Information Transmission in Revision Games

Yves Guéron

Revision games model a situation in which players can prepare their actions during a pre-play phase. We introduce one-sided incomplete information in two coordination games, one of common interest and one of opposing interest, and study how the pre-play phase affects coordination. We find that in the common interest game, the unique Bayesian equilibrium is such that the informed player will signal the state of the world through her prepared action, unless the pre-play phase is about to finish, in which case she seeks to coordinate with the other player. In the opposing interest game, the equilibrium is similar when the informed player is the one receiving less opportunity to revise her actions. When it is the uninformed player who receives less revision opportunities, we show that it is possible no information is revealed if both players are initially coordinated, but some information must be revealed if they are initially miscoordinated.

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I. Introduction

Revision games (see Kamada and Kandori 2017; Calcagno *et al.* 2014) model a situation in which players can prepare their actions during a

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pre-play phase. At the end of the pre-play phase, the action profile last prepared by players is implemented and players receive the corresponding stage-game payoff. Revision opportunities are stochastic and arrive according to independent Poisson processes. There is therefore a positive probability that a player might no longer be able to revise his prepared action before the deadline. While Kamada and Kandori (2017) show that with a continuum of actions the addition of a pre-play phase can increase the set of equilibrium payoffs, Calcagno *et al.* (2014) show that it can also narrow it down in finite games.

In this paper we introduce one-sided incomplete information in two coordination games, one of common interest and one of opposing interest, and study how coordination is affected. In particular we study how information is or is not transmitted through the prepared action of the informed player.

In the common interest game, players wish to coordinate on a risky action which depends on the state of the world, only known by one of the players. We show that there is a unique equilibrium with the following features: first, the informed player will not signal her private information when close to the deadline; second, the uninformed player always coordinates with the informed player.

In the opposing interest game, players wish to coordinate, though not on the same action. Which action is preferred by each player depends on the state of the world, only know to one of the players. There is a tension between cooperation and competition, and revealing too much information might be detrimental to the informed player. We show that when players are initially coordinated, it is possible that no information is transmitted, while when players are initially miscoordinated, some information transmission will occur.

II. Setting

In this section we describe the characteristics of a revision game with one-sided incomplete information in which players seek to coordinate on an action which depends on the state of the world. We first describe the Bayesian game and the preparation stage. We then illustrate how the uninformed player revises his beliefs. Finally we describe histories, strategies, and define the equilibrium concept used in the Bayesian revision game.

$\omega = a$

	A	B
A	d,d	0,1
B	1,0	1,1

$\omega = b$

	A	B
A	1,1	1,0
B	0,1	d,d

FIGURE 1
A BAYESIAN COMMON INTEREST COORDINATION GAME ($d > 1$)

$\omega = a$

	A	B
A	d,1	0,0
B	0,0	1,d

$\omega = b$

	A	B
A	1,d	0,0
B	0,0	d,1

FIGURE 2
A BAYESIAN OPPOSING INTEREST COORDINATION GAME ($d > 1$)

A. The Bayesian Game

We consider in turns two coordination games: a common interest coordination game, presented in Figure 1, and an opposing interest coordination game, presented in Figure 2.

In both games, there are two players, $N = \{1, 2\}$, two states of the world $\Omega = \{a, b\}$ and two actions for each player, $S_1 = S_2 = \{A, B\}$. In both cases, players weakly prefer to be coordinated than not coordinated. In the common-interest game, both players would like to coordinate on the action that corresponds to the state of the world. In the opposing-interest game, Player 1 prefers if both players coordinate on the action that matches the state of the world, while Player 2 prefers coordination on the other action.

We assume that Player 1 (she) knows the state of the world while Player 2 (he) is uninformed and has a uniform prior belief: $p(a) = p(b) = 1/2$.

B. Timing and Revision Opportunities

Time goes from $-T < 0$ to 0. A positive time $t > 0$ denotes the time remaining until the deadline ($t = 0$). At $-T$, an action profile $(s_1, s_2) \in S_1 \times S_2$ is exogenously given. From $-T$ to 0, players then receive stochastic opportunities to revises their actions according to two independent Poisson processes with arrival rates λ_1 and λ_2. In particular, the probability that revision opportunities are simultaneous is zero. At $t = 0$, the last prepared action profile is implemented and payoffs are realized.

It is important to note that a player is not aware of the other player's revision opportunities unless they revised their prepared action.

C. Histories

Let $-t_i^k$ be the time at which Player i receives her k^{th} revision opportunity, and $-t_i^o := -T$ and let $k_i(t)$ be the number of revision opportunities received by Player i in $[-T, -t)$. Let $X_i^k \in \{A, B\}$ be the action prepared by Player i at time $-t_i^k$ and $X_i(t)$ be the action prepared by Player i at time $-t$. Finally let $r_i(t) \in \{0, 1\}$ indicate whether Player i received a revision opportunity at time $-t$.

A history of the revision game at time $-t$ takes the following form:

$$h(t) = \{\omega, (t_1^k, X_1^k)_{k=0}^{k_1(t)}, (t_2^k, X_2^k)_{k=0}^{k_2(t)}, r_1(t), r_2(t)\}.$$

That is, a history indicates the state of the world, all the revision opportunities received up to and including time $-t$ and specifies the prepared actions up to, but not including, time $-t$.

A player only knows about revision opportunities of the other player if she observes a change in the prepared action. Therefore a private history for player i takes the form of:

$$h_i(t) = s_i \cup \{(t_i^k, X_i^k)_{k=0}^{k_i(t)}, (t_j^{f(k)}, X_j^{f(k)})_{k=0}^{k_j(t)}, r_i(t)\}.$$

where, s_i is the signal of player i, $f(0) = 0$ and $t_j^{f(k)}$ is the first time after $t_j^{f(k-1)}$ such that $X_j^{f(k)} \neq X_j^{f(k-1)}$. Since player 1 is the only player informed, we have $s_1(\omega) = \omega$, while $s_2(\omega) = \phi$. The set of all private histories for Player i is denoted by H_i.

