this section we derive in some detail a standard expression for the relationship between long-term and short-term interest rates implied by a macroeconomic model with representative households. In the next section we will specify one such model, but here we simply derive the term structure for the general case.

Define  $Q_{t,t+s}$  to be the dollar price in period  $t$  of an  $s$ -period discount bond that pays one dollar in period  $t+s$ . The implied  $s$ -period net nominal interest rate in period  $t$  then satisfies

$$\left(1 + R_t^{(s)}\right)^s = \frac{1}{Q_{t,t+s}}, \quad (1)$$

that past efforts along these lines have not met with much empirical success, leading to the popularity of the no-arbitrage approach.

where we have expressed the  $s$  period rate in comparable terms (e.g. quarterly) to the one period rate. Now suppose there is a representative household who discounts the future at rate  $\beta$ , and whose marginal (utility) value of real wealth in period  $t$  is  $\lambda_t$ , and let the price level be denoted  $P_t$ . Then the price of an  $s$ -period bond is given by

$$Q_{t,t+s} = \beta^s E_t \left[ \frac{\lambda_{t+s}/P_{t+s}}{\lambda_t/P_t} \right]. \quad (2)$$

Intuitively, it costs  $(\lambda_t/P_t) \cdot Q_{t,t+s}$  in utility terms to purchase one bond in period  $t$ , and in return the bond pays off  $\beta^s E_t [\lambda_{t+s}/P_{t+s}]$  in discounted utility terms in period  $t+s$ . With optimal behavior by households, the current cost of the bond is identical to the future benefit.

We now specialize to the case of  $s=2$ , so as to derive a relationship between the two-period rate, the current one-period rate, and the uncertain future one-period rate. With  $s=2$ , Equations (1) and (2) imply

$$(1+R_t^{(2)}) = \left[ \frac{1}{\beta} \frac{\lambda_t/P_t}{E_t(\lambda_{t+1}/P_{t+1})} \cdot \frac{1}{\beta} \frac{E_t(\lambda_{t+1}/P_{t+1})}{E_t(\lambda_{t+2}/P_{t+2})} \right]^{1/2}. \quad (3)$$

Since the one-period rate satisfies

$$(1+R_t^{(1)}) = \beta^{-1} \frac{\lambda_t/P_t}{E_t(\lambda_{t+1}/P_{t+1})}, \quad (4)$$

we can rewrite (3) as

$$(1+R_t^{(2)}) = \left[ (1+R_t^{(1)}) \cdot \frac{1}{\beta} \frac{E_t(\lambda_{t+1}/P_{t+1})}{E_t(\lambda_{t+2}/P_{t+2})} \right]^{1/2}. \quad (5)$$

The next step is to note that from the expression for the one-period rate in period  $t+1$  (that is, (4) one period ahead) we can express the two-period rate as

$$(1+R_t^{(2)}) = \left[ (1+R_{t+1}^{(1)}) \cdot \frac{E_t(\lambda_{t+1}/P_{t+1})}{E_t\left(\frac{\lambda_{t+1}}{P_{t+1}} \frac{1}{1+R_{t+1}^{(1)}}\right)} \right]^{1/2}, \quad (6)$$

where we have also used the fact that  $E_t(\lambda_{t+2}/P_{t+2}) = E_t(E_{t+1}(\lambda_{t+2}/P_{t+2}))$  by the law of iterated expectations.

Finally, we use the fact that

$$E_t \left[ \frac{\lambda_{t+1}}{P_{t+1}} \cdot \frac{1}{1+R_{t+1}^{(1)}} \right] = E_t \left( \frac{\lambda_{t+1}}{P_{t+1}} \right) E_t \left( \frac{1}{1+R_{t+1}^{(1)}} \right) + \text{cov}_t \left( \frac{\lambda_{t+1}}{P_{t+1}}, \frac{1}{1+R_{t+1}^{(1)}} \right)$$

to express the two-period rate as

$$\begin{aligned} & (1+R_t^{(2)}) \\ &= \left[ (1+R_t^{(1)}) \left\{ E_t \left( \frac{1}{1+R_{t+1}^{(1)}} \right) \right\}^{-1} \right]^{1/2} \left[ 1 + \frac{\text{cov}_t [\lambda_{t+1}/P_{t+1}, (1+R_{t+1}^{(1)})^{-1}]}{E_t(\lambda_{t+1}/P_{t+1}) E_t(1+R_{t+1}^{(1)})^{-1}} \right]^{-1/2}. \end{aligned} \quad (7)$$

A similar expression can be derived for the  $s$ -period rate, but for the sake of simplicity we will restrict attention to two-period rates, and we will refer to them as short and long rates.

The first factor in (7) represents the expectations theory of the term structure; if the covariance between future marginal utility and the future price of a short-term bond is zero, then the current long rate is pinned down by the current short rate and the conditional expectation of the future price of the short term bond. If the aforementioned covariance is nonzero, its presence in the second factor drives a wedge between the long rate and the expectations hypothesis.

To get some intuition for the role of the covariance in (7), compare two strategies for obtaining one dollar in period  $t+2$  by saving in period  $t$ . The first strategy involves buying a one-period bond, selling it in period  $t+1$  and then using the proceeds to buy a one-period bond which will pay off in period  $t+2$ . The second strategy involves buying and holding a two-period bond. If there is positive covariance between future marginal utility of nominal wealth and the future bond price, then rolling over the one-period bond is "expensive." Positive covariance means that future marginal utility tends to be high when the future bond price is high. In that case, there is a high dollar cost of purchasing a one-period bond next period in precisely those circumstances in which the utility

value of a dollar's consumption is high. With positive covariance the consumer is willing to pay a premium (over the pure expectations theory) to buy a long-term bond that avoids the high-cost of rolling over a short-term bond after one period. This price premium from a positive covariance is reflected in a lower value for the interest rate  $R_t^{(2)}$  in (7).

