Entrepreneurial Efficiency: An Empirical Framework and Evidence

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This paper examines a particular aspect of entrepreneurship, namely firms' activities in adapting to idiosyncratic environmental changes by appropriately reallocating resources. It presents an empirical framework that examines the social value of firms' abilities to predict and adapt to the movement of idiosyncratic shocks. In order to demonstrate how to implement our menthod, the quantitative effect of firms' prediction ability on Tatal Factor Productivity (TFP) is investigated using semi-aggregated data from Japan's Census of Manufacturing.

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JEL Classification: D21, D61, O40

I. Introduction

It has long been recognized that adaptation to idiosyncratic shocks is vital in several economic activities. In his well-known paper, Hayek (1945) regards "rapid adaptation to changes in particular circumstances of time and place" as one of the most important economic problems. Recently, much micro-evidence reconfirms the economic significance of both idiosyncratic shocks and adaptations

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to these shocks. For example, Davis and Haltiwanger (1999) review the literature and insist that unobserved idiosyncratic factors play a dominant role in explaining the redistribution of workers, and Hubbard (2003) finds that advanced on-board computers significantly increase capacity utilization in the trucking industry by improving dispatchers' abilities to make resource allocation decisions.

It can be conjectured that improvements in the ability to adapt to idiosyncratic shocks will have significant influences on the aggregate economy. One of the difficulties in examining this in macroeconomic terms is that macroeconomics has a tradition of working with an exogenously given aggregate production function. Because the relation between outputs and inputs is given by the aggregate production function, there is no need for an economic agent who finds a productive use for inputs. As aggregate production function is a cornerstone of the neoclassical growth model, incorporating entrepreneurship in the aggregate production function will give a tractable tool with which to examine entrepreneurship in macroeconomics, where entrepreneurship is defined as the activity of allocating resources in order to adapt to idiosyncratic shocks.

This paper aims to provide a tractable empirical framework to examine how firms' abilities to predict and adapt to idiosyncratic changes in the environment influence an aggregate economy. For this purpose, a firm's entrepreneurial ability is modeled by its ability to predict idiosyncratic changes in productivity and this paper derives an aggregate production function as a result of entrepreneurship.

The concept of prediction ability in this paper aims to capture the soundness of firms' judgments about the economic impacts of idiosyncratic changes. Firms face several idiosyncratic changes every day: local area traffic increases, a new firm takes away their many skillful engineers, a politician connected to a company loses an election. As productivity is estimated by the ratio of measured output to input, these factors can potentially influence measured productivity. Hence, predicting change in the productivity of one's own firm means predicting how various changes in the environment directly influence production or sales.

When changes in productivity occur, the marginal products of inputs deviate from input prices, and this generates opportunities for entrepreneurs to exploit. If entrepreneurs predict change and react to it appropriately, the deviation of the marginal products of the inputs from input prices will be small. Hence, the improvement in firms'

prediction ability raises allocative efficiency and therefore increases productivity in the economy. It is shown that the increased prediction ability of firms raises total factor productivity (TFP) of the aggregate production function in a competitive economy.

A novel part of this paper is that it provides an empirical framework with which to quantitatively examine the effect of entrepreneurship on TFP. It is shown that prediction ability can be measured by the squared correlation between a firm-specific shock and labor input. In other words, appropriate adaptation to idiosyncratic changes can be seen as evidence of better predictions about idiosyncratic changes in the environment.[1]

There are two advantages of using this measure. First, we can separate a firm's prediction ability from its flexibility. An improvement in flexibility increases not only the covariance between unexpected shocks and the firm's response, but also increases the variance of its response. Therefore, the correlation measure is relatively invariant to the firm's flexibility. It allows us to identify how prediction contributes to adaptation. Second, estimation does not require information observed by managers. Note that as most idiosyncratic shocks are unobservable, specifying all information that might influence entrepreneurs' expectations is not feasible. This paper shows that even if we do not know what entrepreneurs observe, we can still infer the economic value of local information from their behavior.

In order to demonstrate how to implement our method, we apply this method to investigate Japanese establishments during 1985-1999. Because it is difficult to access establishment level data, we must rely on semi-aggregated data by city and industry and investigate the average behavior of firms in a city and industry. Although this is an obvious limitation of our empirical study, it is interesting to investigate how well establishments in Japan adapted to changes in this period because it roughly corresponds to the Japanese boom in the 1980s and the long recession of the 1990s. Increasing the ability of firms to adapt to changes in the environment may have been quite an important issue for restructuring the Japanese economy in this period. Because of measurement

[1] Hence, the derived measure can be alternatively interpreted. It measures the appropriateness of a firm's adaption to idiosyncratic changes. The previous version of this paper interpreted this as the measure of adaptability.

problems caused by semi-aggregated data, we can only provide tentative results. But our estimate suggests that a rise in prediction ability had a small but significant positive impact on TFP growth in Japan during that period.

A similar view of entrepreneurs is emphasized by Kirzner (1973). Following Hayek (1945), Kirzner argues that entrepreneurial discovery about previously unknown events is the engine of market equilibrating processes and insists that an equilibrium analysis cannot capture the importance of entrepreneurial discovery. We suggest that using an equilibrium model is beneficial. Particularly, we quantify the effect of entrepreneurial discovery on TFP.[2]

From a different perspective, Schultz (1975) also defines entrepreneurial ability as the ability to process new information and to allocate resources to profitable opportunities. His idea is incorporated into an equilibrium model by Holmes and Schmitz (1990), Hassler and Mora (2000), and Takii (2003). However, no paper quantifies the social values of entrepreneurial ability, which is this paper's purpose.

As the review by Foster, Haltiwanger, and Krizan (1998) found, there are many empirical studies that investigate the impacts of resource allocation on the aggregate production function. This literature typically lacks a solid theoretical foundation. We show how entrepreneurial ability can influence the reallocation of resources. In fact, our estimated measure of prediction ability resembles the covariance term in Foster, Haltiwanger, and Krizan (1998), which they found accounts for a third of aggregate productivity. We hope that our approach suitably complements theirs.

This paper is organized as follows. The next section describes the model. Section III provides an empirical framework with which to examine the effect of firms' prediction abilities on TFP. Section IV implements the methods empirically and reports results from Japanese data. Section V concludes by summarizing the main results and discussing possible extensions.

[2] It may be argued that the method in this paper underestimates the role of entrepreneurial discovery because, as Kirzner (1997) pointed out, the major part of uncertainty may not be measured by Knightian risk. However, we believe that it is an important exercise to find a lower measure of economic value that entrepreneurship can produce from the data. It might clarify the benefits and limits of equilibrium analysis.

II. The Model

This section extends a simple general equilibrium model presented by Takii (2004) as being suitable for empirical study. It lays the foundations for examining the quantitative effects of entrepreneurship in the succeeding sections.

An agent can be an entrepreneur or a worker. Every firm needs an entrepreneur to organize it. Firms are continuously distributed on $[0, mN]$, where $m \in (0, 1)$ is the proportion of entrepreneurs in the total population, N. This implies that agents are assumed to be identical. Although the lack of heterogeneity among entrepreneurs forced us to ignore a size distribution of firms as emphasized by Lucas (1978), it allows us to focus on a different economic problem: the effect of entrepreneurship on productivity in an economy. First the representative entrepreneur's problem is described, then resource constraints are presented.

The Entrepreneur's Problem: An entrepreneur establishes a firm, employs capital stock and workers, and produces output. The entrepreneur faces the following production function:

$$Y_i = z_i A[F(K_i, TL_i)]^{\alpha}, \quad 0 < \alpha < 1,$$

where z_i is a firm-specific productivity shock for the i*th* firm, and Y_i, K_i, and L_i are the amounts of the i*th* firm's output, capital stock and labor input, respectively. The parameters α and A measure the span of control and the management productivity respectively, and T measures the effectiveness of labor, which is assumed to be influenced by common factors. We call this labor-augmenting technology in this paper. It is assumed that F exhibits constant returns to scale in K and L. By defining $f(k) = F(k, 1)$, where $k = K/TL$, we can express $F(K, TL)$ as a function of capital per unit of effective labor in production: $F(K, TL) = f(k)TL$. We assume that $f'(\cdot) > 0$, $f''(\cdot) < 0$, $\lim_{k \to 0} f(k) = 0$, $\lim_{k \to 0} f'(k) = \infty$ and $\lim_{k \to \infty} f'(k) = 0$.

