Hidden Wealth and Incentives

Jin Yong Jung

This paper analyzes the characteristics of optimal menu of contracts in the case that agents' wealth and actions are hidden from the principal. We first consider a benchmark case that the wealth levels of all agents are known to the principal, and we show that the power of incentives for agents with high wealth is greater than the one for agents with low wealth. The reason is that richer agents are less risk averse; providing high incentives for less risk-averse agents is beneficial to the principal. However, in the case that the agents' wealth is hidden, the principal will fail to achieve social efficiency because rich agents will act like poor agents. Thus, the principal must design another menu of contracts to prevent rich agents from mimicking poor agents. Ultimately, compared with the optimal menu of contracts derived when it was known, the power of incentives for rich agents is identical, but the power of incentives for poor agents declines.

Keywords: Principal-agent model, Moral hazard, Adverse selection, Hidden wealth, Incentives.

JEL Classification: D82, D86, J33, D31.

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I. Introduction

Moral hazard problems can be mitigated by providing incentive wage contracts for agents. Therefore, when the principal faces the agents who are risk averse, but with heterogeneous risk preferences by different levels of wealth, the principal is willing to prepare a customized menu of contracts. However, if agents' wealth is hidden from the principal, his/her plan will miscarry, because the rich (or poor) agent may act like a poor (or the rich) agent. Thus, the question about what is the optimal menu of contracts for them would be naturally raised. This is the main question of the paper. Thus, we study the principal–agent model with moral hazard and adverse selection problems.

The existing literature about wealth effects under moral hazard problems has mainly conducted studies on the effects of agent's wealth on the principal's cost when his/her wealth is observable to the principal. Thiele and Wambach (1999) found that agent's wealth positively affects the principal's compensation cost under a reasonable condition on agent's preferences, resulting in a negative effect on her profit. They actually showed that the principal prefers poor agents to rich ones. Later, more extended results are provided by Kadan and Swinkels (2013) and Chade and Serio (2014). Especially, Kadan and Swinkels (2013) not only showed that Thiele and Wambach's results remain valid, but also provided some new results rather than them, assuming the principal–agent model in which the first-order approach is not adopted and the agent's limited liability constraint is also considered.

Contrastingly, some previous results showed that an increase in agent's wealth may be beneficial to the principal. Jung (2017), based on the principal–agent model of Baker and Hall (2004), showed that, as the agent is wealthier, given that the degree of risk aversion decreases, the principal's compensation cost also falls. Thus, the principal prefers rich agents over poor ones. Moreover, he unveiled that, as the agent becomes wealthier, he is provided with low-powered incentives.

Meanwhile, studies on the principal–agent problem considering adverse selection and moral hazard are found in various fields. *In* competitive insurance markets, Stewart (1994) considered the agents who can exert efforts in reducing the probability of the loss but its cost is private information. He showed that high-risk agents purchase the same coverage with the one which is offered in the absence of adverse selection, but that low-risk agents purchase the lower coverage than the one which is offered when adverse selection is absent.

In competitive labor markets, Moen and Rosen (2005) considered the model in which the agents' effort and its cost are private information. They showed that, in a separating equilibrium, the power of incentives for agents with lower cost (or higher productivity) exceeds the efficient level, and that the welfare loss can be eliminated by a tax on high income. Recently, Bénabou and Tirole (2016) considered the labor markets with adverse selection about workers' talents. They showed that, as the competition of the labor market becomes fiercer, the power of incentives for all agents escalates, and that when the market competition exceeds a certain level, high types are over-incentivized relative to social efficiency.

As explained earlier, almost all literature about asymmetric information focused on adverse selection about agents' effort cost or their productivity. However, this paper considers adverse selection about agents' wealth (and risk preference). More concretely, we analyze the principal–agent problem in which the agents' actions and wealth are hidden from the principal. For this, we borrow the settings used in Jung (2017), but consider the case that agents' wealth is two type. We first derive the characteristics of the optimal contracts for agents when the agents' wealth is known to the principal, and then investigate how the optimal contracts change when it becomes hidden.

As shown in Jung (2017), in the case that the principal has information on agents' wealth, the power of incentives for rich agents is greater than the one for poor agents. However, when agents' wealth is hidden, the optimal contracts designed for all agents under moral hazard are no longer valid. The reason is that rich agents can have higher utility by selecting the incentive contract aimed at poor agents. This means that rich agents will act like poor agents when their wealth is hidden. Thus, the principal must design a new menu of contracts targeting rich and poor agents.

Compared with the optimal contracts derived in the case that the agents' wealth is known, the optimal menu of contracts has the following two characteristics: First, for the optimal contract aimed at rich agents, the power of incentives is same with the one under moral hazard, but the fixed wage level is high enough for them to enjoy rent over the reservation utility level. Second, for the optimal contract targeting poor agents, the power of incentives is lower than the one under moral hazard, and the fixed wage level is determined as much as their reservation utility level is satisfied at a minimum. Resultantly, hidden wealth problem harms poor agents, but profits rich agents.

Our result implies that the difference of incentive powers among agents becomes worse by screening when agents' risk preference is private information. This helps us understand the difference of incentive powers for workers within firms. According to some empirical studies, the introduction of performance pay may cause wage inequality from the difference of workers' productivities, as seen in Lemieux, MacLeod, and Parent (2009). The literature has actually concentrated on income inequality because of the gap of productivities among workers. However, our result shows that such a difference can be partially explained by the difference of the risk preferences among risk-averse agents and it will be worse when their risk preference is hidden from firms.

This paper is organized as follows. Section II suggests our basic model. Section III provides our results in the cases that agents' wealth is known and when it is hidden and discusses the loss of the social welfare from hidden wealth problem. Section IV presents the conclusion.

II. Basic Model

We consider one-period principal–agent model in which a risk-neutral principal hires risk-averse agents with different levels of initial wealth. After each agent exerts his/her effort $a \in [0,\infty)$, output x is realized. Output x is normally distributed with mean a and variance σ^2. That is, $x \sim N(a,\sigma^2)$. Moral hazard problem will be resolved by providing wage scheme $s(x)$ depending on his outcome x.

Each agent has the negatively exponential utility: $U(s,a;\rho) = -(1/\rho) e^{-\rho(s-c(a))}$, where s is income and $c(a)$ is his/her cost from exerting effort a. We assume that, for all $a > 0$, $c'(a) > 0$ and $c''(a) > 0$ with $c(0) = c'(0) = 0$ and $c''(0) \geq 0$, and that $c'''(a) \geq 0$ for all $a \geq 0$. These assumptions mean that cost of effort $c(a)$ and marginal cost $c'(a)$ are strictly increasing and convex in a. Moreover, $\rho > 0$ is the agent's degree of absolute risk aversion and it has different values according to his/her initial wealth.

When the agent is provided with linear contract, such as $s(x) = \alpha + \beta x$,[1] his/her expected utility is calculated as follows:

[1] Based on Holmstrom and Milgrom's (1987) result, the optimal contract is a

$$E[U(s(x), a; \rho)] = -\frac{1}{\rho}\exp\{-\rho[\alpha + \beta a - c(a) - \frac{\rho}{2}\beta^2\sigma^2]\}.$$

Here, we define $CE = \alpha + \beta a - c(a) - (\rho/2)\beta^2\sigma^2$, which means the certainty equivalent. Given that function $-(1/\rho)e^{-\rho(s-c(a))}$ is strictly increasing in y, we use the CE as the agent's expected utility. When the agent is compensated with linear contract, he/she will choose an effort level that maximizes his/her expected utility (*i.e.*, certainty equivalent). Given that his/her CE is concave in a, his/her effort choice should satisfy the following condition:

$$\beta = c'(a).$$

Let $a = a(\beta)$ solve the above equation. This function indicates that agent's effort choice is $a(\beta)$ when the slope of linear contract is equal to β. Given that $c'(a(0)) = 0 = c'(0)$, $a(0) = 0$. Given that differentiating $\beta \equiv c'(a(\beta))$ regarding β gives $1 = c''(a(\beta))a'(\beta)$, we have

$$a'(\beta) = \frac{1}{c''(a(\beta))} > 0, \quad \forall\beta > 0, \tag{1}$$

where the strict inequality holds given $c''(a) > 0$ for all $a > 0$ by assumption. At this time, agent's (expected) utility is represented as

$$u(\alpha, \beta; \rho) \equiv \alpha + \beta a(\beta) - c(a(\beta)) - \frac{\rho}{2}\beta^2\sigma^2.$$

The principal should respect the agent's reservation utility level $\bar{u}$. Thus, his/her participation constraint is written as $u(\alpha,\beta;\rho) \geq \bar{u}$. Furthermore, when the linear contract $s(x) = \alpha + \beta x$ is provided, the principal's (expected) profit is

$$\pi(\alpha, \beta) \equiv E[x - s(x)] = E[x] - \{\alpha + \beta E[x]\} = a(\beta) - [\alpha + \beta a(\beta)],$$

where the second equality holds because $E[x] = a(\beta)$.

linear contract in the principal-agent model in which the agent has a negatively exponential utility and the outcome is normally distributed.