D. Strategies

A strategy for Player i is a mapping $\sigma_i : H_i \rightarrow \{\phi\} \times \Delta(S_i)$ such that $\sigma_i(h_i(t)) = \phi$ if $r_i(t) = 0$. (That is, a player can choose an action only when having a revision opportunity.) A pair of strategies (σ_1, σ_2), along with the Poisson processes, generate a measure $\mathbb{P}_{\sigma_1,\sigma_2}$ on the set of prepared actions at the deadline, $(X_1(0), X_2(0))$.

E. Perfect Bayesian Equilibrium

A strategy profile (σ_1^*, σ_2^*) is a perfect Bayesian equilibrium of the revision game if for any history $h_i(t)$ such that $r_i(t) = 1$ and any strategy σ_i, we have that

$$\mathbb{E}_{\sigma_i^*,\sigma_j^*}[u_i(X_i(0), X_j(0)) \mid h_i(t)] \geq \mathbb{E}_{\sigma_i,\sigma_j^*}[u_i(X_i(0), X_j(0)) \mid h_i(t)],$$

$i = 1, 2$, and Player 2's beliefs are derived from Bayes' rule whenever possible.

F. Belief Updating of Player 2

To illustrate how the uninformed player may revise his beliefs, let us assume that Player 1 revises her action only to signal the correct state of the world. Let $X \in \{A, B\}$ denote the initial action of Player 1 and let $p^x(t)$ denote Player 2's belief that the state is $x \in \{a, b\}$ when $t > 0$ is remaining until the deadline. If Player 1 has revised her action, given that this is interpreted as a signal, Player 2's belief falls to $p^x(t) = 0$.

Consider now the case in which Player 1 has not revised her action since the beginning of the game (that is, for a time interval of length $T - t$). If Player 1 has not yet revised her prepared action, it could be because (i) the state is x, which occurs with probability 1/2, or (ii) the state is y and Player 1 did not get a revision opportunity, which occurs with probability $1/2\{e^{-\lambda_1(T-t)}\}$.[1, 2] Therefore from Bayes rule we have that:

$$p^x(t) = \frac{1}{1 + e^{-\lambda_1(T-t)}}. \tag{1}$$

The beliefs of Player 2 when Player 1 has not revised her prepared action are illustrated in Figure F for a preparation phase of length 1 and when Player 1 has on average either 0.5, 1 or 2 revision opportunities per unit of time. Note that when Player 1 has not revised her prepared action we have:

[1] If $x \in \{a, b\}$ we use y to denote $\{a, b\} \setminus \{x\}$, and similarly for X and Y in $\{A, B\}$.

[2] Note that in Calcagno *et al.* (2014) revision opportunities are observed by both players, although it does not matter as there are no information asymmetries.

$$\frac{1-p^x(t)}{p^x(t)} = e^{-\lambda_1(T-t)}, \tag{2}$$

and

$$\frac{\partial p^x(t)}{\partial t} = -\lambda_1 p^x(t)(1-p^x(t)). \tag{3}$$

III. Common Interest Game

In this section we characterize the unique equilibrium of the Bayesian common interest revision game, for which payoffs have been reproduced in Figure 4 below.

Theorem 1. *There exists $\tau_1 > 0$ such that, in the unique equilibrium of the Bayesian common interest revision game:*

- *When $t \geq \tau_1$ (when enough time remains to the deadline), Player 1 signals the state of the world; when $t \leq \tau_1$ (the deadline is close), Player 1 coordinates with Player 2.*
- *On the equilibrium path, Player 2 always coordinates with Player 1. Moreover τ_1 is given by:*

$$\tau_1 = \frac{1}{\lambda_1 + \lambda_2} \ln\left[\frac{\lambda_1 + \lambda_2 d}{(d-1)\lambda_2}\right]. \tag{4}$$

The proof of Theorem 1 consists in three steps. First, we show that in any equilibrium, close to the deadline, Player 1 does not wish to signal the state of the world (Proposition 1). Therefore there is always a positive probability that player 1 chooses to disregard her private information. In particular, if the preparation stage is too short, no information is transmitted.

We then characterize Player 2's behavior when Player 1 chooses to signal the state of the world through her action, and show that Player 2 always wish to be coordinated with Player 1 (Proposition 2).

Finally we show that, when the preparation stage is long enough, Player 1 will always want to signal the state of the world, proving the uniqueness of equilibrium (Proposition 3).

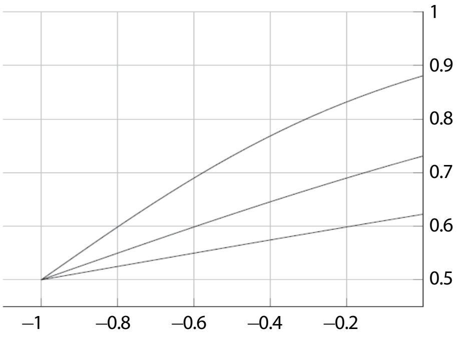

FIGURE 3
PLAYER 2'S BELIEFS WHEN PLAYER 1 HAS NOT REVISED HER PREPARED ACTION
($T = 1$, $\lambda_1 = 0.5, 1, 2$)

	A	B
A	d,d	0,1
B	1,0	1,1

$\omega = a$

	A	B
A	1,1	1,0
B	0,1	d,d

$\omega = b$

FIGURE 4
A BAYESIAN COMMON INTEREST COORDINATION GAME ($d > 1$)

A. No Signalling Close to the Deadline

We first show that close to the deadline, Player 1 will prefer to be coordinated on the wrong action rather signal the correct state of the world. This is because close to the deadline the probability that a future revision opportunity arises is too small relative to the benefit of being coordinated on the correct action.

Proposition 1. *In any equilibrium, there is a length of time $\tau > 0$ such that when $t \leq \tau$ remains until the deadline, Player 1 prefers to be coordinated with Player 2 on the wrong action rather than being miscoordinated. Moreover $\tau \geq \tau_1$, where τ_1 is given by* (4).