It is assumed that management productivity is a function of the effectiveness of the entrepreneur. Because agents are homogeneous, the effectiveness of the entrepreneur is the same as that of workers, which is given by T. Assuming that $A = T^{1-\alpha}$, the production function can be written as:

$$Y_i = z_i[f(k_i)L_i]^{\alpha}T.$$

There are three advantages in assuming that $A=T^{1-\alpha}$. Firstly, this assumption implies that the production function has constant returns to scale in capital stock, labor and managerial input. Hence, it can be shown that the firm's profits are equivalent to the returns to managerial input in a competitive environment (Mas-Colell *et al.* 1995). Secondly, this production function, which has constant returns to scale, results from maximizing total output in a hierarchical organization, as Rosen (1982) has shown. Given Rosen's model, managerial input, T, is required to supervise different tasks. Thirdly, when T grows at a constant rate, this assumption guarantees the existence of a balanced growth path, which is roughly consistent with the movement of macro data.

An entrepreneur has an important task other than a supervisory one. Because the movement of z_i is unpredictable *ex ante*, when z_i changes, the entrepreneur must predict the direction and magnitude of this change in order to respond appropriately. When the entrepreneur makes production decisions, she/he observes a noisy signal, s_i, from which the realization of z_i can be inferred. It is assumed that the entrepreneur's inference is based on a conditional distribution function, $Q^h(z|s) \equiv \int^z q^h_{s/z}(s|x)q_z(x)dx / \int q^h_{s/z}(s|x)q_z(x)dx$, where $q^h_{s/z}(s|z)$ is a conditional density function of s given the realization of z and $q_z(z)$ is a marginal density function of z. Note that $q^h_{s/z}(s|z)$ depends on h, which measures the firms' ability to predict the changes in a firm-specific productivity shock. If the firm can accurately predict the realization of z_i, the firm has a likelihood function endowed with high h and is considered to have precise knowledge about the nature of shifting the shock. As the knowledge makes it possible for the firms to adequately adapt to the changes, the parameter, h, can be interpreted as the index of firms' entrepreneurial ability. The conditional distribution function is the same for all entrepreneurs. It implies that all entrepreneurs share the same knowledge about the relation between the productivity shock and the observable signal. A more detailed information structure is subsequently specified.

Note that z is assumed to be an idiosyncratic shock. Hence, the information required to infer z must be local information. However, as both z and s are idiosyncratic, prices in this model do not depend on them. Hence, prices are predictable without knowing what others

observe. That is, entrepreneurs do not need to know all the local economic information because the price system summarizes the information they need. This is the role of the price mechanism emphasized by Hayek (1945).

It is assumed that the financial market is complete. Therefore, entrepreneurs can hedge against idiosyncratic risks. Entrepreneurs maximize their firms' expected profits:

$$\pi(s_i)=\max_{k,L}\{\int z_i[f(k_i)L_i]^{\alpha}TdQ^h(z_i\mid s_i)-wTL_i-rk_iTL_i\},$$

where w is the wage rate for effective labor and r is the rental price of capital. The first-order conditions are:

$$w=\alpha\int z_i\,dQ^h(z_i\mid s_i)[f(k(s_i))L(s_i)]^{\alpha-1}f(k(s_i))-rk(s_i), \tag{1}$$

$$r=\alpha\int z_i\,dQ^h(z_i\mid s_i)[f(k(s_i))L(s_i)]^{\alpha-1}f'(k(s_i)), \tag{2}$$

for any s, where $k(s)$ and $L(s)$ are the optimal levels of k and L.

Expected profits are derived by substituting the two first-order conditions into $\pi(s_i)$.

$$\int\pi(s)dQ_s^h(s)=(1-\alpha)z(h)^{\frac{1}{1-\alpha}}\left[\frac{\alpha f(k)}{w+rk}\right]^{\frac{\alpha}{1-\alpha}}T \tag{3}$$

$$z(h)=\left[\int\left[\int z_i dQ^h(z_i\mid s_i)\right]^{\frac{1}{1-\alpha}}dQ_s^h(s_i)\right]^{1-\alpha},$$

where $Q_s^h(s)\equiv\int q_{s/z}^h(s\mid x)q_z(x)dx$. Equation (3) shows that expected profit positively depends on $z(h)$, the component examined below. Note that k does not depend on s_i, shown from Equations (1) and (2).

The components of $z(h)$**:** Assume that $\log z$ comprises a predictable component μ and an unpredictable component u:

$$\log z=\mu+u,$$

where u is normally distributed with mean 0 and variance σ_u^2. It is

assumed that the unpredictable component u summarizes an unexpected change in productivity. The entrepreneur cannot observe u before making production decisions, but can observe the signal s:

$$s=u+\varepsilon,$$

where ε is normally distributed with mean 0 and variance $\sigma_{\varepsilon}^2(h)$. We apply Takii's (2003) measure of prediction ability in this context. Let $Q_u^h(u|s)(\equiv \int^u q_{s/u}^h(s|x)q_u(x)dx/\int q_{s/u}^h(s|x)q_u(x)dx)$ denote the conditional distribution of u given s. The measure of an entrepreneur's ability to predict unexpected change, u, is defined as follows.

Definition 1: *The measure of an entrepreneur's ability to predict unexpected change u is defined by:*

$$h=1-\frac{\int Var(u|s)dQ_s^h(s)}{\sigma_u^2},$$

where $Var(u|s)=\int (u-\int u dQ_u^h(u|s))^2 dQ_u^h(u|s)$.

This measure implies that the entrepreneur accurately recognizes u when she/he reduces on average the conditional variance having observed s. To compare ability in different environments, $\int Var(u|s)dQ_s^h(s)$ is divided by σ_u^2, which is the unconditional variance of u. The measure h ranges from 0 to 1. If the entrepreneur perfectly predicts the change, $h=1$, whereas if the entrepreneur does not predict change at all, it is $h=0$.

It shows that:

$$z(h)=z^e\exp[\frac{\alpha\sigma_u^2 h}{2(1-\alpha)}],$$

where $z^e=\exp\{\mu+\sigma_u^2/2\}$. This equation shows that $z(h)$ can be decomposed into the predicted productivity, μ, the risk from unpredicted changes, σ_u^2, and prediction ability, h. It shows that $z(h)$ is an increasing function of h. This means that a rise in h increases expected profits, a proposition empirically supported by Takii (2003). Rather, this paper examines the social value of prediction ability.

The Arbitrage Condition and Resource Constraints: Because entrepreneurs can hedge their risks in the financial market, they do not bear risk. As agents, they are identical and can be entrepreneurs or workers, and expected profits must be equal to the opportunity costs of being an entrepreneur, which is the wage rate in the labor market.

$$\int \pi(s) dQ_s^h(s) = wT. \tag{4}$$

To close the model, the labor and capital markets must clear:

$$K^a = mNkT \int L(s) dQ_s^h(s), \tag{5}$$

$$(1-m)N = mN \int L(s) dQ_s^h(s), \tag{6}$$

where K^a is the aggregate capital stock. Equation (5) is the capital market clearing condition. The left-hand side is the supply of capital, and the right-hand side is the demand for capital: mN is the number of firms, and $kT \int L(s_l) dQ(s_l)$ is the average firm's demand for capital. Equation (6) is the labor market clearing condition. The left-hand side is the supply of labor, and the right-hand side is labor demand.

Aggregate Production Function: Let us define $\theta(k) = f'(k)k/f(k)$. Let Y^a and y^a denote aggregate output and aggregate output per unit of effective labor in an economy, $y^a \equiv Y^a/TN$, respectively. The following proposition is a direct application of the results in Takii (2004) to the model with productivity growth. Hence, I omit the proof.