Each agent has initial wealth w that affects his/her degree of risk aversion, *i.e.*, $\rho = \rho(w)$. In this paper, we consider the two-type case that $w = w_H$ with probability p_H, and $w = w_L$ with probability $p_L = 1 - p_H$. It is assumed that $w_L < w_H$ and that $\rho(w)$ is a decreasing function. Thus, we have $\rho_L \equiv \rho(w_L) > \rho(w_H) \equiv \rho_H$, indicating that the richer agent is less risk averse.

III. Results

In this section, we address the situation wherein agents' wealth is common information, and then the situation wherein it is hidden from the principal.

A. When agents' wealth is known

In this subsection, we begin with the case that the wealth levels of the agents are known to the principal. Thus, we consider the principal–agent relationship only with the moral hazard problem. In this situation, the principal can design a linear contract $s_i(x) = \alpha_i + \beta_i x$ for each agent by selecting (α_i, β_i), $i = L,H$. Therefore, he/she must determine (α_L, β_L) and (α_H, β_H) for agents with low wealth w_L and high wealth w_H, respectively.

The principal's profit maximization problem is

$$\max_{\alpha_L, \beta_L, \alpha_H, \beta_H} \; p_L \pi(\alpha_L, \beta_L) + p_H \pi(\alpha_H, \beta_H),$$

$$\text{s.t. i)} \quad u(\alpha_L, \beta_L; \rho_L) = \alpha_L + \beta_L a(\beta_L) - c(a(\beta_L)) - \frac{\rho_L}{2}\beta_L^2\sigma^2 \geq \bar{u},$$

$$\text{ii)} \quad u(\alpha_H, \beta_H; \rho_H) = \alpha_H + \beta_H a(\beta_H) - c(a(\beta_H)) - \frac{\rho_H}{2}\beta_H^2\sigma^2 \geq \bar{u}.$$

In the above problem, the first and second constraints indicate the participation constraints for the agents with wealth w_i and so ρ_i, $i = L,H$, respectively. As seen in the principal's problem, the objective function is decreasing in α_L and α_H. Hence, the principal's profit can be maximized by determining α_L and α_H to satisfy agents' participation constraints at a minimum:

$$u(\alpha_L, \beta_L; \rho_L) = \alpha_L + \beta_L a(\beta_L) - c(a(\beta_L)) - \frac{\rho_L}{2}\beta_L^2\sigma^2 = \bar{u}, \quad \Rightarrow \alpha_L = \bar{u} - \beta_L a(\beta_L) + c(a(\beta_L)) + \frac{\rho_L}{2}\beta_L^2\sigma^2, \tag{2}$$

and

$$u(\alpha_H, \beta_H; \rho_H) = \alpha_H + \beta_H a(\beta_H) - c(a(\beta_H)) - \frac{\rho_H}{2}\beta_H^2\sigma^2 = \bar{u}, \quad \Rightarrow \alpha_H = \bar{u} - \beta_H a(\beta_H) + c(a(\beta_H)) + \frac{\rho_H}{2}\beta_H^2\sigma^2. \tag{3}$$

By substituting Equations (2) and (3) for α_L and α_H in the objective function, the principal's problem is rewritten as

$$\max_{\beta_L, \beta_H} p_L\phi(\beta_L; \rho_L) + p_H\phi(\beta_H, \rho_H) - \bar{u}, \qquad \text{[MP]}$$

where

$$\phi(\beta; \rho) = a(\beta) - c(a(\beta)) - \frac{\rho}{2}\beta^2\sigma^2.$$

$\phi(0; \rho) = a(0) - c(a(0)) = 0$ because $a(0) = 0$ given that $c'(a(0)) = 0 = c'(0)$.

The following lemma verifies the existence and uniqueness of a solution to the maximization problem [MP] under our assumptions regarding cost function $c(a)$.

Lemma 1. For any given $\rho > 0$, $\phi(\beta; \rho)$ is strictly concave on interval (0,1), and is decreasing on interval $(1,\infty)$. Thus, for any $\rho > 0$, a solution to maximize $\phi(\beta; \rho)$ uniquely exists on interval (0,1).

Proof. Differentiating $\phi(\beta; \rho)$ with respect to β gives

$$\phi_\beta(\beta; \rho) = a'(\beta) - c'(a(\beta))a'(\beta) - \rho\sigma^2\beta = \frac{1-\beta}{c''(a(\beta))} - \rho\sigma^2\beta,$$

where the second equality holds because $c'(a(\beta)) \equiv \beta$ by the definition of $a(\beta)$ and $a'(\beta) = 1/c''$ by Equation (1). For all $\beta > 1$, because $(1 - \beta) / c''(a(\beta)) \leq 0$ and $\rho\sigma^2\beta > 0$, we have $\phi_\beta(\beta; \rho) < 0$, implying that $\phi(\beta; \rho)$ is decreasing in $\beta \in (1,\infty)$ for any given $\rho > 0$. Differentiating $\phi_\beta(\beta; \rho)$ one more regarding β yields

$$\phi_{\beta\beta}(\beta;\rho) = -\frac{1}{c''} - (1-\beta)\frac{c'''a'(\beta)}{[c'']^2} - \rho\sigma^2$$
$$= -\left\{1 + (1-\beta)\frac{c'''}{[c'']^2}\right\} \times \frac{1}{c''} - \rho\sigma^2.$$

where the second equality holds by Equation (1). Because $c''(a(\beta)) > 0$ and $c'''(a(\beta)) \geq 0$ for any $a(\beta) > 0$ by assumption, we have $\phi_{\beta\beta}(\beta;\rho) < 0$ for all $\beta \in (0,1)$, indicating that $\phi(\beta;\rho)$ is strictly concave on interval (0,1).

Given that $a(0) = 0$, and because $c''(a(0)) = c''(0) \geq 0$, we have $\phi_\beta(0;\rho) = 1/c''(0) > 0$ and $\phi_\beta(1;\rho) = -\rho\sigma^2 < 0$. Resultantly, given that $\phi(\beta;\rho)$ is strictly concave on interval (0,1) with $\phi_\beta(0;\rho) > 0$ and $\phi_\beta(1;\rho) < 0$ and because $\phi(\beta;\rho)$ is strictly decreasing on interval $(1,\infty)$, a solution to satisfy $\phi_\beta(\beta;\rho) = 0$ uniquely exists on interval (0,1). Q.E.D.

Let the solution to the principal's problem in [MP] be (β_L^*, β_H^*). Then, β_i^*, $i = L, H$, must satisfy the first-order condition:

$$\phi_\beta(\beta_i;\rho_i) = a'(\beta_i) - c'(a(\beta_i))a'(\beta_i) - \rho_i\sigma^2\beta_i = 0$$
$$\Leftrightarrow \frac{1-\beta_i}{c''(a(\beta_i))} = \rho_i\sigma^2\beta_i, \quad \forall i = L, H, \tag{4}$$

where the equivalence is satisfied because $c'(a(\beta_i)) = \beta_i$ and given that $a'(\beta_i) = 1/c''(a(\beta_i))$. Equation (4) shows that β_i^* depends on ρ_i, for $i = L, H$. Moreover, define

$$\pi^m(p_H, w_L, w_H) = (1-p_H)\phi(\beta_L^*;\rho(w_L)) + p_H\phi(\beta_H^*;\rho(w_H)) - \bar{u},$$

which indicates the principal's maximized profit under moral hazard.

Proposition 1. $\beta_L^* < \beta_H^*$ and $\pi^m(p_H, w_L, w_H)$ is increasing in p_H, w_L and w_H.[2]

Proof. As shown in the proof of Lemma 1, solution β_i^* satisfying Condition (4) is less than one, *i.e.*, $\beta_i^* < 1$. In addition, for all $\beta_i \in (0,1)$, the LHS of Condition (4) is decreasing, but the RHS is increasing. Thus, an increase in the slope of the RHS of Condition (4) by an increase in ρ_i

[2] This proposition is derived in the two-type case. For generalized results, see Proposition 3 in Jung (2017).

reduces the value of β_i^*, implying $\beta_L^* < \beta_H^*$ given that $\rho_L > \rho_H$.