Proof. Let us assume that at any revision opportunity Player 2 seeks to coordinate with Player 1. This is the most favourable case for Player 1 and will therefore gives us the lower bound on τ, $\tau \geq \tau_1$. Given Player

2's behavior, we are interested in two different values for Player 1: (i) the value of being coordinated with Player 2 on the wrong action and (ii) the value of being miscoordinated with Player 2 while preparing the correct action.

Let $V(X_1, X_2, x, t)$ denote the value to Player 1 when she is preparing action X_1, Player 2 is preparing action X_2, the state of the world is x, and there is t that remains until the deadline. In particular, we are interested in the values $V(Y, Y, x, t)$ and $V(X, Y, x, t)$, in which either players are coordinated on the wrong state of the world or players are miscoordinated by Player 1's action matches with the state of the world. Consider now a small interval of time dt:

- Player 1 receives a revision opportunity with probability $1 - e^{-\lambda_1 dt} \sim \lambda_1 dt$. She can then choose between being coordinated on the wrong action or signal the correct state of the world.
- Player 2 receives a revision opportunity with probability $1 - e^{-\lambda_2 dt} \sim \lambda_2 dt$ and will coordinate with Player 1 if players are miscoordinated.

Therefore the value of being coordinated on the wrong action for Player 1 satisfies the following equation:

$$V(Y, Y, x, t) \sim \lambda_1 dt \max\{V(Y, Y, x, t-dt), V(X, Y, x, t-dt)\} + (1-\lambda_1 dt)V(Y, Y, x, t-dt).$$

By subtracting $V(Y, Y, x, t-dt)$. from both sides, dividing by dt, and letting dt go to zero, we obtain the following Bellman equation:

$$V_t(Y, Y, x, t) = \lambda_1 \max\{V(X, Y, x, t) - V(Y, Y, x, t), 0\}, \tag{5}$$

where $V_t(Y, Y, x, t)$ is the total derivative of $V(Y, Y, x, t)$ with respect to t, the time left until the deadline. Note that $V_t(Y, Y, x, t) \geq 0$, so that the value weakly decreases as the deadline approaches.

Similarly, we have the following Bellman equation for $V(X, Y, x, t)$:

$$V_t(X, Y, x, t) = \lambda_1 \max\{V(Y, Y, x, t) - V(X, Y, x, t), 0\} + \lambda_2[d - V(X, Y, x, t)]. \tag{6}$$

The second term corresponds to Player 2 having a revision opportunity, in which case he will coordinate with Player 1 on the correct action, yielding a payoff of d for both players. Note that we also have $V_t(X, Y, x, t) \geq 0$ as d is the highest payoff in the game.

We first note that when it is optimal for Player 1 to be coordinated on the wrong action, it remains so until the deadline. When $V(Y, Y, x, t) - V(X, Y, x, t) \geq 0$, it is optimal for Player 1 to be coordinated with Player 2 on the wrong action. In that case, (5) and (6) become

$$V_t(Y, Y, x, t) = 0, \tag{7}$$

and

$$V_t(X, Y, x, t) = \lambda_1[V(Y, Y, x, t) - V(X, Y, x, t)] + \lambda_2[d - V(X, Y, x, t)]. \tag{8}$$

Because $V(Y, Y, x, t)$ is constant and $V(X, Y, x, t)$ is decreasing as the deadline approaches, if $V(Y, Y, x, t) - V(X, Y, x, t) \geq 0$ then $V(Y, Y, x, t') - V(X, Y, x, t') \geq 0$ for $t' \leq t$: when it is optimal for Player 1 to remain coordinated on the wrong action, it continues to be so until the deadline. This implies that

$$V(Y, Y, x, t) = V(Y, Y, x, 0) = 1, \tag{9}$$

and we can therefore rewrite (8) as

$$V_t(X, Y, x, t) + (\lambda_1 + \lambda_2)V(X, Y, x, t) = \lambda_1 + \lambda_2 d. \tag{10}$$

Along with the terminal condition $V(X, Y, x, 0) = 0$,[3] this gives us (see Appendix A.a) for a calculation)

$$V(X, Y, x, t) = \frac{\lambda_1}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)t}) + \frac{\lambda_2}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)t})d. \tag{11}$$

With probability

$$\frac{\lambda_1}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)t}),$$

[3] See either the top-right entry of the left matrix or the bottom-left entry of the right matrix in Figure 1.

Player 1 gets the first revision opportunity before the deadline and chooses to coordinate with Player 2 on the wrong action, yielding a payoff of 1. With probability

$$\frac{\lambda_2}{\lambda_1+\lambda_2}(1-e^{-(\lambda_1+\lambda_2)t})$$

Player 2 gets the first revision opportunity before the deadline and coordinates with Player 1 on the correct action, yielding a payoff of d. Finally with the complementary probability no player gets a revision opportunity before the deadline and Player 1 gets a payoff of 0.

The time τ_1 which remains until the deadline and for which Player 1 is indifferent between being coordinated on the wrong action or miscoordi-nated while choosing the correct action is then defined by $V(X, Y, x, \tau_1) = V(Y, Y, x, \tau_1) = 1$, that is:

$$\frac{\lambda_1}{\lambda_1+\lambda_2}(1-e^{-(\lambda_1+\lambda_2)\tau_1})+\frac{\lambda_2}{\lambda_1+\lambda_2}(1-e^{-(\lambda_1+\lambda_2)\tau_1})d=1, \tag{12}$$

which gives us

$$\tau_1=\frac{1}{\lambda_1+\lambda_2}\ln\left[\frac{\lambda_1+\lambda_2 d}{(d-1)\lambda_2}\right],$$

as in (4).