***Proposition 2*:** *Suppose that* $\lim_{k\to\infty}\theta(k) < 1$ *and* $\lim_{k\to 0}\theta(k) < 1$. *Then, for any* $k^a \equiv K^a/TN \in (0, \infty)$, *there exists an aggregate production function,* $\phi(k^a)$, *which satisfies Equations (1), (2), (3), (4), (5), and (6)*:

$$y^a = z(h)\phi(k^a) \tag{7}$$

$$\text{where } z(h) = z^e \exp\left[\frac{\alpha \sigma_u^2 h}{2(1-\alpha)}\right].$$

The derived aggregate production function is increasing and concave in $k^a \in (0, \infty)$, *and satisfies the Inada conditions:* $\phi'(k^a) > 0$, $\phi''(k^a) < 0$,

$\lim_{k^a \to 0} \phi(k^a) = 0$, $\lim_{k^a \to \infty} \phi'(k^a) = 0$, *and* $\lim_{k^a \to 0} \phi'(k^a) = \infty$.

In order to understand the intuitive logic of this result, note that firms equate the wage rate (rental price) to the expected marginal product of labor (capital), not to the actual marginal product of labor (capital). Idiosyncratic shocks cause the actual marginal products to deviate from input prices, and these deviations provide opportunities for entrepreneurs to exploit. If entrepreneurs accurately predicted change, the deviations of the actual marginal productivities of inputs from input prices would be small. This means that actual marginal productivities of inputs across firms would be closer. Hence, an improvement in a firm's prediction ability raises allocative efficiency, and therefore increases economic productivity. Proposition 2 shows that an increase in productivity is represented by a rise in the TFP of the derived aggregate production function and that an increase in the firm's prediction ability raises GDP per unit of effective labor in an economy.

III. A Framework for an Empirical Study

This section proposes an empirical framework to quantify the effect of firms' prediction abilities on the aggregate TFP and presents several propositions linking unobserved parameters to observable data. The next section demonstrates how to implement the propositions of this section using semi-aggregated data from Japan's Census of Manufacturing.

The growth rate of the aggregate TFP, g_{TFP}, is usually defined as $g_{TFP} \equiv g_{Y/N} - \phi'(k^a)k^a / \phi(k^a) g_{K/N}$, where $g_{Y/N}$ and $g_{K/N}$ are the growth rates of GDP *per capita* and capital stock *per capita*, respectively. Proposition 2 implies that:

$$g_{TFP} \approx g_{z^e} + (1 - \frac{\phi'(k^a)k^a}{\phi(k^a)}) g_T + \frac{\alpha}{2(1-\alpha)} [hd\sigma_u^2 + \sigma_u^2 dh]. \quad (8)$$

Hence, we are primarily interested in the following regression equation:

$$\Delta \log TFP = \psi_0 + \psi_\mu \Delta\mu + \psi_\sigma^1 \Delta\sigma_u^2 + \psi_T \Delta \log T + \psi_\sigma^2 h_t \Delta\sigma_u^2 + \psi_h \sigma_{ut-1}^2 \Delta h + \varepsilon, \quad (9)$$

where ψ_0, ψ_μ, ψ_T, ψ_σ^1, ψ_σ^2, and ϕ_h are constant parameters. The growth rate of T, $\Delta\log T$, represents labor-augmenting technological progress. It captures an aggregate trend of technological progress. A change in μ, $\Delta\mu$, can be interpreted as an increase in firm-specific productivity. After controlling these two effects, our theory predicts that a change in both risk and prediction ability has a positive effect on the growth rate of TFP. The estimation of this equation requires estimates of the variables, $\Delta\log TFP$, $\Delta\mu$, $\Delta\log T$, σ_u^2, and h, described below.

Estimation of $\Delta\log TFP$ and $\Delta\log T$: First, we derive the equations that relate $\Delta\log TFP$ and $\Delta\log T$ to observable variables, and then provide an interpretation of each equation. The following proposition explains the estimation of $\Delta\log TFP$ and $\Delta\log T$. Proof is given in the Appendix.

Proposition 3: *If w is constant, the growth rate of TFP, $\Delta\log TFP$, and aggregate productivity growth, $\Delta\log T$, can be estimated as follows:*

$$\Delta\log TFP=\Delta\log\frac{Y}{N}-\frac{\phi'(k^a)k^a}{\phi(k^a)}\Delta\log\frac{K}{N} \tag{10}$$

$$\Delta\log T=\Delta\log wT, \tag{11}$$

where $\phi'(k^a)k^a/\phi(k^a)$, $\alpha\theta(k)$ and α are estimated by:

$$\frac{\phi'(k^a)k^a}{\phi(k^a)}=\alpha\theta(k)=\frac{1}{\int\left[\frac{Y(z,s)}{rK(s)}\right]dQ_{zs}^h(z,s)}, \tag{12}$$

$$Y(z,s)\equiv z\left[f(k)L(s)\right]^\alpha T,\ K(s)\equiv kTL(s).$$

Equation (10) is the usual definition of TFP growth, except the method of estimating the elasticity of output with respect to capital is unusual. Equation (12) shows this can be estimated by the average capital share. Note that the definition of the average capital share corresponds to the usual definition of capital share when there is no random component.

Equation (11) shows that labor-augmenting technological progress can be estimated by the growth rate of the average wage. When there is economy-wide technological progress, competition in the labor

market pushes up workers' wage rates. Equation (11) reflects this intuition.

The wage rate per unit of effective labor, w_t, is an endogenous variable. It would change when there is systematic change in μ, σ_u^2, and h. A justification for this assumption of constant w_t is discussed in the next section.

Estimation of $\Delta\mu$ and σ_u^2: Next, we derive the equations for the estimation of $\Delta\mu$ and σ_u^2. For this purpose, two different assumptions are considered separately. The first assumption is that k_t^a is constant over time. This assumption is valid when the economy is in a steady state. The second assumption is that $f(k)$ is Cobb-Douglas. Therefore, two different assumptions bring two different estimates. These estimates are used to check the robustness of the empirical results below.

Let $E[x]$ and $Var(x)$ denote the expectation and the variance of x. First, we assume k_t^a is constant. Given this assumption, we can derive the following proposition. Proof is given in the Appendix.

Proposition 4: *Suppose that w_t and k_t^a are constant. Then a change in firm specific productivity, $\Delta\mu$, and the measure of risk, σ_u^2, can be estimated by a change in expectation and the variance of* $\log Y - \alpha \log wTL - (1-\alpha)\log wT$:

$$\Delta\mu = \Delta E[\log Z_1(z,s)], \tag{13}$$

$$\sigma_u^2 = Var[\log Z_1(z,s)], \tag{14}$$

where:

$$\alpha = \frac{1}{\int [\frac{Y(z,s)}{rK(s)+wTL(s)}]\, dG_{zs}^h(z,s)}, \tag{15}$$

$$\log Z_1(z,s) = \log Y(z,s) - \alpha \log wTL(s) - (1-\alpha)\log wT.$$

If constant k_t^a is restrictive, an alternative method is to specify the production function. Assume that $f(k) = Bk^\beta$. Then we can propose an alternative proposition. As the proof is similar to the proof of proposition 4, it is not repeated.

***Proposition 5*:** *Suppose that* w *is constant and* $f(k)=Bk^{\beta}$. *Then a change in firm specific productivity,* $\Delta\mu$, *and the measure of risk,* σ_u^2, *can be estimated by a change in expectation and variance of* $\log Y-\alpha(1-\beta)\log wTL-\alpha\beta\log K-(1-\alpha)\log wT$:

$$\Delta\mu=\Delta E\,[\log Z_2(z,s)], \tag{16}$$

$$\sigma_u^2=Var\,[\log Z_2(z,s)], \tag{17}$$

where:

$$\alpha(1-\beta)=\frac{1}{\int\left[\frac{Y(z,s)}{wTL(s)}\right]dQ^h_{zs}(z,s)},\quad \alpha\beta=\frac{1}{\int\left[\frac{Y(z,s)}{rK(s)}\right]dQ^h_{zs}(z,s)},$$

$$\log Z_2(z,s)=\log Y(z,s)-\alpha(1-\beta)\log wTL(s)-\alpha\beta\log K(s)-(1-\alpha)\log wT.$$

Note that $\log Z_1(z,s)$ and $\log Z_2(z,s)$ can be interpreted as a firm-specific shock. $\log Z_1(z,s)$ is the component of value added that cannot be explained by the labor expenses or wage rates, and $\log Z_2(z,s)$ is the component of value added that cannot be explained by the labor expense, capital input or wage rates. Hence, both measure the levels of productivity excluding the contribution represented by wage rates. As the movement of aggregate shocks must be captured by that of wage rates, both the estimated $\log Z_1(z,s)$ and $\log Z_2(z,s)$ exclude aggregate shocks in productivity. Hence, the average firm-specific productivity is the component of $E\,[\log Z_1(z,s)]$ and $E\,[\log Z_2(z,s)]$. Similarly, σ_u^2 can be interpreted as the measure of risk due to changes in firm-specific productivity.