Given that β_H^* is a maximizer of function $\phi(\beta;\rho_H)$, we have $\phi(\beta_H^*;\rho_H) > \phi(\beta_L^*;\rho_H)$. Moreover, because $\phi_\rho(\beta;\rho) = -\beta\sigma^2/2 < 0$ for all $\beta > 0$, $\phi(\beta_L^*;\rho)$ is decreasing in ρ, implying $\phi(\beta_L^*;\rho_H) > \phi(\beta_L^*;\rho_L)$. Thus, we have $\phi(\beta_H^*;\rho_H) > \phi(\beta_L^*;\rho_L)$. Given this, differentiating $\pi^m(p_H, w_L, w_H)$ regarding p_H gives

$$\frac{\partial}{\partial p_H}\pi^m(p_H, w_L, w_H) = \phi(\beta_H^*;\rho_H) - \phi(\beta_L^*;\rho_L) > 0.$$

Differentiating $\pi^m(p_H, w_L, w_H)$ with respect to w_i, $i = L, H$, makes

$$\begin{aligned}\frac{\partial}{\partial w_i}\pi^m(p_H, w_L, w_H) &= p_i\phi_\beta(\beta_i^*;\rho(w_i))\times\left(\frac{\partial\beta_i^*}{\partial\rho_i}\right) + p_i\phi_\rho(\beta_i^*;\rho(w_i))\rho'(w_i)\\ &= p_i\phi_\rho(\beta_i^*;\rho_i)\rho'(w_i)\\ &= -p_i\frac{1}{2}(\beta_i^*)^2\sigma^2\rho'(w_i) > 0,\end{aligned}$$

where the second equality is satisfied given that $\phi_\beta(\beta_i^*;\rho_i) = 0$ by the first-order condition. Q.E.D.

Proposition 1 contains three results. The first is that the power of incentives for the rich agent is greater than the one for the poor agent (see Figure 1). This is mainly because the rich agent has a lower

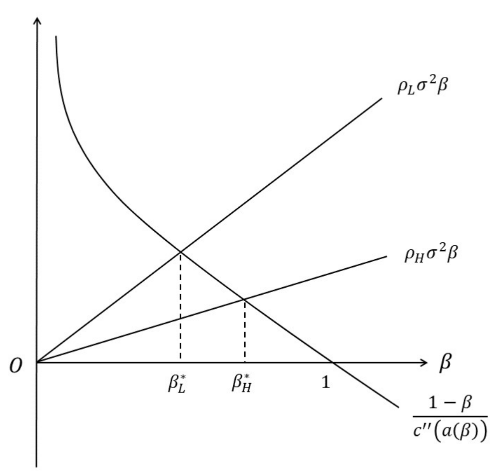

FIGURE 1

COMPARISON OF THE INCENTIVE POWERS UNDER MORAL HAZARD

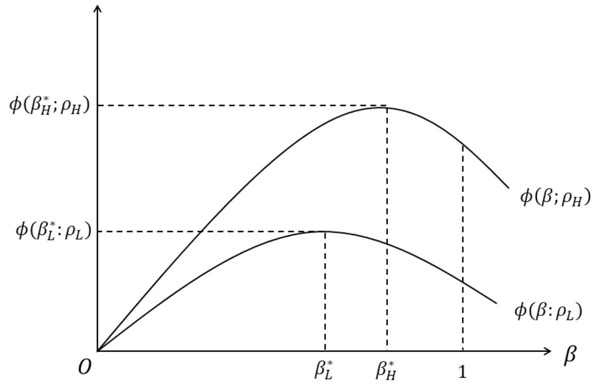

Figure 2
Graphs for $\phi(\beta;\ \rho_L)$ and $\phi(\beta;\ \rho_H)$

degree of absolute risk aversion than the poor agent, *i.e.*, $\rho_L = \rho(w_L) > \rho(w_H) = \rho_H$. Thus, the poor agent will demand more risk premium against accepting a risky incentive wage contract, which increases the principal's cost. Thus, providing low-powered incentives for the poor agent rather than for the rich agent is beneficial to the principal. The second is that the net benefit from the rich agent, $\phi(\beta_H^*;\ \rho_H)$, is greater than the one from the poor, $\phi(\beta_L^*;\ \rho_L)$, implying that an increase in the proportion of rich agents is beneficial to the principal (see Figure 2). Finally, the third is that a decrease in the degree of risk aversion raises principal's profit. Thus, given that an increase in the wealth level of every agent makes him/her become less risk averse, the principal's profit increases with agents' wealth.

B. When agents' wealth is hidden

In this subsection, we consider the case that agents' wealth is hidden from the principal. In this case, each agent can independently choose any contract that is offered by the principal. However, if the principal offers every agent the set of two contracts, such as $(\alpha_L^*,\ \beta_L^*)$ and $(\alpha_H^*,\ \beta_H^*)$, what happens then? The following proposition explains it.

Proposition 2. Consider the situation under which, although agents' wealth is hidden, $(\alpha_L^*,\ \beta_L^*)$ and $(\alpha_H^*,\ \beta_H^*)$ are still offered to agents. Then, all agents select $(\alpha_L^*,\ \beta_L^*)$, but nobody does $(\alpha_H^*,\ \beta_H^*)$.

Proof. Given (α_i^*, β_i^*), because the participation constraint for the agent with wealth w_i must be binding at the optimum, we have

$$u(\alpha_i^*, \beta_I^*; \rho_i) = \alpha_i^* + \beta_i a(\beta_i^*) - c(a(\beta_i^*)) - \frac{\rho_i}{2}(\beta_i^*)^2\sigma^2 = \bar{u}, \quad \forall i = L, H. \tag{5}$$

The above equation shows that when the agent with wealth w_i selects (α_i^*, β_i^*), his/her expected utility level is equal to $\bar{u}$.

Now, if the agent with low wealth w_L selects (α_H^*, β_H^*), his/her utility level is

$$\begin{aligned} u(\alpha_H^*, \beta_H^*; \rho_L) &= \alpha_H^* + \beta_H^* a(\beta_H^*) - c(a(\beta_H^*)) - \frac{\rho_L}{2}(\beta_H^*)^2\sigma^2 \\ &= \bar{u} - \frac{1}{2}(\rho_L - \rho_H)(\beta_H^*)^2\sigma^2 < \bar{u}, \end{aligned}$$

where the second equality holds because $\alpha_H^* + \beta_H a(\beta_H^*) - c(a(\beta_H^*)) = \bar{u} + \frac{\rho_H}{2}(\beta_H^*)^2\sigma^2$ from Equation (5), and where the last strict inequality holds given $\rho_L > \rho_H$. Thus, we have $u(\alpha_H^*, \beta_H^*; \rho_L) < u(\alpha_L^*, \beta_L^*; \rho_L)$, indicating the poor agent still prefers (α_L^*, β_L^*) to (α_H^*, β_H^*).

Meanwhile, if the agent with high wealth w_H selects (α_L^*, β_L^*), his/her utility level is

$$\begin{aligned} u(\alpha_L^*, \beta_L^*; \rho_H) &= \alpha_L^* + \beta_L^* a(\beta_L^*) - c(a(\beta_L^*)) - \frac{\rho_H}{2}(\beta_L^*)^2\sigma^2 \\ &= \bar{u} + \frac{1}{2}(\rho_L - \rho_H)(\beta_L^*)^2\sigma^2 > \bar{u}, \end{aligned}$$

where the second equality holds because $\alpha_L^* + \beta_L^* a(\beta_L^*) - c(a(\beta_L^*)) = \bar{u} + \frac{\rho_L}{2}(\beta_L^*)^2\sigma^2$ from Equation (5). Thus, we have $u(\alpha_L^*, \beta_L^*; \rho_H) > u(\alpha_H^*, \beta_H^*; \rho_H)$, indicating that the rich agent prefers (α_L^*, β_L^*) to (α_H^*, β_H^*). Q.E.D.

Proposition 2 shows that, when agents' wealth is hidden from the principal, rich agents, unlike the poor, have an incentive to select the contract (α_L^*, β_L^*). The reason is simple: The less risk-averse agent with high wealth can extract rent by taking more risk premium. This is the reason why the rich agents mimic the poor. Thus, in this case, the menu of two contracts (α_L^*, β_L^*) and (α_H^*, β_H^*) can no longer be optimal.