Our purpose in this paper is to investigate the effect of the inflation target on the behavior of the term structure, working through the zero bound on nominal interest rates. We will summarize the term structure by the slope of the yield curve ( $y_t$ ), which we define as follows, based on (7):

$$y_t = \frac{R_t^{(2)} - R_t^{(1)}}{1 + R_t^{(1)}}. \quad (8)$$

Also from (7), we have

$$y_t = \left[ \frac{\left(\frac{1}{1 + R_t^{(1)}}\right) \left\{ E_t \left( \frac{1}{1 + R_{t+1}^{(1)}} \right) \right\}^{-1}}{1 + \frac{\text{cov}_t[\lambda_{t+1}/P_{t+1}, (1 + R_{t+1}^{(1)})^{-1}]}{E_t(\lambda_{t+1}/P_{t+1}) E_t(1 + R_{t+1}^{(1)})^{-1}}} \right]^{1/2} - 1. \quad (9)$$

According to the expectations theory of the term structure, the slope of the yield curve is positive if the price of a one-period bond is expected to fall from period  $t$  to period  $t+1$  (this is the effect of the numerator in (9)). The covariance term (risk premium) in the denominator implies that even if the price of a one-period bond is not expected to change, the yield curve can be positively (negatively) sloped if there is negative (positive) conditional covariance between the future price of a one-period bond and the future marginal utility of nominal wealth.

If the zero bound induces an effect of the inflation target on the slope of the yield curve, it must work through some combination of the expected future bond price and the covariance between that price and marginal utility. That the former effect exists seems clear, at least in principle: If the inflation target is low enough that the interest rate on a short-term bond occasionally is zero, then the conditional expectation of future bond prices must differ from the case where the inflation target is high enough for the short-rate

never to hit zero. Although it is less straightforward, we conjecture that the latter effect will also generally be present in a fully specified macroeconomic model with some form of monetary nonneutrality.<sup>2</sup> That effect simply requires that in the low-inflation target case, the economy's equilibrium behavior involves a different covariance between the price of short term securities and the marginal utility of nominal wealth than in the case where interest rates never hit zero. We could continue speculating about how this covariance would change with the inflation target, but instead we will specify a model and then investigate exactly how the covariance changes in that model.

### III. The Macroeconomic Model

The model is one in which a representative household chooses consumption, labor supply to firms, and asset holdings (money and bonds) to maximize utility over an infinite horizon. Money holdings produce direct utility, as a stand-in for money's transactions-facilitating role. Firms are monopolistically competitive and must set their prices for two periods at a time. Each period one-half of the firms are allowed to change their price, so price setting is staggered as in Taylor (1980). There are only small differences between the model in this paper and the model Wolman (2005) uses to study the zero bound's implications for the behavior of real variables at low inflation rates. Households in the model face distinct disutility for labor supplied to each firm, whereas in the earlier paper disutility depended only on total labor supplied. This feature makes marginal cost for firms that adjust their price less sensitive to the output of firms that do not adjust their prices. Because of the different labor supply specification, it is more convenient to use money-in-the-utility function than the shopping time approach of Wolman (2005).

#### A. Consumers

Consumers have preferences over a consumption aggregate ( $c_t$ ), labor supplied to each firm  $z_t(h_t(z))$  and the ratio of real balances to

<sup>2</sup>This does not mean that the covariance-altering effect must be absent in a model with monetary neutrality. It may be present there as well, because the effect is one that concerns nominal variables only.



consumption  $(M_t/(P_t c_t))$  given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(c_t + \vartheta_t)^{1-\sigma} - 1}{1-\sigma} - \gamma \int_0^1 \frac{h_t(z)^{\nu+1}}{\nu+1} dz - g\left(\frac{M_t}{P_t c_t}\right) \right\}. \quad (10)$$

The variable  $\vartheta_t$  is a random preference shock. The consumer's budget constraint is

$$c_t + \frac{M_t}{P_t} + \frac{B_t/P_t}{1+R_t} = \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} + \int_0^1 w_t(z) h_t(z) dz + d_t + \frac{S_t}{P_t}, \quad (11)$$

where  $P_t$  is the price level,  $M_t$  is nominal money balances chosen in period  $t$ , to carry over to  $t+1$ ,  $B_t$  is holdings of one-period nominal zero-coupon bonds maturing at  $t+1$ ,  $R_t$  is the interest rate on nominal bonds,  $w_t(z)$  is the real wage paid by firm  $z$ ,  $l_t$  is leisure time,  $d_t$  is real dividend payments from firms, and  $S_t$  is a lump sum tax or transfer paid to or from the government. The function  $g(M_t/(P_t c_t))$  can be thought of as standing in for time spent transacting. Defining real balances to be  $m_t \equiv M_t/P_t$ , the function  $g(\cdot)$  is

$$g(m_t/c_t) = \phi \cdot (m_t/c_t) - \frac{\zeta}{1+\zeta} A^{-1/\zeta} (m_t/c_t)^{\frac{1+\zeta}{\zeta}} + \Omega, \text{ for } m_t/c_t < A \cdot \phi^\zeta, \quad (12)$$

$$g(m_t/c_t) = \frac{A \phi^{1+\zeta}}{1+\zeta} + \Omega, \text{ for } m_t/c_t \geq A \cdot \phi^\zeta,$$

with  $A > 0$ ,  $\phi \geq 0$  and  $\zeta < 0$ . Utility from real balances is thus increasing in the ratio of real balances to consumption, up to a satiation level of the ratio of real balances to consumption.