Estimation of h**:** Next, we explain the estimation of h. It is subsequently shown that h can be estimated by the correlation between the unexpected shock and the reaction to the shock. If a firm recognizes the change and reacts to it, this correlation must be high. To confirm this intuition, we define the reaction to the shock.

***Definition 6*:** *The firm's reaction to the shock* $R(L(s))$, *is defined as the logarithm of the deviation of labor input,* $L(s)$, *from predicted labor input,* L^*:

$$R(L(s))=\log L(s)-\log L^*,$$

where L^* is estimated from the input level in the absence of an unexpected shock:

$$L^* = \left\{ \frac{z^e}{w} \alpha [f(k) - f'(k)k] \right\}^{\frac{1}{1-\alpha}} f(k). \tag{18}$$

Equation (18) is derived by substituting z^e into the first-order conditions (1) and (2) for $\int z_t dQ^h(z_t | s_t)$. Using the definition of the firm's reaction to the shock, the following theorem is proved in the Appendix.

Theorem 7: *A firm's ability to predict idiosyncratic shocks, h, can be estimated by the correlation between u and R(L(s)):*

$$h = [\rho_{uR(L(s))}]^2, \ \rho_{uR(L(s))} \geq 0,$$

where

$$\rho_{uR(L(s))} = \frac{\int u(R(L(s)) - \int R(L(s)) dQ_s^h(s)) dQ_{us}^h(u,s)}{\sqrt{(\int u^2 dQ_u(u) \int (R(L(s)) - \int R(L(s)) dQ_s^h(s))^2 dQ_s^h(s)}}.$$

Theorem 7 shows that h can be estimated by the squared correlation between the unexpected shock and the firm's reaction to the shock. The proof is based on the first-order condition (1). The entrepreneur employs more than the predicted level of labor input when it is believed that a positive productivity shock has been realized, and employs less than the predicted level of labor when it is believed that a negative one has occurred. When the entrepreneur's belief is accurate, then the correlation must be larger.

To implement this idea, we need to estimate L^*. This involves the estimation of an unknown function $f(\cdot)$. However, if k^a is constant, it is shown that $f(k)$ is also constant. Because the correlation coefficient is invariant to an affine transformation of a variable ($\rho_{XY} = \rho_{X(\eta Y + \iota)}$, where η and ι are constant), the correlation between the unexpected shock and the reaction to the shock can be estimated without using the function $f(\cdot)$. The following corollary can be proved from the definition: u and $R(L(s))$.

Corollary 8: *If w_t and k_t^a are constant, the correlation between the unexpected shock and the reaction to the shock can be estimated by*

the correlation between $\log Y-\alpha\log wTL-(1-\alpha)\log wT$ *and* $\log wTL-\alpha\log wTL-(1-\alpha)\log wT$.

Corollary 8 shows the main factor affecting h is the correlation between value added and labor expenses. This is a fairly crude measure of prediction ability. If we could explicitly model information that entrepreneurs observe, a more accurate measure might be obtained. However, observable data are less likely to reflect the ideas of Hayek (1945) and Kirzner (1973), who emphasized the importance of unobservable local information.

The correlation measure reflects the value of local information. To understand this, it is helpful to modify the equations in corollary 8.

$$\log Y-\alpha\log wTL-(1-\alpha)\log wT=\log Y-\alpha\log L-\log wT, \tag{19}$$

$$\log wTL-\alpha\log wTL-(1-\alpha)\log wT=(1-\alpha)\log L. \tag{20}$$

As argued before, Equation (19) measures firm-specific productivity, as an aggregate productivity shock must also increase wT. The two equations show that h can be measured by the correlation between a firm-specific shock and labor input. It means that prediction ability can be estimated by how appropriate the adaptation to the idiosyncratic changes are. Hence, despite potential problems, it is likely that the correlation measure contains useful information about the ability of firms to process local information.

The correlation measure can be affected by various factors, including talent levels in management groups, education, personal networks, population density, regional transportation costs, and communication costs within organizations. In the absence of a theory that determines firms' predictions, identifying the factors that enhance entrepreneurship is beyond the scope of this paper. However, we doubt that the correlation measure would be greatly affected by factors affecting adjustment costs, as adjustment costs are lower not only in the covariance of value added and labor expenses, but also in their variances.

Similarly to the estimation of $\Delta\mu$ and σ_u^2, if constant k^a is a restrictive assumption, alternatively, we can assume $f(k)=Bk^{\beta}$. Then the following corollary is easily proved.

Corollary 9: *Suppose that* w *is constant and* $f(k)=Bk^{\beta}$. *Then, the correlation between the unexpected shock and the reaction to the shock can be estimated by the correlation between* $\log Y-\alpha(1-\beta)\log wTL-\alpha\beta\log K-(1-\alpha)\log wT$ *and* $\log wTL-\alpha(1-\beta)\log wTL-\alpha\beta\log K-(1-\alpha)\log wT$.

IV. Evidence from Japanese Data

This section implements empirical methodology using data from Japan. Firstly, we describe the data and how to estimate each variable from this data. Secondly, summary statistics are reported. Finally, the regression results are reported.

Data Description: Proxies for Y, K, wTL, wT, and r were constructed mainly from the Census of Manufacturing in Japan for 1985-99, provided by I-N Information Systems, Ltd. Every year the Japanese Ministry of Economy, Trade and Industry releases the Census of Manufacturing by city and industry. It covers all establishments in which four or more persons work as employers or employees. However, because of minor changes in the classification of industries and the integration and division of cities, the data released must be modified for use in panel data analysis. I-N Information Systems, Ltd undertakes this modification and thereby enables panel data analysis of the behavior of the average establishment by city and industry. More details of the data and the construction of variables are given in the Appendix.

We split the data into two periods: 1985-91 and 1992-99. These periods roughly correspond to before and after the burst of Japan's economic bubble. We estimate $E[\log Z_1(z,s)]$, $E[\log Z_2(z,s)]$, σ_u^2, and h using the constructed Y, wTL, wT, r, and K, by city, industry and period. This estimation is based on sampled averages over time. It gives us estimates for the average establishments by city, industry and period. Then, we estimate the representative values for the aggregate production function from the weighted average of $E[\log Z_1(z,s)]$, $E[\log Z_2(z,s)]$, σ_u^2, and h by prefecture, industry and period, with the number of establishments in 1988 and 1996 as weights. Finally, we take the difference of these representative values in two periods. Hence, $\Delta\mu$, $\Delta\sigma_u^2$, and Δh are estimated by prefecture and industry.

In order to estimate TFP growth, we estimate the aggregate output, aggregate capital and aggregate number of workers by prefecture and industry in 1988 and 1996. Then $\Delta \log TFP$ is estimated by prefecture and industry. That is, we estimate the aggregate production function's TFP growth between 1988 and 1996 by prefecture and industry. We also estimate the weighted average of wT by prefecture and industry in 1988 and 1996. Then we estimate $\Delta \log wT$ from the difference between 1988 and 1996 by prefecture and industry. Our regression analyses use these estimated variables, $\Delta \log TFP$, $\Delta \log wT$, $\Delta \mu$, σ_u^2, h, $\Delta \sigma_u^2$, and Δh, which span a cross-section of prefecture and industry. Precise definitions of variables are presented in the Appendix.