Now, by the Revelation Principle, the principal must design a new menu of contracts under which all agents are transparent. For this, the principal's problem is

$$\max_{\alpha_L,\beta_L,\alpha_H,\beta_H} \; p_L\pi(\alpha_L, \beta_L) + p_H\pi(\alpha_H, \beta_H) \qquad \text{[AMP]}$$

$$s.t. \text{ i) } u(\alpha_L, \beta_L; \rho_L) = \alpha_L + \beta_L a(\beta_L) - c(a(\beta_L)) - \frac{\rho_L}{2}\beta_L^2\sigma^2 \geq \bar{u},$$

$$\text{ii) } u(\alpha_H, \beta_H; \rho_H) = \alpha_H + \beta_H a(\beta_H) - c(a(\beta_H)) - \frac{\rho_H}{2}\beta_H^2\sigma^2 \geq \bar{u},$$

$$\text{iii) } \alpha_L + \beta_L a(\beta_L) - c(a(\beta_L)) - \frac{\rho_L}{2}\beta_L^2\sigma^2 \geq \alpha_H + \beta_H a(\beta_H) - c(a(\beta_H)) - \frac{\rho_L}{2}\beta_H^2\sigma^2,$$

$$\text{iv) } \alpha_H + \beta_H a(\beta_H) - c(a(\beta_H)) - \frac{\rho_H}{2}\beta_H^2\sigma^2 \geq \alpha_L + \beta_L a(\beta_L) - c(a(\beta_L)) - \frac{\rho_H}{2}\beta_L^2\sigma^2.$$

The above problem still contains the participation constraints for the poor and the rich agents, but the last two constraints are newly added. The two constraints indicate that all agents should prefer the contract which is prepared for themselves by the principal to the one for others. More precisely, they mean that the principal should design the menu of contracts under which the rich agent prefers (α_H, β_H) to (α_L, β_L), and simultaneously, the poor agent prefers (α_L, β_L) to (α_H, β_H).

Let (α_L^0, β_L^0) and (α_H^0, β_H^0) be a solution to [AMP]. The following proposition reveals their characteristics.

Proposition 3. $\beta_H^0 = \beta_H^*$ but $\beta_L^0 < \beta_L^*$, resulting in $\beta_H^0 > \beta_L^0$. β_L^0 is decreasing in ρ_L, but increasing in ρ_H. Moreover, β_L^0 is decreasing in p_H with

$$\lim_{p_H \to 0} \beta_L^0 = \beta_L^* \text{ and } \lim_{p_H \to 1} \beta_L^0 = 0.$$

Under the menu of optimal contracts, the expected utility of poor and rich agents is

$$u(\alpha_L^0, \beta_L^0; \rho_L) = \bar{u}, \text{ and } u(\alpha_H^0, \beta_H^0; \rho_H) = \bar{u} + \frac{1}{2}(\rho_L - \rho_H)(\beta_L^0)^2\sigma^2,$$

respectively.

The formal proof of the above proposition is presented in the Appendix. As shown in the proof of Proposition 3, the two constraints i) and iv) must be binding, but the two constraints ii) and iii) must be non-binding at the optimum. The reason that the participation constraint for the poor agent should be binding but the one for the rich agent should be non-binding at the optimum, is because the poor agent tells the truth, but the rich agent has an incentive to lie. Thus, the principal compensates the poor agent at a minimum, but inevitably permits the rich agent to enjoy rent over $\bar{u}$. Moreover, constraint iv) should be binding, because the principal should provide an incentive to tell the truth for the bad guy (*i.e.*, rich agent), while constraint iii) should be non-binding because the good guy (*i.e.*, poor agent) has no incentive to lie. These results enable us to deal with the problem [AMP] subject to only the two binding constraints i) and iv). Thus, the problem [AMP] is rewritten by

$$\max_{\beta_L,\beta_H} p_L[\phi(\beta_L;\rho_L) - \frac{p_H}{2p_L}(\rho_L-\rho_H)\beta_L^2\sigma^2] + p_H\phi(\beta_H;\rho_H). \qquad \text{[RP]}$$

The solution β_H^0 to [RP] should satisfy the first-order condition regarding β_H, that is,

$$\phi_\beta(\beta_H;\rho_H) = a'(\beta_H) - c'(a(\beta_H))a'(\beta_H) - \rho_H\sigma^2\beta_H = 0$$
$$\Leftrightarrow \frac{1-\beta_H}{c''(a(\beta_H))} = \rho_H\sigma^2\beta_H,$$

which is equivalent to Condition (4) for $i = H$. Thus, we have $\beta_H^0 = \beta_H^*$. Meanwhile, the solution β_L^0 to [RP] should satisfy

$$\phi_\beta(\beta_L;\rho_L) - \frac{p_H}{p_L}(\rho_L-\rho_H)\beta_L\sigma^2$$
$$= a'(\beta_L) - c'(a(\beta_L))a'(\beta_L) - \rho_L\beta_L\sigma^2 - \frac{p_H}{p_L}(\rho_L-\rho_H)\beta_L\sigma^2 = 0$$
$$\Leftrightarrow \frac{1-\beta_L}{c''(a(\beta_L))} = \left[\rho_L + \frac{p_H}{p_L}(\rho_L-\rho_H)\right]\sigma^2\beta_L, \qquad (6)$$

which has the same form with Condition (4) for $i = L$, except that ρ_L in the RHS of Condition (4) is replaced with term $\rho_L + (p_H / p_L)(\rho_L - \rho_H)$ in Condition (6). Given that $(p_H / p_L)(\rho_L - \rho_H)\sigma^2 > 0$, we have $\beta_L^0 < \beta_L^*$ (see Figure 3). Furthermore, an increase in the slope of β_L in the RHS of condition (6), $[\rho_L + (p_H / p_L)(\rho_L - \rho_H)]\sigma^2$, decreases the value of β_L^0. Thus, an increase in ρ_L, a decrease in ρ_H, or an increase in p_H lowers the value

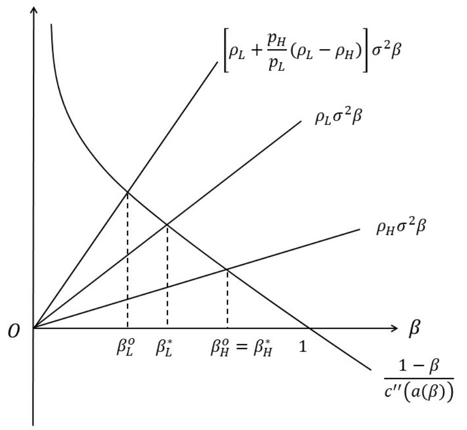

Figure 3
Comparison between the incentive powers under adverse selection problem

of β_L^0 by increasing the value of $\rho_L + (p_H / p_L)(\rho_L - \rho_H)$. In particular, ρ_H positively affects β_L^0 by shortening the gap between the degrees of risk aversion of rich and poor agents (*i.e.*, $\rho_L - \rho_H$).[3]

As shown in Proposition 3, when the agents' wealth is hidden, the incentive power for agents being richer and less risk averse remains unchanged, but poorer agents' incentive power becomes lower. This is because the rich agents watch thirstily for a chance to profit additionally by picking up the incentive contract prepared for poor agents. Hence, for the principal to prevent the less risk-averse agents from mimicking the more risk-averse agents, the poor agents' incentive power is getting lower.

The results in the Proposition 3 imply that the difference of the incentive powers between rich and poor agents widens further when agents' wealth becomes hidden. The reason is that $\Delta\beta^* \equiv \beta_H^* - \beta_L^* < \beta_H^0 - \beta_L^0 \equiv \Delta\beta^0$, where the strict inequality is satisfied given $\beta_H^0 = \beta_H^*$ and $\beta_L^0 < \beta_L^*$. More importantly, Proposition 3 reveals that the difference $\Delta\beta^0$ decreases with (ρ_L, p_H) but increases with ρ_H. This means that, given that the degree of risk aversion $\rho(w)$ is a decreasing function of wealth

[3] An increase in $(\rho_L - \rho_H)$ expands the difference $\beta_H^0 - \beta_L^0$, which worsen the social welfare. This issue will be discussed in the next subsection.

w, the gap of incentive powers is expanded by a decrease in the poor agent's wealth w_L, by an increase in the rich agent's wealth w_H, and/or by an increase in the proportion of rich agents, p_H. Because a change in the distribution of wealth in the sense of mean-preserving spread can be made by a decrease in w_L, an increase in w_H, and/or an increase in p_H, our results in the Proposition 3 imply that the gap of incentive powers among heterogenous agents is escalated when wealth inequality increases.