*Goods market structure.* As has become standard in the sticky-price literature, we assume that every producer faces a downward sloping demand curve, with constant elasticity  $\varepsilon$ .<sup>3</sup> The composite consumption good is an aggregate of the differentiated products produced by a continuum of firms,  $c_t = [\int c_t(\omega)^{(\varepsilon-1)/\varepsilon} d\omega]^{1/(\varepsilon-1)}$ , as in Dixit and Stiglitz (1977). The constant elasticity demands for each of the goods take the form:

<sup>3</sup>We assume  $\varepsilon = 10$ .

$$c_t(\omega) = \left( \frac{P_t(\omega)}{P_t} \right)^{-\varepsilon} \cdot c_t, \quad (13)$$

where  $P_t(z)$  is the nominal price at time  $t$  of good  $z$ , and  $P_t$  is the price index at time  $t$ , given by

$$P_t = \left[ \int P_t(\omega)^{1-\varepsilon} d\omega \right]^{\frac{1}{1-\varepsilon}}. \quad (14)$$

The staggered pricing assumption introduced above, together with an absence of other heterogeneity, means that all firms that adjust their prices in a given period choose the same price, and hence will face the same demand. It is then easier to write the consumption aggregate as

$$c_t = c(c_{0,t}, c_{1,t}) = \left( \frac{1}{2} \cdot c_{0,t}^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{2} \cdot c_{1,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad (15)$$

where  $c_{j,t}$  is the quantity consumed in period  $t$  of a good whose price was set in period  $t-j$ . The demands for each of the goods take the form:

$$c_{j,t} = \left( \frac{P_{t-j}^*}{P_t} \right)^{-\varepsilon} \cdot c_t, \quad (16)$$

where  $P_{t-j}^*$  is the nominal price at time  $t$  of any good whose price was set  $j$  periods ago, and the price index can now be written

$$P_t = \left[ \frac{1}{2} \cdot (P_t^*)^{1-\varepsilon} + \frac{1}{2} \cdot (P_{t-1}^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \quad (17)$$

Likewise, since all producers that adjust their prices in a given period hire the same quantity of labor input, we can write the disutility of labor as

$$\gamma \int_0^1 \frac{h_t(z)^{\nu+1}}{\nu+1} dz = \left( \frac{\gamma}{\nu+1} \right) \left( \frac{1}{2} \cdot h_{0,t}^{\nu+1} + \frac{1}{2} \cdot h_{1,t}^{\nu+1} \right), \quad (18)$$

where  $h_{0,t}$  is labor input at firms with newly set prices, and  $h_{1,t}$  is labor input at firms with one-period-old prices.

*Optimization.* If we attach Lagrange multiplier  $\lambda_t$  to the budget constraint, so that  $\lambda_t$  is the marginal value of real wealth,

$$L_0 = E_0 \sum_{t=0}^{\infty} \beta_t \left[ \frac{(c_t + \vartheta_t)^{1-\sigma} - 1}{1-\sigma} - \frac{1}{2} \cdot \left( \frac{\gamma}{\nu+1} \right) (h_{0,t}^{\nu+1} + h_{1,t}^{\nu+1}) - g \left( \frac{M_t}{P_t c_t} \right) \right] \quad (19)$$

$$+ \lambda_t \left[ \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} + \frac{1}{2} \cdot (w_{0,t} h_{0,t} + w_{1,t} h_{1,t}) + d_t + \frac{S_t}{P_t} - c_t - \frac{M_t}{P_t} - \frac{B_t/P_t}{1+R_t} \right],$$

the first order conditions for the individual's maximization problem, with respect to  $c_t$ ,  $h_{j,t}$ ,  $B_t$ , and  $M_t$  are

$$c_t : (c_t + \vartheta_t)^{-\sigma} = \lambda_t - \left( \frac{m_t}{c_t^2} \right) \cdot g'(\cdot), \quad (20)$$

$$h_{j,t} : \gamma h_{j,t}^{\nu} = \lambda_t w_{j,t}, \quad j=0,1, \quad (21)$$

$$B_t : \frac{\lambda_t}{P_t} = \beta \cdot (1+R_t) \cdot E_t \frac{\lambda_{t+1}}{P_{t+1}}, \quad (22)$$

and

$$M_t : \frac{\lambda_t}{P_t} + \left( \frac{1}{P_t c_t} \right) g'(\cdot) = \beta E_t \frac{\lambda_{t+1}}{P_{t+1}}. \quad (23)$$

### B. Firms

Each firm produces with an identical technology:

$$c_{j,t} = a_t h_{j,t}, \quad j=0,1. \quad (24)$$

Given the price that a firm is charging, it hires enough labor to meet the demand for its product at that price. Firms that do not adjust their price in a given period can thus be thought of as passive. Given that it has set a relative price  $(P_{t-j}^*)/P_t$ , real profits

for a firm of type  $j$  are

$$\frac{P_{t,j}^*}{P_t} \cdot c_{j,t} - w_{j,t} \cdot h_{j,t}, \quad (25)$$

that is, revenue minus cost.

Firms choose  $P_t^*$  optimally, to maximize the present discounted value of profits over the two periods that the price will be charged. Maximization of present value implies that a firm chooses its current relative price taking into account the effect on current and expected future profits. Substituting into (25) the demand curve (16) and the technology (24), the present discounted value of expected profits is given by

$$c_t \cdot \left[ \left( \frac{P_t^*}{P_t} \right)^{1-\varepsilon} - \frac{w_{0,t}}{a_t} \cdot \left( \frac{P_t^*}{P_t} \right)^{-\varepsilon} \right] + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \cdot c_{t+1} \cdot \left[ \left( \frac{P_t^*}{P_{t+1}} \right)^{1-\varepsilon} - \frac{w_{1,t+1}}{a_{t+1}} \cdot \left( \frac{P_t^*}{P_{t+1}} \right)^{-\varepsilon} \right]. \quad (26)$$

for the two periods over which a price will be in effect. Differentiating (26) with respect to  $P_t^*$  and setting the resulting expression equal to zero, one sees that the optimal relative price satisfies

$$\frac{P_t^*}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \cdot \frac{\lambda_t (w_{0,t}/a_t) c_t + \beta E_t \{ \lambda_{t+1} \cdot (w_{1,t+1}/a_{t+1}) \cdot (P_{t+1}/P_t)^\varepsilon \cdot c_{t+1} \}}{\lambda_t c_t + \beta E_t \{ \lambda_{t+1} \cdot (P_{t+1}/P_t)^{\varepsilon-1} \cdot c_{t+1} \}} \quad (27)$$

### C. Monetary and Fiscal Policy

The policy variables - broadly defined - are  $M_t$ ,  $R_t$ ,  $B_t$  and  $S_t$ . Of these four variables, policy can exogenously choose at most two independent functions, but we will need two such independent functions in order to completely describe an equilibrium. We will also need the government budget constraint, which is

$$S_t = M_t - M_{t-1} + \frac{B_t}{1+R_t} - B_{t-1}. \quad (28)$$

We assume there is a policy rule for the short-term nominal interest rate, given by

$$R_t = \max\left(\frac{\bar{\pi}}{\beta} - 1 + f_\pi \cdot (\ln(P_t/P_{t-1}) - \ln(1 + \bar{\pi})) + f_c \cdot (\ln c_t - \ln \bar{c}), 0\right), \quad (29)$$

where  $\bar{c}$  is the steady state level of consumption. We also assume that the quantity of bonds is fixed at zero. This leaves the money supply to be determined by money demand, and lump sum taxes to be determined by the government budget constraint (28).