We need discussions for our empirical strategies. Firstly, our aggregate production function is based on prefecture and industry. This is necessary for an empirical study. Although we assume that parameters, α, β, μ, σ_u^2, h are the same in theory for all firms, it is less likely. The chemical industry demands more capital than the textile industry. Hence, the capital share in the chemical industry would be larger. It will certainly influence α and β. The measure of h is supposed to capture the information advantages of a firm. A big city may have this advantage. Hence, it is more reasonable to aggregate variables by prefecture and industry. There is also an alternative reason. For the purpose of estimation, we must assume that w is constant. There could be objections that a constant w is not consistent with our theory, because shifts in z^e, h, and σ_u^2 change w. However, as workers can move between prefectures and industries, w is approximately the same in all prefectures and industries in a competitive labor market. To the extent that z^e, h, and σ_u^2 do not change on average, a constant w is justified. In fact, the data support this assumption, as is subsequently shown. Hence, to satisfy the assumption of constant w, this level of aggregation is preferable.

Secondly, for the purpose of estimating unobserved variables, we split the data into two periods. One of the assumptions of this estimation is the constant k^a. It requires the economy to be in a steady state. However, the steady-state assumption is not consistent with the regression Equation (9), because if the economy is in a steady state, $g_{TFP} \approx (1 - \phi'(k^a)k^a / \phi(k^a))g_T$. In order to maintain consistency in our analysis, we implicitly assume that the economy was in a steady state during 1985-91, and 1992-99. A large shock is

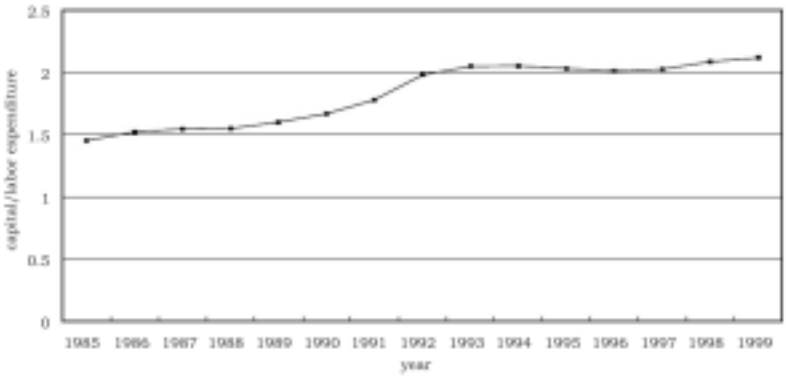

FIGURE 1
CAPITAL PER LABOR EXPENDITURE

assumed to have occurred around 1991, which caused the economy to move away from a steady state. The values of $E[\log Z_1(z,s)]$, $E[\log Z_2(z,s)]$, h, and σ_u^2 during the transition period are approximated by the steady-state values of $E[\log Z_1(z,s)]$, $E[\log Z_2(z,s)]$, h, and σ_u^2. Because the Japanese economic bubble burst in 1991, it is not unreasonable to assume that the steady state changed around this year. In fact, the ratio of aggregate capital to aggregate labor expenditure was roughly constant during the period 1985-91, and 1992-99, and it increased around 1991 (Figure 1). Given the assumption of constant w, a change in aggregate capital per aggregate labor expenditure captures a shift in k^a. Hence, this evidence is consistent with our argument above. We also examine the alternative assumptions of $f(k)=Bk^\beta$ in order to examine the robustness of our argument.

Summary Statistics: Table 1 reports the summary statistics of our estimates for $\Delta \log TFP$, $\Delta \mu$, $\Delta \log T$, $\Delta \sigma_u^2$, and Δh. The annual TFP growth rate is about 2% and the growth rate of labor-augmenting technology is 1.7%. This means that aggregate productivity growth accounts for most growth in TFP, which is broadly consistent with the steady-state assumption.

On average, firm-specific productivity declined slightly (-0.0018--0.0058), and the level of idiosyncratic risk remained constant. The average of this measure of firms' prediction ability increased modestly

TABLE 1
SUMMARY STATISTICS 1 (1988-1995)

Variable	Obs.	Mean (1988-1996)	Std. Dev.	Mean (annual)
$\Delta \log TFP$	800	0.165*	0.298	0.0206
$\Delta \mu(f(k)=Bk^{\beta})$	800	-0.046*	0.287	-0.0058
$\Delta \mu(steady)$	800	-0.014*	0.294	-0.0018
$\Delta \log T$	800	0.139*	0.088	0.0174
$\Delta \rho_{uR(L(s))}(f(k)=Bk^{\beta})$	800	0.053*	0.322	0.0066
$\Delta \rho_{uR(L(s))}(steady)$	800	0.024*	0.315	0.003
$\Delta h(f(k)=Bk^{\beta})$	587	0.028*	0.152	0.0035
$\Delta h(steady)$	592	0.023*	0.153	0.0029
$\Delta \sigma_u^2(f(k)=Bk^{\beta})$	800	-0.003	0.070	-0.0004
$\Delta \sigma_u^2(steady)$	800	-0.003	0.070	-0.0004

Notes: 1) "*Steady*" means that the steady state is assumed, and "$f(k)=Bk^{\beta}$" means that $f(k)$ is assumed to be Cobb-Douglas for the purposes of estimation.
2) * indicates significance at the 5% level.

(0.0029-0.0035). This modest increase in prediction ability is confirmed by a simple correlation between an unexpected shock and firms' reactions to the shock (0.003-0.0066). The increase in the prediction ability measured during the 1990s is interesting. Although Miyagawa (2003) argues that both the labor and capital markets became more rigid during the 1990s, evidence suggests that firms' ability to appropriately adapt to changes in the environment did not decrease.

This result also means that the measure of prediction ability is not greatly affected by adjustment cost. Although adjustment is expected to be more difficult during a recession, the data show that, on average, Japanese firms become better able to adapt to idiosyncratic changes. As already discussed, the use of the correlation measure corresponds broadly to excluding the effect of adjustment costs. The data lend some support to this argument.

Note that the movement of firm-specific productivity growth, risk and prediction ability is much smaller than the movement of $\log T$. Moreover, changes in firm-specific productivity and prediction ability are opposite, and there is no movement in risk. These observations imply that even though there may be shifts in the industry and prefecture-level aggregate production functions, the economy-wide

TABLE 2
SUMMARY STATISTICS 2

Variable	Obs.	Mean	Std. Dev.	Period
$\rho_{uR(L(s))t}$ ($f(k)=Bk^{\beta}$)	800	0.258	0.242	1992-1999
$\rho_{uR(L(s))t}$ (*steady*)	800	0.243	0.241	1992-1999
h_t ($f(k)=Bk^{\beta}$)	587	0.132	0.124	1992-1999
h_t (*steady*)	592	0.128	0.125	1992-1999
σ^2_{ut-1}($f(k)=Bk^{\beta}$)	800	0.034	0.063	1985-1991
σ^2_{ut-1}(*steady*)	800	0.035	0.063	1985-1991

Note: "*Steady*" means that the steady state is assumed, and "$f(k)=Bk^{\beta}$" means that $f(k)$ is assumed to be Cobb-Douglas for the purposes of estimation.

aggregate production function would be roughly stable, which guarantees that w does not vary. This provides empirical justification for our assumption that w is constant.

There are fewer observations on Δh than on $\Delta\rho_{uR(L(s))}$. To estimate Δh, we require a positive correlation in both periods. Twenty-six percent of our observations do not satisfy this condition. This apparent irrationality may indicate that some assumptions might be unrealistic. In particular, we assume that all firms know the unconditional mean of the shock, and we define an unexpected change as the deviation from the unconditional mean. If a firm's subjective belief about the unconditional mean of the shock differs from the objective one, a negative correlation might be possible. To check the robustness of the results, we used the simple correlation as an alternative measure of prediction ability. Using this simple correlation, we can use all the observations.