Note that Player 1 is willing to remain miscoordinated longer as revision opportunities become more frequent, that is

$$\frac{\partial\tau_1}{\partial\lambda_1}<0$$

and

$$\frac{\partial\tau_1}{\partial\lambda_2}<0.$$[4]

[4] We have

$$\frac{\partial\tau_1}{\partial\lambda_2}=-\frac{1}{(\lambda_1+\lambda_2)^2}\ln\left(\frac{\lambda_2 d+\lambda_1}{\lambda_2 d-\lambda_2}\right)-\frac{\lambda_1}{\lambda_2(\lambda_1+\lambda_2)(\lambda_2 d+\lambda_1)}<0.$$

If λ_1 increases then Player 1 will have more opportunities to coordinate with Player 2 in the future and is therefore willing to remain uncoordinated longer. Similarly, if λ_2 increases then there are more chances that Player 2 will be able to coordinate on the correct action with Player 1 in the future and therefore Player 1 is willing to signal the correct action longer.

Proposition 1 tells us that close to the deadline Player 1 will prefer to be coordinated with Player 2 on the wrong action rather than signal the correct action through miscoordination. This is because close to the deadline the risk of miscoordination becomes too important, as it is unlikely that Player 2 will have a revision opportunity and be able to coordinate with Player 1 on the correct action. Therefore if given a revision opportunity Player 1 will prefer to coordinate with Player 2 on the wrong action.

Proposition 1 also characterizes Player 2's best reply off the path of an informative equilibrium, that is when Player 1 first signals the state of the world by changing her prepared action and then changes her prepared action a second time before τ_1 is left until the deadline:

Corollary 1. *In an informative equilibrium, when Player 1 has signalled the state of the world, Player 2 will choose to prepare the corresponding action until τ_2 is left to the deadline, irrespective of Player 1's prepared action, where τ_2 is given by*

Furthermore

$$\frac{\partial \tau_1}{\partial \lambda_1} = \frac{1}{(\lambda_1 + \lambda_2)^2}\left[\frac{\lambda_1 + \lambda_2}{\lambda_1 + \lambda_2 d} + \ln\frac{(d-1)\lambda_2}{\lambda_1 + \lambda_2 d}\right].$$

The term in brackets is a strictly increasing function of d for any $\lambda_1 > 0$, $\lambda_2 > 0$ and

$$\lim_{d\to\infty} \frac{\lambda_1 + \lambda_2}{\lambda_1 + \lambda_2 d} + \ln\frac{(d-1)\lambda_2}{\lambda_1 + \lambda_2 d} = 0.$$

Therefore

$$\frac{\partial \tau_1}{\partial \lambda_1} < 0$$

for any $d > 1$, $\lambda_1 > 0$, and $\lambda_2 > 0$.

$$\tau_2 = \frac{1}{\lambda_1 + \lambda_2} \ln\left[\frac{\lambda_2 + \lambda_1 d}{(d-1)\lambda_1}\right]. \tag{13}$$

B. The Uninformed Player Always Prefers To Be Coordinated

We now characterize Player 2's behaviour in an equilibrium where Player 1 signals the state of the world until τ_1 and show that Player 2 always prefers to be coordinated with Player 1, whether or not the state has been revealed.

Note that if players are miscoordinated, when Player 2 does not know the state, only one revision opportunity is required for players to coordinate on the correct action. On the other hand, if players are coordinated, it might require either zero or two revision opportunities for players to coordinate on the correct action. What Proposition 2 tells us is that there is no option value in being miscoordinated, when the prior is uniform. If the prior was not uniform then the uninformed player might have an incentive to choose miscoordination while waiting for new information to unfold.

Proposition 2. *In an equilibrium where Player 1 signals the state of the world with her action then, on the equilibrium path, Player 2 always wishes to be coordinated with Player 1.*

Proof. Consider the following strategy for Player 1: prepare the correct action until τ_1 remains to the deadline, after which try to coordinate with Player 2. That is,

$$\sigma_1(h^t) = \begin{cases} \text{match the state of the world} & \text{if } t \geq \tau_1 \text{ and } r_1(t) = 1, \\ X_2(t) & \text{if } t < \tau_1 \text{ and } r_1(t) = 1. \end{cases}$$

Note that this strategy is optimal for Player 1 as long as Player 2 is willing to coordinate with Player 1 for $t \in [\tau_1, \tau_1 + \varepsilon)$, $\varepsilon > 0$.[5]

Let

$$U(X_1(t), X_2(t), p(t), t)$$

denote Player 2's value function when

[5] This is because from Proposition 1 τ_1 is the last time Player 1 is willing to prepare the correct action given that Player 2 will coordinate with Player 1.

- t is left until the deadline,
- the prepared action profile is $(X_1(t), X_2(t))$,
- Player 2's belief about $X_1(t)$, the action prepared by Player 1, is $p(t)$.

Let X denote Player 1's initial action. We first find Player 2's best response when the deadline is close, and then when it is far.

Case 1: *the deadline is close* $(t \leq \tau_1)$

When $t \leq \tau_1$ we know that Player 1 will try to coordinate with Player 2 irrespective of the state of the world. When Player 1 has revised her prepared action before τ_1 and therefore signalled the state of the world to Player 2, we have

$$U(Y, Y, 1, t) = d, \tag{14}$$

and

$$U(Y, X, 1, t) = \frac{\lambda_2}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)t})d + 1 - \frac{\lambda_2}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)t}). \tag{15}$$

With probability

$$\frac{\lambda_2}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)t})$$

Player 2 is the first to obtain a revision opportunity and will coordinate to get a payoff of d. With the complementary probability either Player 1 revises her action first or no player receives a revision opportunity, in which case Player 2's payoff at the deadline is 1.