#### D. Driving Processes

The exogenous variables in the model are the preference shock  $\vartheta_t$  and the productivity shock  $a_t$ ; they are assumed to follow independent Markov processes. For now we shut down the productivity shock, so the only uncertainty involves  $\vartheta_t$ .

#### E. Solving the Model

We solve the model using the finite element method (see McGrattan (1996)). This involves picking a grid of points for the model's endogenous state variable(s), and then finding values of the other endogenous variables numerically for each grid point and each value of the discrete random forcing variable such that the model's equations are satisfied. The solution consists of mappings from the state variable to each of the other variables. Those mappings can be used in conjunction with the driving process for  $\vartheta_t$  to simulate the model.

There is one natural endogenous state variable, the ratio of the lagged price set by adjusting firms to the lagged price level ( $P_{t-1}^*/P_{t-1}$ ), and to economize on characters we denote it by  $s_t$  :

$$s_t \equiv P_{t-1}^*/P_{t-1}. \quad (30)$$

We also detrend the price level by the lagged price level, and define the detrended price (which is the inflation rate) by  $\pi_t$  :

$$\pi_t \equiv P_t/P_{t-1}. \quad (31)$$

With these definitions, it can be shown that there are three fundamentally nonlinear equations that need to be solved for the three variables  $c_t$ ,  $m_t$  and  $s_{t+1}$ , at a grid of points for  $s_t$  and for each realization of the shocks. Adopting recursive notation, the equations are the first-order condition for money holding (23),

$$\lambda + \frac{g' \left( \frac{m(s; a, \vartheta)}{c(s; a, \vartheta)} \right)}{c(s; a, \vartheta)} = \beta E \frac{\lambda'}{\pi'}; \quad (32)$$

the first-order condition for bond-holding (22),

$$\lambda = \beta \cdot (1 + R) \cdot E \frac{\lambda'}{\pi'}; \quad (33)$$

and the optimal pricing condition (27),

$$s'(s; a, \vartheta) = \frac{\varepsilon}{\varepsilon - 1}.$$

$$\frac{\lambda \cdot (w_0/a) \cdot c(s; a, \vartheta) + \beta E \{ \lambda' \cdot (w'_1/a') \cdot (\pi')^\varepsilon \cdot c(s'; a', \vartheta') \}}{\lambda \cdot c(s; a, \vartheta) + \beta E \{ \lambda' \cdot (\pi')^{\varepsilon-1} \cdot c(s'; a', \vartheta') \}}. \quad (34)$$

We can solve for each of the other variables as explicit functions of  $c$ ,  $m$  and  $s'$  follows. The price index (17) yields the inflation rate:

$$\pi = \left( \frac{s^{1-\varepsilon}}{2 - (s'(s; a, \vartheta))^{1-\varepsilon}} \right)^{\frac{1}{1-\varepsilon}}.$$

The policy rule yields the nominal interest rate,

$$R_t = \max \left( \frac{\bar{\pi}}{\beta} - 1 + f_\pi \cdot (\ln \pi - \ln \bar{\pi}) + f_c \cdot (\ln c - \ln \bar{c}), 0 \right). \quad (35)$$

The first order condition for consumption yields the marginal utility of wealth.

$$\lambda = \frac{m(s;a,\vartheta)}{c(s;a,\vartheta)^2} \cdot g' \left( \frac{m(s;a,\vartheta)}{c(s;a,\vartheta)} \right) + (c(s;a,\vartheta) + \vartheta)^{-\sigma}. \quad (36)$$

The firm's technologies, together with the demand functions they face yield labor input,

$$h_0 = \frac{c(s;a,\vartheta)}{\alpha} (s) ^{-\varepsilon}. \quad (37)$$

and

$$h_1 = \frac{c(s;a,\vartheta)}{\alpha} \left( \frac{s}{\pi} \right)^{-\varepsilon}. \quad (38)$$

Finally, the first order conditions for labor supply yield real wages,

$$w_0 = \gamma h_0^\nu / \lambda \quad (39)$$

and

$$w_1 = \gamma h_1^\nu / \lambda. \quad (40)$$

#### IV. Results

After describing how we calibrate the model, we present two sets of results. The first set compares how the equilibrium relationships described above vary across different inflation targets. While our emphasis is on the fact that these relationships may vary because of the zero bound on nominal interest rates, they will also vary because the nominal interest rate is a tax on money holding; at higher inflation targets this tax is greater, leading households to economize on money holding and thereby making consumption more costly. The second set of results compares how simulated time series for the model vary across the different inflation targets; for this purpose we hold constant the sequence of shocks and the initial condition for the endogenous state variable.