Table 2 reports the summary of statistics for σ^2_{ut-1} and h_t. Subscript $t-1$ means the variables are estimated data during 1985-1991, and subscript t means the variables are estimated data during 1992-1999. The level of prediction ability, h, is about 0.13 during 1992-1999. It seems like a small number, which is confirmed from the simple correlations, $\rho_{uR(L(s))t}$, which is around 0.25. The simple correlations $\rho_{uR(L(s))t}$ are larger than h because h is the square of the simple correlations. This small number might imply that the adaptation to shocks are not always appropriate. On the other hand, it may be simply a result of measurement errors. The issue of the

TABLE 3

THE EFFECT OF FIRMS' ADAPTABILITY ON TFP

	$f(k)=Bk^{\beta}$	$f(k)=Bk^{\beta}$	*Steady*	*Steady*
$\Delta \log T$	0.802***	0.981***	0.769***	0.954***
	(0.096)	(0.118)	(0.096)	(0.114)
$\Delta \mu$	0.558***	0.603***	0.534***	0.597***
	(0.029)	(0.035)	(0.029)	(0.034)
$\Delta \sigma_u^2$	0.018	0.303	0.110	0.954***
	(0.139)	(0.302)	(0.141)	(0.281)
$\rho_{uR(L(s))t} \Delta \sigma_u^2$	-0.399		-0.681	
	(0.388)		(0.364)	
$\sigma^2_{ut-1} \Delta \rho_{uR(L(s))}$	0.866*		1.236***	
	(0.345)		(0.330)	
$h_t \Delta \sigma_u^2$		-1.799		-4.17***
		(1.440)		(1.029)
$\Delta \sigma^2_{ut-1} \Delta h$		1.620*		6.680***
		(0.775)		(0.977)
constant	0.079***	0.062***	0.066***	0.045*
	(0.016)	(0.019)	(0.016)	(0.018)
Adj. R-squared	0.389	0.413	0.397	0.426
Obs.	800	587	800	592

Notes: 1) "*Steady*" means that the steady state is assumed, and "$f(k)=Bk^{\beta}$" means that $f(k)$ is assumed to be Cobb-Douglas for the purposes of estimation.

2) * indicates significance at the 5% level, ** indicates significance at the 1% level, and *** indicates significance at the 0.5% level.

3) Standard errors are in parentheses.

measurement errors is discussed below.

Regression Results and Their Interpretation: Table 3 reports our regression results. All regressions show that a change in firms' prediction ability increases TFP.[3] This is consistent with the predictions of our theory.[4]

[3] This result is robust. For robustness checks, we included employment or the number of establishments to control scale effects. We also added the growth rate of employment to check whether high correlation picks up the effect of growing firms. Including these variables did not change results. We also used weighted least squares estimations using the square root of the number of cities as weights. Some regression results are insignificant, but the coefficients were always positive.

[4] Takii (2004) shows that if political shocks are important, improvements in

$$\Delta \log TFP = \psi_0 + \psi_\mu \Delta\mu + \psi_\sigma^1 \Delta\sigma_u^2 + \psi_T \Delta \log T + \psi_\sigma^2 h_t \Delta\sigma_u^2 + \psi_h \sigma_{ut\text{-}1}^2 \Delta h + \varepsilon$$

In order to examine its importance, we took the largest value of our estimate, $\psi_h = 6.7$, for our exercise. As average risk is 0.035, a coefficient of 6.7 implies that if h changes from 0 to 1, TFP increases by 23%. At first glance, it does not look like a small number. However, note that Δh increases annually by 0.0035. This means that an increase in h raised annual TFP growth rate by 0.08% in Japan. As annual TFP growth is about 2%, the contribution of an increase in prediction ability on TFP growth is less than 5%. This is a small number.

Although we implicitly assume that estimating a representative h for the aggregate production function eliminates measurement errors, this assumption might be questionable. A measurement error typically causes effects to be underestimated. This might be a reason for this small number. In order to examine this concern, we estimated the coefficient using a different approach. Note the theory predicts:

prediction ability cause negative externality: if somebody is good at seizing opportunities, it reduces the opportunities of others. In this case, Takii (2004) shows that μ is endogenously chosen by:

$$\mu = \frac{-\alpha\sigma_u^2 h}{(1-\alpha)} - \frac{\sigma_u^2}{2} .$$

Although individual entrepreneurs react to political shocks given μ, as these reactions do not produce a new value in the economy, prediction ability lowers μ. If this indirect effect is important, although we find positive coefficient of ψ_h, then the overall effect of an increase in h could be negative. This possibility is examined by the following regression equation:

$$\Delta\mu = \phi + \phi_\sigma^1 \Delta\sigma_u^2 + \phi_T \Delta \log T + \phi_\sigma^2 h_t \Delta\sigma_u^2 + \phi_h \sigma_{ut-1}^2 \Delta h + \varepsilon.$$

We find that ϕ_h is typically positive and not significant. Hence, this concern is dismissed. Although Peek and Rosengren (2005) emphasize the existence of a misallocation of credit in the 1990s, this evidence suggests that the distortion may not be sufficiently strong to change the direction of adaptability in the TFP. In fact, although Nishimura, Nakajima, and Kiyota (2005) find that unproductive firms survive in the Japanese market, this occurred only during the period 1996-1997.

$$\psi_h = \frac{\alpha}{2(1-\alpha)} .$$

We estimated average α from our sample. Our estimated average $\alpha=0.54$ when we assume $f(k)=Bk^{\beta}$ and 0.51 when we assume the steady state. Using this number, $\psi_h=0.64$ when we assume $f(k)=Bk^{\beta}$ and 0.56 when we assume the steady state. Both numbers are much smaller than 6.7. Hence, different approaches do not change results and the impact is small.

The estimated $\alpha=0.54$ is smaller than the commonly used number in the literature: Atkeson and Kehoe (2001) use 0.85 and Chang (2000) uses 0.8. Probably, it is because Japanese establishments are much smaller than in the U.S. However, it is interesting to estimate our ψ_h using a common estimate. Take $\alpha=0.85$. It means that $\psi_h=2.8$. Again this is smaller than 6.7. Hence, our tentative conclusion is that the effect of entrepreneurship on productivity growth is small.

This result is tentative because our estimates probably understate real effects. Firstly, a large measurement error is likely to lower estimated Δh and σ_u^2. As the main reasons for the small contribution of h comes from small Δh and σ_u^2, the measurement error is likely to lower the contribution of h. Secondly, if h is heterogeneous in an industry and a prefecture, the estimated relation between aggregate productivity and the average value of h would understate the real relation. Although heterogeneous h is an attractive assumption, it requires estimating h from a smaller sample and amplifies measurement errors. Hence, to solve these problems, more disaggregated data are required.[5]

[5] The existence of large adjustment costs is a possible alternative explanation of why the impacts of h are small. If there were substantial adjustment costs of inputs, and even if a firm could appropriately predict changes in the environment, a large reaction would be costly. Large adjustment costs reduce the impact of firms' prediction ability. Although this is a plausible explanation, we have some reservations. The main reason for small estimates was a relatively small value of Δh and σ_u^2 and the estimated value of the coefficient, 6.7, implies that $\alpha=0.93$, which is large. The large value of α implies that a firm can flexibly respond to firm-specific shocks if it perfectly knows about the shock. If the true model contains large adjustment costs, as the estimated value of ψ_h is also influenced by the impacts of adjustment costs, the true value of α would be larger than 0.93, which is less likely.

V. Conclusions and Extensions

This paper has presented an empirical framework investigating the social value of the ability to predict and adapt to idiosyncratic productivity shocks, which aims to capture the soundness of firms' judgments about economic impacts of idiosyncratic changes. It provides a novel method of investigating the economic value of unobserved information in macroeconomics.

Several extensions can be considered. Firstly, more accurate data are called for. Because of possible measurement errors, much disaggregate data is required for precise estimations. This data requirement is one of the difficulties in implementing our method. Recently, much evidence has been presented by plant-level data. Use of this data will improve understanding about entrepreneurship.

Secondly, incorporating an adjustment cost of investment and firms' entry and exit might be important. As the main purpose of this paper is to position entrepreneurship in the aggregate production function, these factors are not considered in the model. However, the consideration of these factors allowed the examination of the dynamics of industry (Lucas and Prescott 1971; Jovanovic 1982; Hopenhayn and Rogerson 1993). Once we introduce the adjustment costs of investment and firms' entry and exit, a difference between persistent shock and independent shock becomes important. A persistent shock together with the adjustment costs of investment raises the importance of prediction. This point is partially examined in Takii (2000), who finds a positive impact of prediction ability on the average Tobin's Q. However, its macro impacts have not yet been investigated.