If Player 1 has not revised her action before τ_1 then learning stops at τ_1 and Player 2's belief is given by

$$p^x(t) = p^x(\tau_1) = \frac{1}{1 + e^{-\lambda_1(T-\tau_1)}} \in (1/2, 1)$$

and Player 2's value is

$$U(X, X, p^x(\tau_1), t) = p^x(\tau_1)d + p^y(\tau_1). \tag{16}$$

We now check that when Player 1 has not revised her action prior to τ_1 then Player 2 prefers to be coordinated with Player 1 for $t \le \tau_1$. Assume that this is the case. We then have

$$\begin{aligned} U(X, Y, p^x(\tau_1), t) = & \frac{\lambda_2}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)t})\left[p^x(\tau_1)d + p^y(\tau_1)\right] \\ & + \frac{\lambda_1}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)t})\left[p^x(\tau_1) + p^y(\tau_1)d\right] \\ & + e^{-(\lambda_1+\lambda_2)t} p^x(\tau_1). \end{aligned} \tag{17}$$

If Player 2 is the first to have a revision opportunity then he will coordinate with Player 1 on X and have an expected payoff of $p^x(\tau_1)d + p^y(\tau_1)$. If Player 1 is the first to have a revision opportunity then she will coordinate with Player 2 and the expected payoff will be $p^x(\tau_1) + p^y(\tau_1)d < p^x(\tau_1)d + p^y(\tau_1)$ since $p^x(\tau_1) > 1/2$. Finally if no player receives a revision opportunity then Player 2 gets a payoff of 1 only if the state is x. Hence $U(X, Y, p^x(\tau_1), t) < U(X, X, p^x(\tau_1), t)$ for any $t \le \tau_1$.

Note that from the continuity of the value functions, it will also be strictly better for Player 2 to coordinate with Player 1 if Player 1 has not yet revised her action and if $t \in [\tau_1, \tau_1 + \varepsilon)$, for some $\varepsilon > 0$ sufficiently small. This will turn out to be of importance when showing uniqueness of equilibrium.

Case 2: *the deadline is far* ($t \ge \tau_1$)

As above, we still have $U(Y, Y, 1, t) = d$. When Player 1 has signalled the state by changing her prepared action, we know that she will wait until τ_1 is left before trying to coordinate with Player 2 on the wrong action again. Therefore if Player 2 obtains a revision opportunity before $-\tau_1$ his payoff will be d. If not his payoff will be $U(Y, X, 1, \tau_1)$, where $U(Y, X, 1, \tau_1)$ is given by (15), evaluated at $t = \tau_1$, and we have

$$U(Y, X, 1, t) = (1 - e^{-\lambda_2(t-\tau_1)})d + e^{-\lambda_2(t-\tau_1)}U(Y, X, 1, \tau_1). \tag{18}$$

To find Player 2's value functions when Player 1 has not yet revised her prepared action, $U(X, X, p^x(t), t)$ and $U(X, Y, p^x(t), t)$, we first assume that if Player 2 has a revision opportunity he will choose to be coordinated with Player 1: $U(X, X, p^x(t), t) \ge U(X, Y, p^x(t), t)$. We know that this is true for $t = \tau_1$ and by continuity it will also be true in a neighborhood of τ_1.

We then get the following first-order ordinary differential equation for $U(X, X, p^x(t), t)$:

$$U_t(X, X, p^x(t), t) + \lambda_1 p^y(t) U(X, X, p^x(t), t) = \lambda_1 p^y(t) U(Y, X, 1, t),$$

where $U_t(X, X, p^x(t), t)$ is the total derivative of $U(X, X, p^x(t), t)$ with respect to t.[6] This is because in a small time interval dt, Player 2 expects Player 1 to revise her prepared action if the state is y and if she receives a revision opportunity, which occurs with probability $\sim p^y(t)\lambda_1 dt$. If that is the case then Player 2 gets the value $U(Y, X, 1, t)$, which is given by (18). Given the the terminal condition (16) we obtain (see Appendix A.b)):[7]

$$\begin{aligned} U(X, X, p^x(t), t) = {} & p^x(t)d \\ & + p^y(t)\left\{\left[1 - \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2(t-\tau_1)} + \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1(t-\tau_1)}\right] d \right. \\ & \left. + \frac{\lambda_1}{\lambda_1 - \lambda_2}(e^{-\lambda_2(t-\tau_1)} - e^{-\lambda_1(t-\tau_1)}) U(Y, X, 1, \tau_1) + e^{-\lambda_1(t-\tau_1)} \right\}. \end{aligned} \tag{19}$$

When the state is x, which occurs with probability $p^x(t)$, Player 2 will receive a payoff of d, as none of the players will revise their actions. The term within braces corresponds to the payoff of Player 2 when the state is y, which occurs with probability $p^y(t)$. It it composed of three terms:

- The first term in brackets is the probability that during a time interval of length $t - \tau_1$ Player 1 obtains a revision opportunity and Player 2 obtains a subsequent revision opportunity, in which case both players receive a payoff of d.
- With probability

[6] That is, let $h(t) = U(X, X, p^x(t), t)$. Then $U_t(X, X, p^x(t), t)$ denotes $h'(t)$.

[7] Note that this expression is well defined when $\lambda_1 = \lambda_2$ as

$$\lim_{\lambda_1 \to \lambda} \frac{1}{\lambda_1 - \lambda}(\lambda_1 e^{-\lambda z} - \lambda e^{-\lambda_1 z}) = e^{-\lambda z}(1 + \lambda_z)$$

and

$$\lim_{\lambda_1 \to \lambda} \frac{\lambda_1}{\lambda_1 - \lambda}(e^{-\lambda z} - e^{-\lambda_1 z}) = \lambda_{ze}^{-\lambda z}.$$

$$\frac{\lambda_1}{\lambda_1-\lambda_2}(e^{-\lambda_2(t-\tau_1)}-e^{-\lambda_1(t-\tau_1)}),$$

Player 1 will have a revision opportunity while Player 2 will not get a subsequent revision opportunity until τ_1, in which case Player 2's value at τ_1 is $U(Y, X, 1, \tau_1)$.

- Finally, with probability $e^{-\lambda_1(t-\tau_1)}$ Player 1 does not get a revision opportunity until τ_1 and players remain coordinated on the wrong action, which yields a payoff of 1.