The principal's maximized profit is represented by

$$\pi^a(p_H, w_L, w_H) \equiv (1 - p_H)\phi(\beta_L^0; \rho(w_L)) - \frac{p_H}{2}[\rho(w_L) - \rho(w_H)](\beta_L^0)^2\sigma^2 + p_H\phi(\beta_H^0; \rho(w_H)).$$

The following proposition uncovers the characteristics of profit function $\pi^a(p_H, w_L, w_H)$.

Proposition 4. The profit function $\pi^a(p_H, w_L, w_H)$ is increasing in w_L and w_H. Moreover, $\pi^a(p_H, w_L, w_H)$ is strictly convex in p_H, implying that $\pi^a(p_H, w_L, w_H)$ is increasing in p_H.

Proof. $\phi_\rho(\beta_i^0; \rho_i) = -\frac{1}{2}(\beta_i^0)^2\sigma^2$ for all $i = L,H$. Although, as seen in Condition (6), β_L^0 is a function of (ρ_L, p_H), the use of the Envelope Theorem makes

$$\pi^a_{w_L}(p_H, w_L, w_H) = (1 - p_H)\phi_\rho(\beta_L^0; \rho_L)\rho'(w_L) - \frac{p_H}{2}(\beta_L^0)^2\sigma^2\rho'(w_L) = -\frac{1}{2}(\beta_L^0)^2\sigma^2\rho'(w_L) > 0,$$

where strict inequality holds given $\rho'(w_L) < 0$, and

$$\pi^a_{w_H}(p_H, w_L, w_H) = \frac{p_H}{2}(\beta_L^0)^2\sigma^2\rho'(w_H) + p_H\phi_\rho(\beta_H^0; \rho(w_H))\rho'(w_H) = -\frac{p_H}{2}[(\beta_H^0)^2 - (\beta_L^0)^2]\sigma^2\rho'(w_H) > 0,$$

where the strictly inequality holds because $\beta_H^0 > \beta_L^0$ by Proposition 3. Thus, $\pi^a(p_H, w_L, w_H)$ is increasing in w_L and w_H.

Similarly, even if β_L^0 is a function of p_H, the Envelope Theorem yields

$$\pi_p^a(p_H, w_L, w_H) = \phi(\beta_H^0;\rho_H) - \phi(\beta_L^0;\rho_L) - \frac{1}{2}(\rho_L - \rho_H)(\beta_L^0)^2\sigma^2,$$

which differentiating one more with respect to p_H gives

$$\pi_{pp}^a(p_H, w_L, w_H) = -[\phi_\beta(\beta_L^0;\rho_L) + (\rho_L - \rho_H)\beta_L^0\sigma^2] \times \frac{\partial \beta_L^0}{\partial p_H}.$$

For any given $\rho_L > 0$, because $\phi(\beta;\rho_L)$ is strictly concave on interval (0,1), because β_L^* is the unique maximizer of function $\phi(\beta;\rho_L)$ (*i.e.*, $\phi_\beta(\beta_L^*;\rho_L) = 0$), and because $\beta_L^0 < \beta_L^* < 1$, we have $\phi_\beta(\beta_L^0;\rho_L) > 0$. Thus, given that β_L^0 is decreasing in p_H by Proposition 3, the sign of $\pi_{pp}^a(p_H, w_L, w_H)$ is positive, meaning that $\pi^a(p_H, w_L, w_H)$ is strictly convex in p_H, or equivalently $\pi_p^a(p_H, w_L, w_H)$, is increasing in p_H. If $\pi_p^a(p_H{=}0, w_L, w_H){>}0$ is satisfied, it implies $\pi_p^a(p_H, w_L, w_H) > 0$ for all $p_H \in [0,1]$. By Proposition 3, $\beta_H^0 = \beta_H^*$ and when $p_H = 0$, $\beta_L^0 = \beta_L^*$. Thus, because $\phi(\beta_H^*;\rho_H) > \phi(\beta_L^*;\rho_H)$ as shown in the proof of Proposition 1, we have

$$\pi_p^a(p_H = 0, w_L, w_H) = \phi(\beta_H^*;\rho_H) - \phi(\beta_L^*;\rho_L) - \frac{1}{2}(\rho_L - \rho_H)(\beta_L^*)^2\sigma^2$$

$$> \phi(\beta_L^*;\rho_H) - \phi(\beta_L^*;\rho_L) - \frac{1}{2}(\rho_L - \rho_H)(\beta_L^*)^2\sigma^2 = 0.$$

Therefore, because $\pi_p^a(p_H = 0, w_L, w_H){>}0$, $\pi^a(p_H, w_L, w_H)$ is increasing in p_H. Q.E.D.

The above Proposition shows that the principal's profit $\pi^a(p_H, w_L, w_H)$ is increasing in wealth of all agents. For the poor agent, an increase in w_L lowers his degree of risk aversion $\rho(w_L)$. This results in two effects: It decreases his/her risk premium, which increases the profit. Also, it lessens the gap between the degrees of risk aversion of poor and rich agents (*i.e.*, $\rho_L - \rho_H$), which makes rent over $\bar{u}$ for rich agents drop, so that the profit increases. Hence, the principal's profit always increases with w_L. Similarly, for the rich agent, an increase in w_H decreases his/her degree of risk aversion $\rho(w_H)$. This first widens the gap $(\rho_L - \rho_H)$, which increases the rent for rich agents, which reduces the profit. However, a decrease in $\rho(w_H)$ simultaneously reduces risk premium of rich agents, which increases the profit. Given that the latter effect dominates the former by $\beta_H^0 > \beta_L^0$, an increase in w_H positively impacts the principal's profit.

Furthermore, the Proposition 4 shows that the principal's profit function $\pi^a(p_H, w_L, w_H)$ is strictly convex in p_H. This means that the first derivative of $\pi^a(p_H, w_L, w_H)$ with respect to p_H, $\pi^a_p(p_H, w_L, w_H)$, is an increasing function of p_H. Thus, because $\pi^a_p(p_H=0, w_L, w_H)>0$ as shown in the proof of the Proposition 4, it implies that $\pi^a_p(p_H, w_L, w_H)>0$ for all $p_H \in [0,1]$. From this, we conclude that $\pi^a(p_H, w_L, w_H)$ is also an increasing function of p_H.

Proposition 4 indicates that the principal prefers an increase in wealth level of any agent and an increase in the proportion of rich agents. The increases in the levels of agents' wealth and/or in the proportion of high wealth mean that a distribution of agents' wealth is changed in the sense of the first-order stochastic dominance (FOSD). Consequently, what Proposition 4 really says is that the principal prefers an improvement of the wealth distribution in the FOSD sense.

C. Welfare Loss

In this subsection, we address changes in social welfare. We assume that social welfare is the sum of the principal's profit and agent's expected utility. So, define

$$SW = \sum_{i=L,H} p_i[\pi(\alpha_i, \beta_i) + u(\alpha_i, \beta_i; \rho_i)] = \sum_{i=L,H} p_i\phi(\beta_i; \rho_i),$$

where the second equality holds because

$$\pi(\alpha_i, \beta_i) + u(\alpha_i, \beta_i; \rho_i) = a(\beta_i) - c(a(\beta_i)) - \frac{\rho_i}{2}\beta_i^2\sigma^2 \equiv \phi(\beta_i; \rho_i).$$

When the agents' wealth is known, the social welfare is

$$SW^m(p_H, w_L, w_H) = (1 - p_H)\phi(\beta_L^*; \rho(w_L)) + p_H\phi(\beta_H^*; \rho(w_H)),$$

and when it is hidden, the welfare is

$$SW^a(p_H, w_L, w_H) = (1 - p_H)\phi(\beta_L^0; \rho(w_L)) + p_H\phi(\beta_H^0; \rho(w_H)).$$

Then, because $\beta_H^0 = \beta_H^*$, the welfare loss is written by

$$\begin{aligned} L(p_H, w_L, w_H) &\equiv SW^m(p_H, w_L, w_H) - SW^a(p_H, w_L, w_H) \\ &= (1 - p_H)[\phi(\beta_L^*; \rho(w_L)) - \phi(\beta_L^0; \rho(w_L))]. \end{aligned}$$

This equation indicates that the loss is determined by the powers of incentives for the agent with low wealth w_L, (*i.e.*, β_L^* and β_L^0) and his/her degree of risk aversion $\rho(w_L)$. β_L^* depends on ρ_L which is a decreasing function of w_L as shown in Condition (4), whereas β_L^0 depends on w_i through $\rho(w_i)$, $i = L, H$, and on p_H as shown in the Proposition 3. Thus, the loss is affected by (p_H, w_L, w_H).