Finally, investigating factors that enhance entrepreneurship is interesting. Although average prediction ability is estimated by industry and prefecture in this paper, we have said nothing about why prediction ability differs between industries and prefectures. Prediction ability may be affected by various factors, including inherited ability, education, social networks, connections, region density, regional communication systems, and organizations. As we have a well-defined measure of prediction ability, it may be possible to empirically identify factors influencing entrepreneurship.

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Appendix

***Proof of Proposition 3*:** The derivation of Equations (10) and (11) is straightforward. We explain the derivation of Equation (12). Takii (2004) shows that:

$$\phi(k^a)=f\left(\frac{k^a}{1-m(k^a)}\right)^{\alpha} m(k^a)^{(1-\alpha)}[1-m(k^a)]^{\alpha},$$

and $m(k^a)\in(0,1)$ is a solution of:

$$\frac{\alpha}{(1-\alpha)}\left[1-\theta\left(\frac{k^a}{1-m}\right)\right]=\frac{1-m}{m}\ . \tag{A.1}$$

The definition of $\phi(k^a)$ implies that:

$$\phi'(k^a)=\alpha f(k)^{\alpha-1}\left(\frac{m}{1-m}\right)^{(1-\alpha)} f'(k)D, \tag{A.2}$$

$$where\ D\equiv\left[1+k\frac{dm}{dk^a}\right]+\frac{f(k)}{f'(k)}\left[\frac{(1-\alpha)(1-m)}{\alpha m}-1\right]\frac{dm}{dk^a}\ .$$

Equation (A.1) implies $D=1$. Hence, it is shown that:

$$\frac{\phi'(k^a)k^a}{\phi(k^a)}\ =\ \alpha\theta(k).$$

In addition, the first-order condition (2) implies that:

$$\alpha\theta(k)=\frac{1}{\int\frac{z[f(k)L(s)]^{\alpha}T}{rkTL(s)}dQ_z(z)}\ .$$

Q.E.D.

***Proof of Proposition 4*:** Equations (13) and (14) are immediately derived from the definition of the firm's production functions. We utilized the fact that random variables do not affect k. Equation (15) is easily derived from the two first-order conditions (1) and (2). *Q.E.D.*

***Proof of Theorem 7*:** Applying the standard Bayesian updating technique, it can be shown that:

$$\int u dQ_u^h(u \mid s) = hs,$$
$$Var(u \mid s) = (1-h)\sigma_u^2.$$

Using these results, $\int z dF(z \mid s)$ can be expressed as follows:

$$\int u dQ^h(z \mid s) = z^e \exp\left[hs - \frac{\sigma_u^2 h}{2}\right], \tag{A.3}$$

where $z^e = \exp(\mu + \sigma_u^2/2)$. Applying Equation (A.3) to Equation (18), it can be shown that $L^* = \{z^e / w\,\alpha[f(k^a) - f'(k^a)k^a]\}^{1/1-\alpha} f(k^a)$:

$$L(s) = L^* \exp\left(hs - \frac{\sigma_u^2 h}{2}\right)^{\frac{1}{1-\alpha}}.$$

Hence, the firm's reaction to the shock $R(L(s))$ is given by $1/(1-\alpha)[hs - \sigma_u^2 h/2]$. Hence, the definition of the correlation coefficient implies that:

$$\rho_{uR(L(s))} = \sqrt{h}.$$

Q.E.D.

Data Appendix:

The Detail of Data Description: Although the census covers all establishments in which four or more persons worked as employers or employees, by city, industry and year, if there are fewer than three establishments for an industry in a city in any given year, data are not reported by the census, to maintain the privacy of establishments. To improve estimations of the correlation, we exclude entities for which there are missing variables in any period.

The industries covered by the census are food, drink/tobacco/manimal feed, textiles, apparel, lumber/wood products, furniture/fixtures, pulp/paper, printing, chemicals, petroleum/coal products, plastic products, rubber, leather/leather products, pottery/glass products, iron/steel, nonferrous metals, metal products, general machinery, electrical machinery, transportation machinery, precision tools, weapons, and other industries.

Data Construction: As shown below, the estimation of variables

requires several steps. This might cause the reader to be suspicious. Although reported results are based on the following estimation method, we conducted several regressions to gain preliminary results using different methods of estimations. The results are more or less the same as our reported results.

***Y* and *wTL*:** Gross value added and labor expenses are divided by the number of establishments. These values are then deflated by the GDP price deflator.

K: Fixed tangible assets are divided by the number of establishments. The replacement cost of the capital stock is then estimated from the following equation:

$$K_{ciy+1} = K_{ciy} + (F_{ciy+1} - F_{ciy})/p_{Iy+1}, \ \textit{if} \ F_{ciy+1} > F_{ciy},$$
$$= K_{ciy} + F_{ciy+1} - F_{ciy}, \ \textit{if} \ F_{ciy+1} \leq F_{ciy},$$

where $K_{ci85} = F_{ci85}/p_{I85}$ and F_{ciy} are average fixed tangible assets per establishment of c*th* city and i*th* industry in year y, and p_{Iy} is a price deflator for investment goods in year y, which is taken from Keizai Tokei Nenkan (2002) (Annual Economic Statistics 2002) by Toyo Keizai Shinpo Sya. As we do not have data on investment, this simplified estimation method was used as an approximation, which is the approach taken by Nishimura, Nakajima, and Kiyota (2003). The subscript y spans the period between 1985 and 1999.

The Census of Manufacturing only reports fixed tangible assets for establishments in which the sum of employers and employees is at least 10 [size group 2]. Hence, the capital stock of establishments in which the sum of employers and employees is between four and nine workers [size group 1] is estimated as follows. First, average labor expenses per establishment in size groups 1 and 2 are estimated by city, industry and year. Average fixed assets per establishment are then regressed on average labor expenses per establishments and city dummies in size group 2 for each industry. The parameters of this regression are used to estimate the capital stock in size group 1.

The Census of Manufacturing reports labor expenses for size groups 1 and 2 by industry and year, but not by city. Hence, we need to estimate this by city, industry and year. For this purpose, the following estimation method is used. First, the average labor expenses per establishment are estimated by size group, industry and year. Average labor expenses per establishment for each size

group are estimated by city, industry and year from the following equation:

$$wTL^{s}_{city}=\frac{wTL^{s}_{iy}}{wTL^{1}_{iy}+wTL^{2}_{iy}}\ wTL_{city}\ ,$$

where wTL^{s}_{city} is the average labor expense per establishment in the c*th* city, i*th* industry and size group s in year y, wTL_{city} is the average labor expense per establishment in the c*th* city and i*th* industry in year y, and wTL^{s}_{iy} is the average labor expense per establishment in the i*th* industry and size group s in year y.

wT: wT is estimated by prefecture, industry and year, using the weighted average of deflated labor expenses over the number of employees. The number of establishments is the weight. Because the number of employees is not reported, this is estimated. The Census of Manufacturing reports the number of employees, the sum of employees and employers, and the number of establishments by industry and year. Assuming that the ratio of employees per establishment to the sum of employers and employees per establishment is the same in each industry and year, the number of employees was estimated by city, industry and year:

$$L_{city}\equiv\left(\frac{L}{E+L}\right)_{iy}\ (E+L)_{city}\ ,$$

where L_{city} is the number of employees per establishment in the c*th* city and i*th* industry in year y, $(E+L)_{city}$ is the sum of employers and employees per establishment in the c*th* city and i*th* industry in year y, and $(L/(E+L))_{iy}$ is the ratio of employees per establishment to the sum of employers and employees per establishment in the i*th* industry in year y.

r: The return to the capital stock is estimated by using:

$$r_{iy}=p_{Iy}(i_y+\delta_i),$$

where i_y is the yield on 10-year government bonds in year y, and δ_i is the average depreciation rate over average fixed tangible assets of the i*th* industry. The yield data are from the website of The Bank of Japan. Average depreciation and average fixed tangible assets are

taken from the Census of Manufacturing. As in Nishimura *et al.* (2003), changes in the price deflator for investment goods are ignored, as this index increased so much during the bubble in Japan that the user cost of capital became negative. Because r is only used to estimate the average capital share over time, this simplification is unlikely to affect our results. To check for robustness, we also used the return to the capital stock in Hayashi and Prescott (2002), which is taken from Hayashi's website. As this did not change the results, we do not report them in this paper.