**TABLE 1**  
CALIBRATION

Parameter	Description	Value
$\beta$	Discount factor	0.995
$\varepsilon$	Goods demand elasticity	10
$\sigma$	Risk aversion	2
$\gamma$	Labor disutility coefficient	1
$\nu$	Inverse of labor supply elasticity	2
$\phi$	Money in the utility function parameter	0.9588
$A$	Money in the utility function parameter	0.99801
$\zeta$	Money in the utility function parameter	-31.33099
$f_{\pi}$	Policy rule coefficient on inflation	1.5
$f_c$	Policy rule coefficient on output	0.125
$\{\vartheta_1, \vartheta_2, \vartheta_3\}$	Demand shock realizations	$\{-0.0125, 0, 0.0125\}$
	( $\vartheta$ process is symmetric, remains in current state with prob = 0.6)	
$\{a, \bar{a}\}$	Productivity realizations	$\{1, 1\}$

### A. Calibration

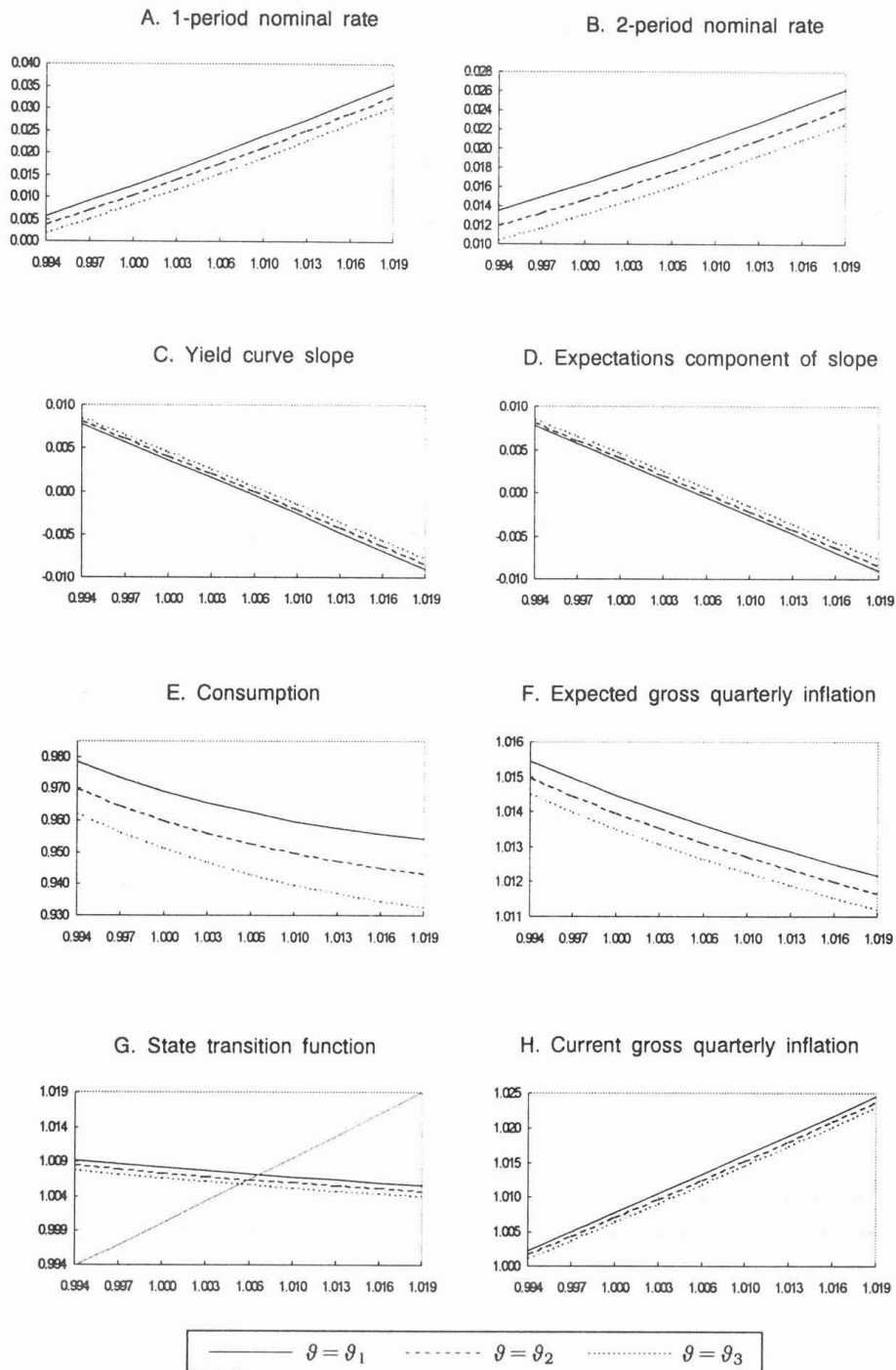
Table 1 contains the parameter values we use to generate numerical results. The parameters  $\sigma$ ,  $\varepsilon$ ,  $\beta$  and  $\nu$  are set to standard values in the literature;  $\gamma$  is not important for any of our results, so we set it to one. The parameters  $\phi$ ,  $A$  and  $\zeta$  are obtained by matching the model's money demand function to U.S. data for M1 and nondurables consumption, over the period 1959 to 2004.<sup>4</sup> The demand shock process is chosen arbitrarily, in order to help illustrate the model's behavior.