We now find $U(X, Y, p^x(t), t)$, while still assuming that $U(X, X, p^x(t), t) \geq U(X, Y, p^x(t), t)$, so that if Player 2 has a revision opportunity he will change his prepared action and coordinate with Player 1. Again with probability ~ $p^y(t)\lambda_1 dt$ Player 1 will change his prepared action to y, this time giving a payoff of d. We therefore obtain the following first-order ordinary differential equation for $U(X, Y, p^x(t), t)$:

$$U_t(X, Y, p^x(t), t) + (\lambda_1 p^y(t) + \lambda_2)U(X, Y, p^x(t), t) \\ = \lambda_1 p^y(t)d + \lambda_2 U(X, X, p^x(t), t),$$

where $U_t(X, Y, p^x(t), t)$ is the total derivative of $U(X, Y, p^x(t), t)$ with respect to t and $U(X, X, p^x(t), t)$ is given by (19). Along with the terminal condition for $U(X, Y, p^x(\tau_1), \tau_1)$ given by (17) and evaluated at $t = \tau_1$, we find (see Appendix A.c))

$$\begin{aligned} U(X,Y,p^x(t),t) &= U(X,X,p^x(t),t) \\ &+ e^{-\lambda_2(t-\tau_1)}\Big\{p^x(t)\Big\{\frac{1}{p^x(\tau_1)}\big[U(Y,X,1,\tau_1)-U(X,X,p^x(\tau_1),\tau_1)\big] \\ &+ \frac{p^y(\tau_1)}{p^x(\tau_1)}\big[U(X,Y,0,\tau_1)-d\big]\Big\} + p^y(t)\big[d-U(Y,X,1,\tau_1)\big]\Big\}. \end{aligned} \quad (20)$$

We now focus on the sign of the difference $U(X, Y, p^x(t), t) - U(X, X, p^x(t), t)$, which is of the same sign as

$$\begin{aligned} & p^x(t)\Big\{\frac{1}{p^x(\tau_1)}\big[U(Y,X,1,\tau_1)-U(X,X,p^x(\tau_1),\tau_1)\big] \\ &+ \frac{p^y(\tau_1)}{p^x(\tau_1)}\big[U(X,Y,0,\tau_1)-d\big]\Big\} + p^y(t)\{d-U(Y,X,1,\tau_1)\}. \end{aligned} \quad (21)$$

Equation (21) is the average of two terms (independent of t) weighted by $p^x(t)$ and $p^y(t)$. Since $U(X, Y, p^x(\tau_1), \tau_1) - U(X, X, p^x(\tau_1), \tau_1) < 0$ and $d - U(Y, X, 1, \tau_1) > 0$, the term weighted by $p^x(t)$ must be negative.

Therefore if $U(X, Y, p^x(t), t) - U(X, X, p^x(t), t) < 0$ then $U(X, Y, p^x(t'), t') - U(X, X, p^x(t'), t') < 0$ for any $t' < t$. That is, if the uninformed player prefers to be coordinated with the informed player, he will continue to do so in the future. This is because as t decreases, the weight on the negative term in (21), $p^x(t)$, increases.

We now look for the sign of $U(X, Y, p^x(t), t) - U(X, Y, p^x(t), t)$ at the start of the game, when $t = T$. Note that at $t = T$, we have $p^x(t) = p^y(t) = 1/2$. Therefore we need to determine the sign of

$$\begin{aligned}&\frac{1}{p^x(\tau_1)}\left[U(Y, X, 1, \tau_1) - U(X, X, p^x(\tau_1), \tau_1)\right]\\&+\frac{p^y(\tau_1)}{p^x(\tau_1)}\left[U(X, Y, 0, \tau_1) - d\right] + \left[d - U(Y, X, 1, \tau_1)\right],\end{aligned} \tag{22}$$

which can be simplified into[8]

$$-\frac{p^y(\tau_1)}{p^x(\tau_1)} e^{-(\lambda_1+\lambda_2)\tau_1} d < 0. \tag{23}$$

This shows that $U(X, Y, p^x(t), t) - U(X, X, p^x(t), t) < 0$ for any $t \in [0, T]$, so that Player 2 will always prefer to remain coordinated with Player 1.

Propositions 1 and 2 therefore prove the existence of an informative equilibrium when $T \geq \tau_1$, with the following strategies:

- Player 1:
  - At any revision opportunity such that $t \geq \tau_1$, prepare the action

[8] Recall that $U(X, X, p^x(\tau_1), \tau_1) = p^x(\tau_1)d + p^y(\tau_1)$,

$$U(Y, X, 1, \tau_1) = \frac{\lambda_2}{\lambda_1+\lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1})d + 1 - \frac{\lambda_2}{\lambda_1+\lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1})$$

and

$$U(X, Y, 0, \tau_1) = \frac{\lambda_2}{\lambda_1+\lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1}) + \frac{\lambda_1}{\lambda_1+\lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1})d.$$

that corresponds to the correct state of the world.
- At any revision opportunity such that $t \leq \tau_1$, coordinate with Player 2.

- Player 2
  - If his belief about a state of the world is 1, play the corresponding action until τ_2 remains to the deadline. After that coordinate with Player 1.
  - If Player 2's beliefs are interior, always coordinate with Player 1.
- Beliefs of Player 2: Player 2's beliefs jump to zero or one when Player 1 changes her prepared action before τ_1 is left until the deadline. When Player 1 does not revise her prepared action then Player 2's belief is given by (1) until τ_1 and then remains constant.

Moreover the payoffs from such an informative equilibrium converge to the efficient payoff, d, as T becomes arbitrarily large. This is because as T becomes arbitrarily large then so does $T - \tau_1$.