Let us analyze the effects of (p_H, w_L, w_H) on the welfare loss. First, differentiating $L(p_H, w_L, w_H)$ with respect to w_H gives

$$\frac{\partial L}{\partial w_H} = -(1-p_H)\phi_\beta(\beta_L^0;\rho(w_L)) \times \frac{\partial \beta_L^0}{\partial \rho_H}\rho'(w_H) > 0,$$

where the strict inequality holds because $\phi_\beta(\beta_L^0;\rho(w_L)) > 0, \partial\beta_L^0 / \partial\rho_H > 0$ and $\rho'(w_H) < 0$. This means that an increase in the wealth level of rich agent raises the welfare loss. The reason is as follows: when w_H increases, the difference $(\rho_L - \rho_H)$ widens by lowering ρ_H, which reduces β_L^0. Given β_L^* and ρ_L, it always raises the welfare loss.

However, the effects of w_L and p_H on the loss are ambiguous. To see the reason, let us differentiate the loss function with respect to w_L. The envelope theorem gives

$$\begin{aligned}\frac{\partial L}{\partial w_L} &= (1-p_H)\left\{\left[\phi_\rho(\beta_L^*;\rho(w_L)) - \phi_\rho(\beta_L^0;\rho(w_L))\right]\rho'(w_L) - \phi_\beta(\beta_L^0;\rho(w_L)) \times \frac{\partial\beta_L^0}{\partial\rho_L} \times \rho'(w_L)\right\} \\ &= (1-p_H)\left\{\underbrace{-\frac{\sigma^2}{2}\left[(\beta_L^*)^2 - (\beta_L^0)^2\right] \times \rho'(w_L)}_{=C} + \underbrace{\left[-\phi_\beta(\beta_L^0;\rho(w_L))\right] \times \frac{\partial\beta_L^0}{\partial\rho_L} \times \rho'(w_L)}_{=D}\right\}. \end{aligned} \tag{6}$$

Given that $\beta_L^* > \beta_L^0$, term C in Equation (6) is positive. However, because $\phi_\beta(\beta_L^0;\rho_L) > 0$ and because $\partial\beta_L^0 / \partial\rho_L < 0$ by Proposition 3, term D is negative. Hence, the sign of Equation (6) is positive if $C + D \geq 0$, but is negative otherwise.

An increase in w_L lowers the degree of risk aversion $\rho(w_L)$ by assumption. The two terms C and D capture direct and indirect effects of w_L on the welfare loss by lowering the degree of risk aversion, respectively. First, a decrease in ρ_L by an increase in w_L directly expands the difference $\phi(\beta_L^*;\rho_L) - \phi(\beta_L^0;\rho_L) = \int_{\beta_L^0}^{\beta_L^*}\phi_\beta(\beta;\rho_L)d\beta$ because $\phi_\beta = (1 - \beta) / c''(a(\beta)) - \rho\beta\sigma^2$ is a decreasing function of ρ. Thus, an increase in w_L raises the welfare loss directly. Meanwhile, as shown in the Proposition 3, a decrease in ρ_L increases β_L^0, which reduces the difference $\phi(\beta_L^*;\rho_L)$

$-\ \phi(\beta_L^0;\rho_L)$. Thus, an increase in w_L reduces the welfare loss indirectly. Consequently, if the direct effect dominates the indirect one, an increase in w_L positively affects the welfare loss, but otherwise it has a negative effect.

Next, differentiating the loss function with respect to p_H gives

$$\frac{\partial L}{\partial p_H} = \underbrace{-\left[\phi(\beta_L^*;\rho(w_L)) - \phi(\beta_L^0;\rho(w_L))\right]}_{=F} + \underbrace{\left[-(1-p_H)\phi_\beta(\beta_L^0;\rho(w_L)) \times \frac{\partial \beta_L^0}{\partial p_H}\right]}_{=G}. \quad (7)$$

Term F in Equation (7) is negative because $\beta_L^* > \beta_L^0$, but term G is positive because $\phi_\beta(\beta_L^0;\rho_L) > 0$ and $\partial\beta_L^0 / \partial p_H < 0^{\cdot}$ Similarly, terms F and G indicate
direct and indirect effects of p_H, respectively. Directly, an increase in p_H lowers the loss by decreasing p_L. However, it also drops β_L^0, which increases the welfare loss by widening the difference $\phi(\beta_L^*;\rho_L) - \phi(\beta_L^0;\rho_L)$. Consequently, if the former effect dominates the latter, an increase in p_H reduces the welfare loss, but otherwise it expands the loss.

The following proposition analyzes the behaviors of the loss function when effort cost is quadratic.

Proposition 5. The loss function $L(p_H, w_L, w_H)$ is increasing in w_H. However, the effects of p_H and w_L are ambiguous. Nevertheless, when $c(a) = \frac{k}{2}a^2$, the loss function $L(p_H, w_L, w_H)$ is increasing in p_H and w_L under the following conditions, respectively:

$$\rho_L - \rho_H \leq \frac{(2-p_H)(1-p_H)}{p_H^2}\left(\frac{1}{k\sigma^2} + \rho_L\right),$$

and

$$\rho_L - \rho_H \leq K\left(\frac{1}{k\sigma^2} + \rho_L\right),$$

where

$$K \equiv \frac{(1-p_H)\left(3 + p_H - \sqrt{(1-p_H)(9-p_H)}\right)}{p_H\left(1 - p_H + \sqrt{(1-p_H)(9-p_H)}\right)}.$$

Proof. When $c(a)=\frac{k}{2}a^2$, because $c'(a) = ka$, we have $a(\beta) = \beta/k$. Hence, solving Equations (4) and (6) separately makes

$$\beta_L^* = \frac{1}{1+k\rho_L\sigma^2} \equiv \frac{1}{A}, \quad \text{and} \quad \beta_L^0 = \frac{1}{1+k\left[\rho_L + \frac{p_H}{p_L}(\rho_L - \rho_H)\right]\sigma^2} \equiv \frac{1}{B}.$$

Fundamentally, $B > A$ for all $p_H \in (0,1)$, and as $p_H \to 0$, $B \to A$. Then, we have

$$\phi_\rho(\beta_L^*;\rho_L) - \phi_\rho(\beta_L^0;\rho_L) = -\frac{\sigma^2}{2}[(\beta_L^*)^2 - (\beta_L^0)^2] = \frac{\sigma^2}{2A^2}\left(\frac{A}{B}+1\right)\left(\frac{A}{B}-1\right),$$

and

$$\phi_\beta(\beta_L^0;\rho_L) \times \frac{\partial \beta_L^0}{\partial \rho_L} = -\frac{\sigma^2}{(1-p_H)B^2}\left(1-\frac{A}{B}\right).$$

Thus, Equation (6) becomes

$$\begin{aligned}\frac{\partial L}{\partial w_L} &= (1-p_H)\left[\frac{\sigma^2}{2A^2}\left(\frac{A}{B}+1\right)\left(\frac{A}{B}-1\right) + \frac{\sigma^2}{(1-p_H)B^2}\left(1-\frac{A}{B}\right)\right]\rho'(w_L) \\ &= \frac{\sigma^2}{2A^2}\left(1-\frac{A}{B}\right) \times f\left(\frac{A}{B}\right)\rho'(w_L),\end{aligned}$$

where $f(t) = 2t^2 - (1-p_H)t - (1-p_H)$. For $t_1 < 1$ satisfying $f(t_1) = 0$, if $A/B \le t_1$, we have $\partial L/\partial w_L \ge 0$. Solving $f(t_1) = 0$ yields $t_1 = (1-p_H + \sqrt{(1-p_H)(9-p_H)})/4$ and then solving inequality $A/B \le t_1$ makes

$$\rho_L - \rho_H \ge \frac{(1-t_1)(1-p_H)}{t_1 p_H}\left(\frac{1}{k\sigma^2} + \rho_L\right).$$