### B. Equilibrium Functions

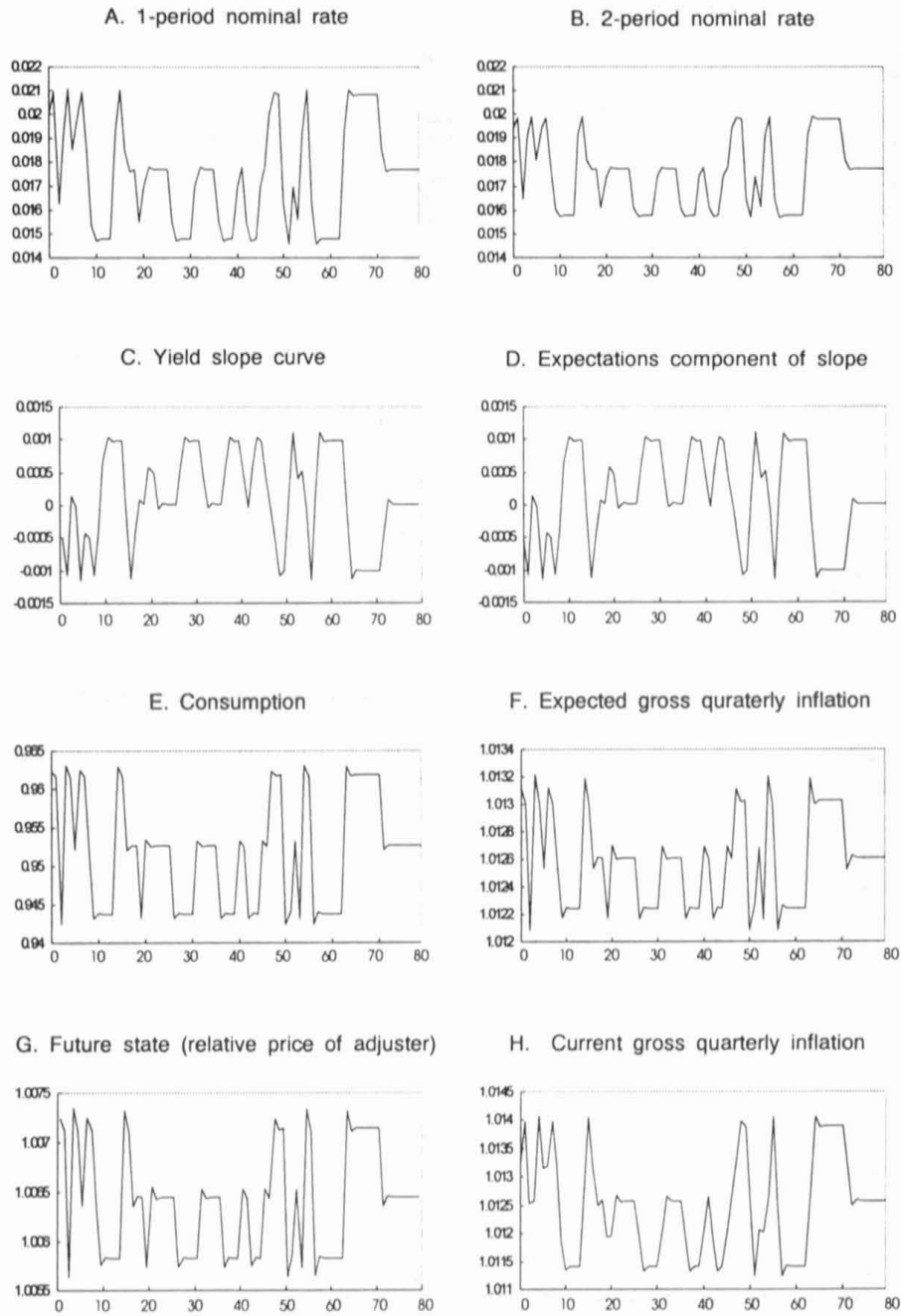
In Figures 1 and 3 we plot the equilibrium relationships between selected endogenous variables and the single endogenous state variable, which is  $s_t$ , the ratio of the price set by adjusting firms in

<sup>4</sup>There is little information in the data about the behavior of money demand at very low nominal interest rates. Thus there are many other parameter values for the  $g(\cdot)$  function that fit the data about as well as those in Table 1.