$\Delta\log$TFP: We estimated the average capital share of each firm from the sample average of Y/rK over time by city and industry. Then the weighted average of the capital share was estimated by prefecture, industry and period, with the number of establishments in 1988 and 1996 as weights. Unless otherwise stated, the same weights are used to estimate the prefecture average. The average of the capital share over the period was chosen to estimate $\phi'(k^a)k^a/\phi(k^a)$. Value added, capital stock and the sum of employees and employers were aggregated by industry and prefecture in 1988 and 1996. Then $\Delta\log TFP$ was estimated as defined.

$$\Delta\log TFP_{pi}=\left[\log\frac{Y_{pi96}}{N_{pi96}}-\log\frac{Y_{pi88}}{N_{pi88}}\right]-\left(\frac{\phi'(k^a)k^a}{\phi(k^a)}\right)_{pi}\left[\log\frac{K_{pi96}}{N_{pi96}}-\log\frac{K_{pi88}}{N_{pi88}}\right],$$

$$\left(\frac{\phi'(k^a)k^a}{\phi(k^a)}\right)_{pi}=\frac{1}{2}\left[\sum_{c\in C_p}\frac{1}{\frac{\sum_{(85\le y\le 99)}\frac{Y_{ciy}}{r_{iy}K_{ciy}}}{15}}\frac{n_{ci96}}{\sum_{c\in C_p}n_{ci96}}+\sum_{c\in C_p}\frac{1}{\frac{\sum_{(85\le y\le 99)}\frac{Y_{ciy}}{r_{iy}K_{ciy}}}{15}}\frac{n_{ci88}}{\sum_{c\in C_p}n_{ci88}}\right],$$

where Y_{piy}, N_{piy}, K_{piy} are aggregate output, the sum of employees and employers, the capital stock of the p*th* prefecture and i*th* industry in year y, and n_{ciy} is the number of establishments in the c*th* city and i*th* industry in year y. Moreover, r_{iy} is the return to the capital stock of the i*th* industry in year y, and C_p is the set of cities in prefecture p.

$\Delta\log$wT: wT in 1988 and 1996 is chosen for this estimation.

$$\Delta\log(\mathbf{wT})_{pi}=\log(\mathbf{wT})_{pi96}-\log(\mathbf{wT})_{pi88},$$

where $(\mathbf{wT})_{piy}$ is the weighted average of the wage rate in the p*th* prefecture and i*th* industry in year y.

$\Delta\mu$: We estimate α, $\alpha(1-\beta)$, and $\alpha\beta$ from the sample averages of $Y/(rK+wTL)$, Y/wTL and Y/rK over time by city and industry,

respectively. Using the estimated α, $\alpha(1-\beta)$, and $\alpha\beta$, $E[\log Z_1(z,s)]$ and $E[\log Z_2(z,s)]$ are estimated by their sample averages over time by city, industry and period. The weighted averages of these values are calculated by prefecture, industry and period. Using these values, $\Delta\mu$ is then estimated by prefecture and industry:

$$\Delta\mu_{pi}=\sum_{c\in C_p}\left\{\frac{\sum_{[92\le y\le 99]}[\log Z]_{ciy}}{8}\ \frac{n_{ci96}}{\sum_{c\in C_p} n_{ci96}}\right\}$$
$$-\sum_{c\in C_p}\left\{\frac{\sum_{[85\le y\le 91]}[\log Z]_{ciy}}{7}\ \frac{n_{ci88}}{\sum_{c\in C_p} n_{ci88}}\right\},$$

where $[\log Z]_{ciy}=\log Y_{ciy}-\alpha_{ci}\log[wTL]_{ciy}-(1-\alpha_{ci})\log[wT]_{piy}$ or $[\log Z]_{ciy}=\log Y_{ciy}-[\alpha(1-\beta)]_{ci}\log[wTL]_{ciy}-[\alpha\beta]_{ci}\log K_{ciy}-(1-\alpha_{ci})\log[wT]_{piy}$ and:

$$\alpha_{ci}=\frac{1}{\frac{\sum_{(85\le y\le 99)}\frac{Y_{ciy}}{r_{ty}K_{ciy}+(wTL)_{ciy}}}{15}},\ [\alpha(1-\beta)]_{ci}=\frac{1}{\frac{\sum_{(85\le y\le 99)}\frac{Y_{ciy}}{(wTL)_{ciy}}}{15}},\ [\alpha\beta]_{ci}=\frac{1}{\frac{\sum_{(85\le y\le 99)}\frac{Y_{ciy}}{r_{ty}K_{ciy}}}{15}}.$$

$[wTL]_{ciy}$ means the average wage payments of the average establishments in the c*th* city and i*th* industry in year y, and $[wT]_{piy}$ is the weighted average of the wage rate in the p*th* prefecture and i*th* industry in year y. C_p is the set of cities in prefecture p.

σ_u^2: To estimate the variance, the standard deviation is estimated by city, industry and period. Then the weighted average of the standard deviation is estimated by prefecture, industry and period. The square of the average standard deviation is then estimated.

$$(\sigma_u^2)_{pit}=\left\{\sum_{c\in C_p}\left[\sqrt{\frac{\sum_{y\in\theta_t}\left[[\log Z]_{ciy}-\frac{\sum_{y\in\theta_t}[\log Z]_{ciy}}{m_t}\right]^2}{m_t}}\ \frac{n_{cix}}{\sum_{c\in C_p} n_{cix}}\right]\right\}^2,$$

where m_t is the number of years during period t, n_{cix} is the number of establishments in the c*th* city and i*th* industry in year x. If $t=1$ then $x=88$, and if $t=2$ then $x=96$. θ_t is the set of years during period t, where $t\in\{1,2\}$ and $\theta_1=\{85,86,87,88,89,90,91\}$ and $\theta_2=\{92,93,94,95,96,97,98,99\}$. See the construction of $\Delta\mu$ for the definition of $[\log Z]_{ciy}$, and C_p.

h: To implement corollaries 8 and 9, the correlations were estimated from the sample averages over time by city, industry and

period. Then, weighted averages of the correlations are estimated by prefecture, industry and period. The squared correlations are then calculated when they are positive.

$$[\rho_{uR(L(s))}]_{pit} = \sum_{c \in C_p} \left\{ \frac{\sum_{y \in \theta_t} \left[\log Z_{ciy} - \frac{\sum_{y \in \theta_t} \log Z_{ciy}}{m_t}\right]\left[\log W_{ciy} - \frac{\sum_{y \in \theta_t} \log W_{ciy}}{m_t}\right]}{\sqrt{\frac{\sum_{y \in \theta_t}\left[\log Z_{ciy} - \frac{\sum_{y \in \theta_t} \log Z_{ciy}}{m_t}\right]^2}{m_t}} \frac{\sum_{y \in \theta_t}\left[\log W_{ciy} - \frac{\sum_{y \in \theta_t} \log W_{ciy}}{m_t}\right]^2}{m_t}} \frac{n_{cix}}{\sum_{c \in Cp} n_{cix}} \right\},$$

$$h_{pit} = [\rho_{uR(L(s))}]^2_{pit}, \text{ if } [\rho_{uR(L(s))}]_{pit} \geq 0.$$

where $\log W_{ciy} = \log [wTL]_{ciy} - \alpha_{ci} \log [wTL]_{ciy} - (1 - \alpha_{ci}) \log [wT]_{piy}$ or $\log W_{ciy} = \log [wTL]_{ciy} - [\alpha(1-\beta)]_{ci} \log [wTL]_{ciy} - [\alpha\beta]_{ci} \log K_{ciy} - (1 - \alpha_{ci}) \log [wT]_{piy}$. See the construction of $\Delta\mu$ for the definition of $[\log Z]_{ciy}$, α_{ci}, $[\alpha(1-\beta)]_{ci}$, $[\alpha\beta]_{ci}$, and C_p, and the construction of σ_u^2 for the definition of n_{cix}, θ_t and m_t.

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