Similarly, we have

$$\phi(\beta_L^*;\rho_L) - \phi(\beta_L^0;\rho_L) = \frac{1}{2kA} - \frac{1}{2kB^2}(2B-A) = \frac{1}{2kA}\left[1 - 2\left(\frac{A}{B}\right) + \left(\frac{A}{B}\right)^2\right],$$

and

$$p_L\phi_\beta(\beta_L^0;\rho_L)\frac{\partial\beta_L^0}{\partial p_H}=-\frac{1}{kp_HB}\left(1-\frac{A}{B}\right)^2.$$

Thus, Equation (7) becomes

$$\frac{\partial L}{\partial p_H}=-\frac{1}{2kA}\left[1-2\left(\frac{A}{B}\right)+\left(\frac{A}{B}\right)^2\right]+\frac{1}{kp_HB}\left(1-\frac{A}{B}\right)^2=\frac{1}{2kp_HA}\times h\left(\frac{A}{B}\right),$$

where $h(t)=(t-1)^2(2t-p_H)$. For $t_2=p_H/2<1$ satisfying $h(t_2)=0$, if $A/B\geq t_2=p_H/2$, because $h(A/B)\geq 0$, we have $\partial L/\partial p_H\geq 0$. Solving inequality $A/B\geq p_H/2$ makes

$$\rho_L-\rho_H\leq\frac{(2-p_H)(1-p_H)}{p^2}\left(\frac{1}{k\;\;{}^{2}}+\rho_L\right)$$

It completes the proof. Q.E.D.

IV. Conclusion

We examined how the incentive contracts for agents with low and high wealth are changed from when their wealth is known to the principal to when it is hidden. For it, we first considered the case that the wealth levels which all agents have are known to the principal. In this case, the power of incentives for rich agents is greater than the one for poor agents, because the rich agents' degree of risk aversion is less than the poor agents. This means that, for a given agent, an increase in his wealth increases the principal's profit by lowering his/her risk premium. Thus, the increases in wealth of any agent and in the proportion of rich agents raises the principal's profit.

However, when agents' wealth is hidden from the principal, if the menu of the optimal contracts designed for rich and poor agents when it is known is still offered, rich agents have an incentive to select the contract targeting poor agents. Fundamentally, rich agents want to mimic poor agents. Therefore, the principal must design a new menu of contracts under which rich agents are transparent.

Compared with the case wherein agents' wealth is known, the power of incentives for rich agents remains unchanged, but the one for poor agents declines. It means that rich agents should be prevented from mimicking poor agents by lowering the power of incentive for the poor and then reducing the risk premium which rich agents may enjoy. Resultantly, the power of incentives for poor agents decreases when the principal concerns adverse selection problem.

We have shown that the power of incentives for poor agents is decreased by the increases in the degree of risk aversion of poor agents and in the proportion of rich agents, but increased by an increase in the degree of risk aversion of rich agents. However, even under these properties, we have shown that the principal's profit is increasing in wealth of any agent and in the proportion of rich agents. This means that the principal prefers an improvement of wealth distribution in the sense of the first-order stochastic dominance. Furthermore, we have shown that the welfare loss from hidden wealth increases with the wealth level of rich agents. The reason is that an increase in their wealth expands the gap of the degrees of risk aversion between rich agent and poor agents, so that the power of incentives aimed at poor agents further declines. Ultimately, it aggravates the welfare loss. However, the effects of poor agents' wealth and the proportion of rich agents on the welfare loss is unclear.

In reality, the wealth levels of agents are rarely open to the principal. Our study is meaningful in that we suggest theoretical results considering such situation seriously. However, our results have a limitation in the sense that we assume the wealth level of agents is two type. Thus, the studies in the case that agents' wealth is multi-type will be needed in the future.

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Appendix

The proof of Proposition 3. We start with the following lemma.

Lemma A. Constraint ii) should be non-binding at the optimum. By constraint iv), we have

$$
\begin{aligned}
u(\alpha_H, \beta_H; \rho_H) &= \alpha_H + \beta_H a(\beta_H) - c(a(\beta_H)) - \frac{\rho_H}{2}\beta_H^2\sigma^2 \\
&\geq \alpha_L + \beta_L a(\beta_l) - c(a(\beta_L)) - \frac{\rho_H}{2}\beta_L^2\sigma^2 \\
&> \alpha_L + \beta_L a(\beta_l) - c(a(\beta_L)) - \frac{\rho_L}{2}\beta_L^2\sigma^2 \geq \bar{u}.
\end{aligned}
$$

where the strict inequality holds because $\rho_L > \rho_H$ and the last inequality holds by constraint i). Thus, because constraints iv) and i) imply that $u(\alpha_H,\beta_H;\rho_H) > \bar{u}.$, constraints ii) should be non-binding at the optimum. Q.E.D.

The above Lemma A allows us to solve [AMP] without constraint ii). Thus, the Lagrange function is

$$
\begin{aligned}
L = {} & p_L[(1-\beta_L)a(\beta_L) - \alpha_L] + p_H[(1-\beta_H)a(\beta_H) - \alpha_H] \\
& +\lambda_L[\alpha_L + \beta_L a(\beta_L) - c(a(\beta_L)) - \frac{\rho_L}{2}\beta_L^2\sigma^2 - \bar{u}] \\
& +\mu_L[\alpha_L + \beta_L a(\beta_L) - c(a(\beta_L)) - \frac{\rho_L}{2}\beta_L^2\sigma^2 - \alpha_H - \beta_H a(\beta_H) + c(a(\beta_H)) + \frac{\rho_L}{2}\beta_H^2\sigma^2] \\
& +\mu_H[\alpha_H + \beta_H a(\beta_H) - c(a(\beta_H)) - \frac{\rho_H}{2}\beta_H^2\sigma^2 - \alpha_L - \beta_L a(\beta_L) + c(a(\beta_L)) + \frac{\rho_H}{2}\beta_L^2\sigma^2],
\end{aligned}
$$

where λ_L, μ_L and μ_H are Lagrange multipliers for constraints i), iii) and iv), respectively. The Kuhn-Tucker conditions are the followings:

$$L_{\alpha_L} = -p_L + \lambda_L + \mu_L - \mu_H = 0, \tag{A.1}$$

$$L_{\alpha_H} = -p_H - \mu_L + \mu_H = 0, \tag{A.2}$$

$$
\begin{aligned}
L_{\beta_L} = {} & p_L[-a(\beta_L) + (1-\beta_L)a'(\beta_L)] + (\lambda_L + \mu_L)[a(\beta_L) - \rho_L\sigma^2\beta_L] \\
& -\mu_H[a(\beta_L) - \rho_H\sigma^2\beta_L] = 0,
\end{aligned} \tag{A.3}
$$

$$L_{\beta_H} = p_H[-a(\beta_H) + (1-\beta_H)a'(\beta_H)] - \mu_L[a(\beta_H) - \rho_L\sigma^2\beta_H] + \mu_H[a(\beta_H) - \rho_H\sigma^2\beta_H] = 0, \quad \text{(A.4)}$$

$$\lambda_L[u(\alpha_L, \beta_L; \rho_L) - \bar{u}] = 0, \quad \lambda_L \geq 0, \quad u(\alpha_L, \beta_L; \rho_L) - \bar{u} \geq 0, \quad \text{(A.5)}$$

$$\mu_L[u(\alpha_L, \beta_L; \rho_L) - u(\alpha_H, \beta_H; \rho_L)] = 0, \quad \mu_L \geq 0, \quad u(\alpha_L, \beta_L; \rho_L) - u(\alpha_H, \beta_H; \rho_L) \geq 0, \quad \text{(A.6)}$$

$$\mu_H[u(\alpha_H, \beta_H; \rho_H) - u(\alpha_L, \beta_L; \rho_H)] = 0, \quad \mu_H \geq 0, \quad u(\alpha_H, \beta_H; \rho_H) - u(\alpha_L, \beta_L; \rho_H) \geq 0. \quad \text{(A.7)}$$