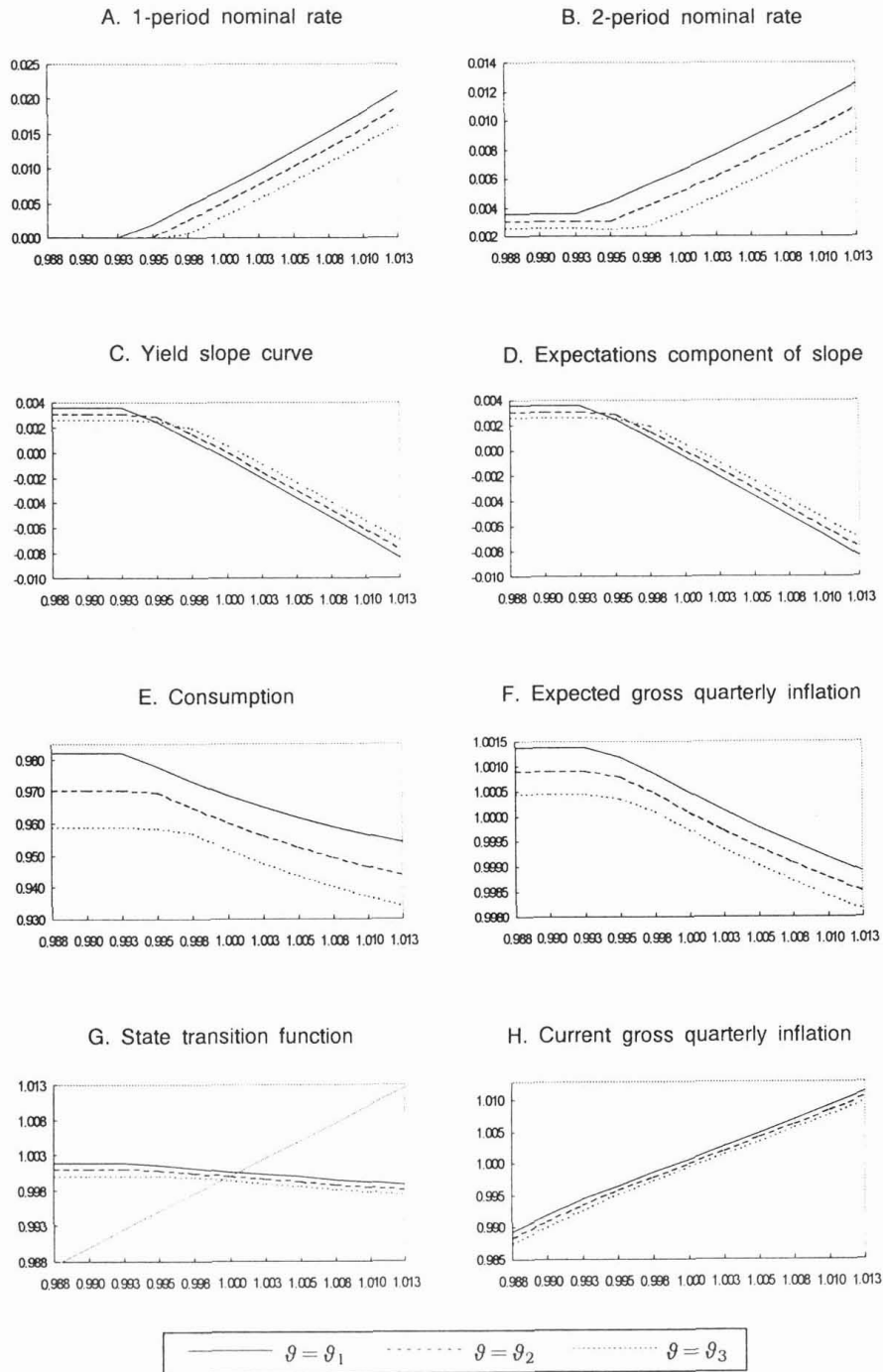




**FIGURE 1**  
 HIGH INFLATION TARGET  
 EQUILIBRIUM AS FUNCTION OF THE ENDOGENOUS STATE



**FIGURE 2**  
HIGH INFLATION TARGET  
SIMULATED TIME SERIES



**FIGURE 3**  
 LOW INFLATION TARGET  
 EQUILIBRIUM AS FUNCTION OF THE ENDOGENOUS STATE

period  $t-1$  to the price level in period  $t-1$ . Figure 1 is constructed using an annual inflation target of five percent, whereas Figure 3 is constructed using an inflation target of zero.<sup>5</sup> Each figure contains eight panels each figure contains eight panels. In each panel there are three lines, corresponding to the three realizations of the demand shock. Solid corresponds to the lowest realization of  $\vartheta$ , dashed to the middle realization, and dotted to the highest realization. The zero bound shows up immediately in Panel A of the two figures: When the inflation target is low, the equilibrium functions for the nominal interest rate are constant at zero for low enough values of the state variable. This translates in panel E to consumption behavior which is "distorted" at low values of the state variable when the inflation target is low. As for our main concern, the term structure, as expected panel C shows that for low values of the state variable (where the short rate is zero) the yield curve is flatter at the low inflation target. Panel D reveals that for both inflation targets, the behavior of the yield curve is driven entirely by the expectations component. That is, the slope of the yield curve is not noticeably affected by covariance between future marginal utility and the future short rate.<sup>6</sup>

Moving away for the moment from the term structure, we draw the reader's attention to panel G in Figures 1 and 3. This panel displays the transition function for the endogenous state variable, together with the 45 degree line. One can use this panel to trace out the evolution of the state variable for a given sequence of  $\vartheta_t$ , by repeatedly going left or right to the 45 degree line and then up or down to the appropriate state transition function. Then, armed with a sequence for  $s_t$  we can use the equilibrium functions in the other panels to derive corresponding sequences for the other endogenous variables. This is the method we use to generate the simulation results discussed next.

<sup>5</sup>The horizontal axis values are different in the two figures because the stationary distribution of  $s$  varies with the inflation target. In both cases we use the same percentage range for  $s$ .

<sup>6</sup>This should perhaps not be surprising, given the lack of success in using standard macroeconomic models for asset pricing. In interesting recent work, Ravenna and Seppälä (2005) find that a sticky-price model with habit-persistent preferences can generate an important covariance effect on the yield curve.

### C. Simulated Time Series

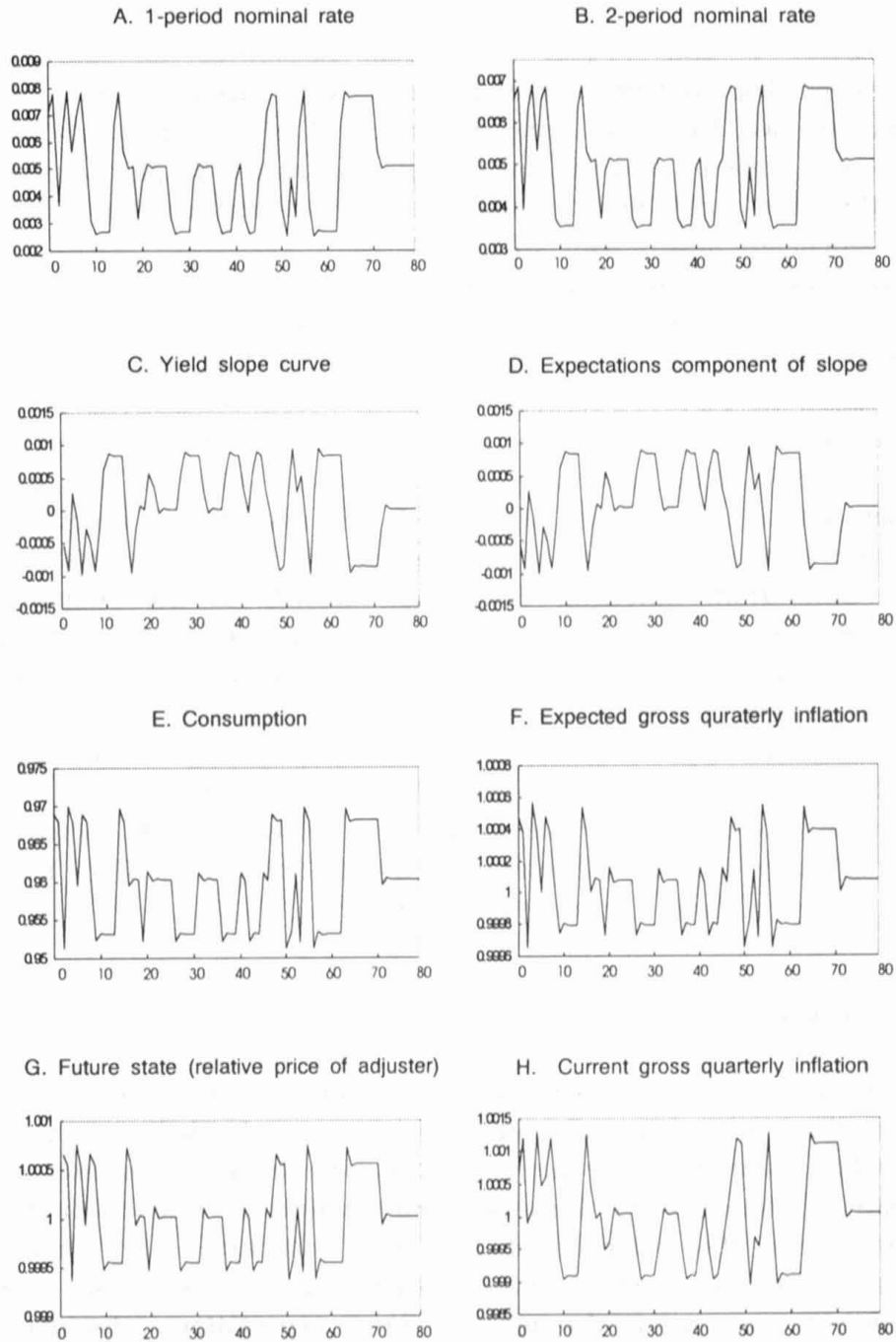
Figures 2 and 4 plot equilibrium sequences for the same variables displayed in Figures 1 and 3, for a given sequence of  $\vartheta_t$ . That is, we feed the same sequence of shocks into the high inflation economy and the low inflation economy and plot the paths of the seven variables. In both cases the initial condition is the nonstochastic steady state with  $\vartheta=0$ .