C. Uniqueness of Equilibrium

In this section we point out why the equilibrium characterized in the previous sections is unique:

Proposition 3. *The informative equilibrium described in the previous sub-section is the unique equilibrium of the Bayesian revision game with one-sided incomplete information.*[9]

Proof. We argue now that there cannot be another equilibrium than the informative equilibrium described previously. From equations (16) and (17), we saw that $U(X, Y, p^x(\tau_1), \tau_1) < U(X, X, p^x(\tau_1), \tau_1)$. That is, Player 2 will coordinate with Player 1 at $t = \tau_1$, and by continuity of the value functions, for $t \in [\tau_1, \tau_1 + \varepsilon)$, for $\varepsilon > 0$ small enough. Since Player 1 is willing to stay miscoordinated with Player 1 until $t = \tau_1$ in order to signal the correct state of the world, she will find it optimal to do so as this gives her the highest expected payoff. Therefore, there cannot be another equilibrium.

[9] Note that even though we call it an informative equilibrium, there is always a phase, close to the deadline, where no information is transmitted. If the preparation phase is too short there is therefore no information transmission at all.

D. Discussion

In a common interest coordination game, one-sided incomplete information does not affect the informed player's behavior. She will be willing to stay miscoordinated until τ_1 remains until the deadline, in order to signal the correct state of the world. The uninformed player will, when given the opportunity, always coordinate with the informed player in particular, there is no option value from being miscoordinated, even when the beliefs of the uninformed player have not been updated.

However, this will not be the case if we depart from a uniform prior belief; for example, if player 2 puts a very strong prior probability on x being the state of the world, he will prepare action X even though this might induce miscoordination.

IV. Opposing Interest Games

We now study a coordination game with opposing interests, reproduced below in Figure 5. In this game both players would like to coordinate and receive a payoff of either $d > 1$ or 1, as being miscoordinated yields a payoff of zero. When the state of the world is a, Player 1 prefers to be coordinated on action A while Player 2 prefers to be coordinated on action B. When the state of the world is b, it is the reverse, where Player 1 prefers to be coordinated on action B, while Player 2 prefers to be coordinated on action A. The state of the world therefore represents the preference of Player 1.

When there is no incomplete information, then Calcagno *et al.* (2014, Theorem 3) show that which action profile players coordinate on depends on the relative arrival rates. They determine that the "stronger" player is the one with the lowest arrival rate.[10] For example, when $\lambda_1 < \lambda_2$, Player 1 is the strong player and if the preparation phase is sufficiently long, the prepared action profile at the deadline will be the one preferred by Player 1. The reason is that close to the deadline, both players will try to coordinate with each other, irrespective of the action profile currently prepared. If Player 2 has on average more revision opportunities than Player 1, then Player 1 is willing to remain miscoordinated longer, which forces Player 2 to coordinate on her own preferred action.

[10] The condition also depends on payoffs, which do not enter into consideration here because of symmetry.

	A	B
A	d,1	0,0
B	0,0	1,d

$\omega = a$

	A	B
A	1,d	0,0
B	0,0	d,1

$\omega = b$

Figure 5
A Bayesian Opposing Interest Coordination Game ($d > 1$)

With one-sided incomplete information, things remain the same if the informed player is the stronger player. That is, when $\lambda_1 < \lambda_2$, Player 1 will always signal the state of the world by preparing her preferred action, and Player 2 will always coordinate with Player 1. (Note that in the case of perfect information, the weak player has an incentive to miscoordinate with the strong player in anticipation of a future change of the strong player's prepared action. This is no longer the case here.)

However when $\lambda_1 > \lambda_2$ this is no longer an equilibrium. Indeed Player 2 is now the strong player, and if he is informed of the state by Player 1's action, he will prepare his preferred action and force Player 1 to coordinate with him. Therefore when $\lambda_1 > \lambda_2$ information is no longer fully transmitted. We show that there is an equilibrium in which, when players start the game coordinated, they remain so until the end, without revising their prepared actions. We then show that in any equilibrium, if players start the game miscoordinated, there must be some degree of information transmission.

A. Player 1 is the "Strong" Player: $\lambda_1 < \lambda_2$

When Player 1 is the strong player, we argue that equilibrium is similar to the perfect information case (see Calcagno *et al.* 2014, Theorem 3), so that player will coordinate on the preferred action of Player 1. However, unlike in the perfect information case, the weak player (in this case player 2) has no incentives to miscoordinate with the strong player in anticipation of a future revision of the strong player's prepared action. This would not be the case if the prior beliefs were initially skewed rather than uniform. Namely, equilibrium takes the following form:

Proposition 4. *In an opposing interest coordination game, when $\lambda_1 < \lambda_2$, the unique perfect Bayesian Nash equilibrium of the revision game is as* *follows:*

- *Player 1 always prepares her preferred action until τ_1, after which she coordinates with Player 2 if players are miscoordinated;*
- Player 2 always coordinates with Player 1.

Proof. Let us first assume that both players know the state of the world (for example if it has been revealed by Player 1's action). At the deadline, both players strictly prefer to be coordinated. By continuity of the value functions, this is again true close to the deadline. Moreover, assuming Player 2 always coordinates with Player 1, we know that Player 1 is willing to stick to her preferred action until

$$\tau_1 = \frac{1}{\lambda_1 + \lambda_2} \ln \frac{\lambda_1 + \lambda_2 d}{\lambda_2 (d-1)}$$

remains until the deadline before coordinating with Player 2. Similarly, if Player 1 always coordinates with Player 2, then Player 2 is willing to stick to his preferred action until

$$\tau_2 = \frac{1}{\lambda_1 + \lambda_2} \ln \frac{\lambda_2 + \lambda_1 d}{\lambda_1 (d-1)}$$

remains until the deadline. As $\lambda_1 < \lambda_2$, we have that $\tau_1 < \tau_2$. That is, Player 1 is willing to wait closer to the deadline than Player 2 before coordinating on Player 2's preferred action. Therefore Player 1 will always signal the state of the world by preparing her preferred action, and Player 2 will coordinate with Player 1.

Let us now determine the behavior of Player 2 when Player 1 has not yet revised her action, so that the state of the world is unknown to him. Given Player 1's strategy, the beliefs of Player 2 are given by (1):

$$p^x(t) = \frac{1}{1 + e^{-\lambda_1 (T-t)}}$$

until τ_1 remains until the deadline, after which the belief no longer changes.