The combination of Conditions (A.1) and (A.2) gives

$$-(p_L + p_H) + \lambda_L = 0 \Leftrightarrow \lambda_L = 1,$$

where the above equivalence is satisfied because $p_L + p_H = 1$. Thus, because $\lambda_L > 0$, we have $u(\alpha_L,\beta_L;\rho_L) = \bar{u}$ by Condition (A.5). Moreover, by Condition (A.2), we have

$$\mu_H = p_H + \mu_L \geq p_H > 0,$$

where the first inequality is satisfied by $\mu_L \geq 0$, implying $u(\alpha_H,\beta_H;\rho_H) = u(\alpha_L,\beta_L;\rho_H)$ by Condition (A.7). Rearranging Equations (A.2) and (A.3) yields

$$\lambda_L + \mu_L = p_L + \mu_H \quad \text{and} \quad -\mu_L = p_H - \mu_H.$$

By using these conditions, Conditions (A.3) and (A.4) become respectively

$$\begin{aligned} L_{\beta_L} &= p_L[-a(\beta_L) + (1-\beta_L)a'(\beta_L)] + (p_L + \mu_H)[a(\beta_L) - \rho_L\sigma^2\beta_L] - \mu_H[a(\beta_L) - \rho_H\sigma^2\beta_L] \\ &= p_L[(1-\beta_L)a'(\beta_L) - \rho_L\sigma^2\beta_L] - \mu_H(\rho_L - \rho_H)\sigma^2\beta_L = 0, \end{aligned} \quad \text{(A.8)}$$

and

$$\begin{aligned} L_{\beta_H} &= p_H[-a(\beta_H) + (1-\beta_H)a'(\beta_H)] + (p_H + \mu_H)[a(\beta_H) - \rho_L\sigma^2\beta_H] \\ &\quad + \mu_H[a(\beta_H) - \rho_H\sigma^2\beta_H] \\ &= p_H[(1-\beta_H)a'(\beta_H) - \rho_L\sigma^2\beta_H] + \mu_H(\rho_L - \rho_H)\sigma^2\beta_H = 0. \end{aligned} \quad \text{(A.9)}$$

Lemma B. Constraint iii) should be non-binding at the optimum.
Suppose that constraint iii) is binding at the optimum, that is,

$$u(\alpha_L,\beta_L;\rho_L) = u(\alpha_H,\beta_H;\rho_L). \tag{A.10}$$

As shown earlier, because $\mu_H > 0$, constraint iv) must be binding, that is,

$$u(\alpha_H,\beta_H;\rho_H) = u(\alpha_L,\beta_L;\rho_H). \tag{A.11}$$

Combining Equations (A.10) and (A.11) gives

$$\begin{aligned}
&u(\alpha_L, \beta_L; \rho_L) + u(\alpha_H, \beta_H; \rho_H) = u(\alpha_H, \beta_H; \rho_L) + u(\alpha_L, \beta_L; \rho_H)\\
&\Leftrightarrow -\frac{\rho_L}{2}\sigma^2\beta_L^2 - \frac{\rho_H}{2}\sigma^2\beta_H^2 = -\frac{\rho_L}{2}\sigma^2\beta_H^2 - \frac{\rho_H}{2}\sigma^2\beta_L^2\\
&\Leftrightarrow \frac{\sigma^2}{2}(\rho_L - \rho_H)(\beta_H^2 - \beta_L^2) = 0,
\end{aligned}$$

from which we have $\beta_H = \beta_L$ because $\rho_L > \rho_H$. Hence, rearranging Equations (A.8) and (A.9) gives

$$(1-\beta_L)a'(\beta_L) - \rho_L\sigma^2\beta_L = \frac{\mu_H}{p_L}(\rho_L - \rho_H)\sigma^2\beta_L, \tag{A.12}$$

and

$$(1-\beta_L)a'(\beta_L) - \rho_L\sigma^2\beta_L = -\frac{\mu_H}{p_H}(\rho_L - \rho_H)\sigma^2\beta_L, \tag{A.13}$$

respectively. Thus, combining Equations (A.12) and (A.13) makes

$$\frac{\mu_H}{p_L}(\rho_L - \rho_H)\sigma^2\beta_L = -\frac{\mu_H}{p_H}(\rho_L - \rho_H)\sigma^2\beta_L \Leftrightarrow \left(\frac{1}{p_L} + \frac{1}{p_H}\right)\mu_H(\rho_L - \rho_H)\sigma^2\beta_L = 0,$$

which implies $\beta_L = 0$ because $\mu_H > 0$ and $\rho_L > \rho_H$. Thus, because $a'(0) = 1 / c''(a(0)) = 1 / c''(0) > 0$ by $c''(0) \geq 0$, when $\beta_L = 0$, we have from Equation (A.8)

$$L_{\beta_L} = p_L a'(0) > 0,$$

which is a contradiction to $L_{\beta_L} = 0$. Therefore, constraint iii) must be non-binding at the optimum, implying $\mu_L = 0$ by Condition (A.6). Q.E.D.

Given that $\mu_L = 0$ by the Lemma B, from (A.2), we have $\mu_L = p_H$, Thus, from Condition (A.9), we have

$$\begin{aligned} & p_H[(1-\beta_H)a'(\beta_H) - \rho_L\sigma^2\beta_H] + p_H(\rho_L - \rho_H)\sigma^2\beta_H \\ & = p_H[(1-\beta_H)a'(\beta_H) - \rho_H\sigma^2\beta_H] = 0 \\ & \Leftrightarrow \frac{1-\beta_H}{c''(a(\beta_H))} = \rho_H\sigma^2\beta_H, \end{aligned} \tag{A.14}$$

where the equivalence holds because $a'(\beta_H) = \dfrac{1}{c''(a(\beta_H))}$. Moreover, from Condition (A.8), we have

$$\begin{aligned} & p_L[(1-\beta_L)a'(\beta_L) - \rho_L\sigma^2\beta_L] + p_H(\rho_L - \rho_H)\sigma^2\beta_L = 0 \\ & \Leftrightarrow (1-\beta_L)a'(\beta_L) = \left[\rho_L + \frac{p_H}{p_L}(\rho_L - \rho_H)\right]\sigma^2\beta_L \\ & \Leftrightarrow \frac{1-\beta_L}{c''(a(\beta_L))} = \left[\rho_L + \frac{p_H}{p_L}(\rho_L - \rho_H)\right]\sigma^2\beta_L, \end{aligned} \tag{A.15}$$

where the last equivalence holds because $a'(\beta_L) = \dfrac{1}{c''(a(\beta_L))}$.

Condition (A.14) is identical to Condition (4) when $i = H$, indicating $\beta_H^0 = \beta_H^*$. And, because term $\rho_L + p_H(\rho_L - \rho_H) / p_L$ in the RHS of Condition (A.15) is greater than ρ_L in the RHS of Condition (4) when $i = L$, we have $\beta_L^0 < \beta_L^*$. Thus, because $\beta_H^0 = \beta_H^* > \beta_L^* > \beta_L^0$, we have $\beta_H^0 > \beta_L^*$.

Furthermore, β_L^0 is a decreasing function of $g(p_H,\rho_L,\rho_H) \equiv \rho_L + p_H(\rho_L - \rho_H) / (1 - p_H)$ in the bracketed term in the RHS of (A.15). Given that $g(p_H, \rho_L, \rho_H)$ increases with p_H and ρ_L but decreases with ρ_H, β_L^0 is decreasing in p_H and ρ_L and increasing in ρ_H. Especially, as $p_H \to 0$, $g(p_H, \rho_L, \rho_H) \to \rho_L$. This makes Condition (A.15) for β_L^0 be identical with Condition (4) for β_L^*, resulting in $\beta_L^0 = \beta_L^*$. Meanwhile, as $p_H \to 1$, because $p_H / p_L = p_H / (1 - p_H)$ goes to ∞, the slope of the right hand side of (A.15) goes to ∞, implying $\beta_L^0 = 0$.

Given that constraint i) must be binding at the optimum, the expected utility of agents with wealth w_L is

$$u(\alpha_L^0, \beta_L^0; \rho_L) = \alpha_L^0 + \beta_L^0 a(\beta_L^0) - c(a(\beta_L^0)) - \frac{\rho_L}{2}(\beta_L^0)^2\sigma^2 = \bar{u}. \qquad \text{(A.16)}$$

Meanwhile, because constraint iv) must be binding at the optimum, the expected utility of agents with wealth w_H is

$$\begin{aligned} u(\alpha_H^0, \beta_H^0; \rho_H) &= \alpha_L^0 + \beta_L^0 a(\beta_L^0) - c(a(\beta_L^0)) - \frac{\rho_H}{2}(\beta_L^0)^2\sigma^2 \\ &= \bar{u} + \frac{1}{2}(\rho_L - \rho_H)(\beta_L^0)^2\sigma^2, \end{aligned}$$

where the second equality holds because $\alpha_L^0 + \beta_L^0 a(\beta_L^0) - c(a(\beta_L^0)) = \bar{u} + \frac{\rho_L}{2}(\beta_L^0)^2\sigma^2$ from Equation (A.16). Q.E.D.