We highlight three features of these figures. First, even though the equilibrium functions in Figure 3 show zero nominal interest rates occurring when the inflation target is low, the time path displayed in Figure 4-A does not display any realizations of zero nominal rates; the explanation is that there is an ergodic set for the state variable, and zero nominal interest rates do not occur when  $s$  is in the ergodic set. Second, and almost as a corollary to the first point, the effect of inflation on the yield curve visible in the equilibrium functions essentially vanishes in the simulation. This is not simply because zero rates do not occur in the simulation, but because there is no probability attached to their occurring in the future.

Finally, note that the average level of consumption is higher under the low inflation target. This reflects the fact that low nominal interest rates make money less costly to hold, decreasing the effective price of consumption. We do not plot welfare, but it too is higher under the low inflation target.

## V. Conclusion

Because the average level of the nominal interest rate will be lower the lower is the inflation target, a central bank that targets a very low inflation rate is more likely to occasionally encounter zero nominal interest rates. Because nominal interest rates cannot go below zero, the behavior of the term structure may then be sensitive to the level of the inflation target. In principle, such sensitivity may work through the pure expectations channel or through the conditional covariance between future marginal utility and the future price of a short-term bond. This paper has begun to investigate the effect of the inflation target on the term structure in a simple staggered pricing model. The model is simple enough that



**FIGURE 4**  
 LOW INFLATION TARGET  
 SIMULATED TIME SERIES

we can study nearly exact solutions, and we can easily disentangle the pure expectations effect from the covariance effect.

For the basic calibration studied thus far, there is essentially no action from the covariance channel. The first order of business now is to study a calibration where the ergodic set for the endogenous state variable includes realizations for which nominal interest rates are zero. Other modifications of interest would involve incorporating mechanisms which add persistence to the model. One can see from Figures 3 and 4 that the state variable is negatively autocorrelated, and this property is inherited by other variables in the model, chiefly among them inflation. Adding many bells and whistles is unappealing however: Virtues of the model are that it is simple enough (i) to be solved nonlinearly to a high degree of accuracy without large computing requirements, and (ii) that one can work through exactly the mechanisms driving particular results.

*(Received 17 September 2005; Revised 10 November 2005)*

## References

- Bernanke, Ben S., Reinhart, Vincent R., and Sack, Brian P. "Monetary Policy Alternatives at the Zero Bound: An Empirical Assessment." In William C. Brainard and George L. Perry (eds.), *Brookings Papers on Economic Activity* (No. 2 2004): 1-100.
- Dixit, Avinash, and Stiglitz, Joseph. "Monopolistic Competition and Optimum Product Diversity." *American Economic Review* 67 (June 1977): 297-308.
- Eggertsson, Gauti B., and Woodford, Michael. "The Zero Bound on Interest Rates and Optimal Monetary Policy." In William C. Brainard and George L. Perry (eds.), *Brookings Papers on Economic Activity* (No. 1 2003): 139-211.
- Fujiki, Hiroshi, and Shiratsuka, Shigenori. Policy Duration Effect under the Zero Interest Rate Policy in 1999-2000: Evidence from Japan's Money Market Data. Manuscript, 2001. *Journal of Money, Credit and Banking*. Special Issue on Monetary Policy in a Low Inflation Environment Vol. 32, 2000.
- McGrattan, Ellen R. "Solving the Stochastic Growth Model with a Finite Element Method." *Journal of Economic Dynamics and*

- Control* 20 (January-March 1996): 19-42.
- Nagayasu, Jun. "The Term Structure of Interest Rates and Monetary Policy during a Zero Interest Rate Period." *Bank of Japan Monetary and Economic Studies* 22 (May 2004): 19-44.
- Oda, Nobuyuki, and Ueda, Kazuo. The Effects of the Bank of Japan's Zero Interest Rate Commitment and Quantitative Monetary Easing on the Yield Curve: A Macro-Finance Approach. Manuscript, 2005.
- Ravenna, Federico, and Seppälä, Juha. Monetary Policy and the Term Structure of Interest Rates. Manuscript, 2005.
- Ruge-Murcia, Francisco. The Expectations Hypothesis of the Term Structure When Interest Rates Are Close to Zero. Forthcoming in *Journal of Monetary Economics*.
- Skallsjö, Sven. Optimal Monetary Policy, the Zero Bound, and the Term Structure of Interest Rates. Manuscript, 2004.
- Svensson, Lars E. O. "Escaping from a Liquidity Trap and Deflation: The Foolproof Way and Others." *Journal of Economic Perspectives* 17 (Fall 2003): 145-66.
- Taylor, John B. "Aggregate Dynamics and Staggered Contracts." *Journal of Political Economy* 88 (February 1980): 1-24.
- Wolman, Alexander L. "Real Implications of the Zero Bound on Nominal Interest Rates." *Journal of Money, Credit and Banking* 37 (March 2005): 273-96.