Case 1*: the deadline is close* ($t \le \tau_1$)

Consider first the case $t \le \tau_1$. Because $\tau_1 \le \tau_2$, we also have $t \le \tau_2$. By definition of τ_2 we therefore have

$$\frac{\lambda_1}{\lambda_1+\lambda_2}(1-e^{-(\lambda_1+\lambda_2)t})d+\frac{\lambda_2}{\lambda_1+\lambda_2}(1-e^{-(\lambda_1+\lambda_2)t})\le 1,$$

which can be rewritten as

$$\frac{\lambda_1}{\lambda_1+\lambda_2}(1-e^{-(\lambda_1+\lambda_2)t})d\le 1-\frac{\lambda_2}{\lambda_1+\lambda_2}(1-e^{-(\lambda_1+\lambda_2)t}). \tag{24}$$

Let $U(X, X, p^x(t), t)$ and $U(X, Y, p^x(t), t)$ denote the value to Player 2 of being coordinated and miscoordinated with Player 1 respectively, when Player 1 has not yet revised her action. Assuming $U(X, X, p^x(t), t) \ge U(X, Y, p^x(t), t)$, and given Player 1's strategy, we have:

$$U(X, X, p^x(\tau_1), t) = p^x(\tau_1) + p^y(\tau_1)d$$

and

$$\begin{aligned}U(X,Y,p^x(\tau_1),t) &= \frac{\lambda_2}{\lambda_1+\lambda_2}(1-e^{-(\lambda_1+\lambda_2)t})\left[p^x(\tau_1)+p^y(\tau_1)d\right]\\ &+\frac{\lambda_1}{\lambda_1+\lambda_2}(1-e^{-(\lambda_1+\lambda_2)t})\left[p^x(\tau_1)d+p^y(\tau_1)\right].\end{aligned}$$

Therefore $U(X, X, p^x(t), t) \ge U(X, Y, p^x(t), t)$ is equivalent to

$$\begin{aligned}\frac{\lambda_1}{\lambda_1+\lambda_2}(1-e^{-(\lambda_1+\lambda_2)t})\left[p^x(\tau_1)d+p^y(\tau_1)\right] &\le \left[1-\frac{\lambda_2}{\lambda_1+\lambda_2}(1-e^{-(\lambda_1+\lambda_2)t})\right]\\ &\quad \left[p^x(\tau_1)+p^y(\tau_1)d\right],\end{aligned}$$

which is implied by (24). This is because the left-hand side of (24) has decreased ($p^x(\tau_1)d + p^y(\tau_1) \le d$) while the right-hand side has increased ($p^x(\tau_1) + p^y(\tau_1)d \ge 1$).

***Case 2**: the deadline is far* ($t \ge \tau_1$)

Assuming that $U(X, X, p^x(t), t) \ge U(X, Y, p^x(t), t)$, and following calculations similar to the ones in Appendices A.b) and A.c), we find:

$$U(X, X, p^x(t), t) = p^x(t) + p^y(t)\left\{\left[1 - \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2(t-\tau_1)} + \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1(t-\tau_1)}\right]\right.$$

$$\left. + \frac{\lambda_1}{\lambda_1 - \lambda_2}(e^{-\lambda_2(t-\tau_1)} - e^{-\lambda_1(t-\tau_1)})U(Y, X, 1, \tau_1) + e^{-\lambda_1(t-\tau_1)}d\right\},$$

and

$$\begin{aligned} U(X, Y, p^x(t), t) &= U(X, X, p^x(t), t) \\ &+ e^{-\lambda_2(t-\tau_1)}\left\{p^x(t)\left\{\frac{1}{p^x(\tau_1)}\left[U(Y, X, 1, \tau_1) - U(X, X, p^x(\tau_1), \tau_1)\right]\right\}\right. \\ &\left. + p^y(t)\left[1 - U(Y, X, 1, \tau_1)\right]\right\}, \end{aligned}$$

where

$$U(Y, X, 1, \tau_1) = \frac{\lambda_1}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1})d + \frac{\lambda_2}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1}).$$

We now argue that the difference $U(X, Y, p^x(t), t) - U(X, X, p^x(t), t)$ is non-positive. For $t = T$, this difference is proportional to

$$\begin{aligned} &\frac{1}{p^x(\tau_1)}\left[U(Y, X, 1, \tau_1) - U(X, X, p^x(\tau_1), \tau_1)\right] + 1 - U(Y, X, 1, \tau_1) \\ &= e^{-(T-\tau_1)}\left[U(Y, X, 1, \tau_1) - d\right] < 0, \end{aligned}$$

where we used the fact that $U(X, X, p^x(\tau_1), \tau_1) = p^x(\tau_1) + p^y(\tau_1)d$. It then suffice to notice that the difference is decreasing as t decreases, as the term in front of $p^x(t)$ is negative and $p^x(t)$ increases as t decreases.

B. Player 2 is the "Strong" Player: $\lambda_1 > \lambda_2$

When Player 1 is the weak player, she no longer has incentives to fully reveal the state of the world. This is because Player 2, being the strong player and knowing the state of the world, will prepare his preferred action until τ_2 remains to the deadline, and Player 1 will yield as $\tau_1 > \tau_2$.

For example, if the initial prepared action profile is (X, Y) and the state is y, if Player 1 revises her action to Y, then Player 2 will, if given the opportunity before τ_2, prepare the action X, giving rise to the prepared action profile (Y, X). However, if Player 2 does not get a revision opportunity in time, the prepared action profile at the deadline will be (Y, Y), which is preferred by Player 1. Therefore, though Player 1 might have incentives to conceal the true state of the world, she will do so if she gets a revision opportunity close to τ_2, which will leave little opportunity for Player 2 to revise his action and try to impose his preferred action profile.

We show two results. First, that there is an equilibrium in which, when players are initially coordinated, no player ever revises their action. That is, there is no information transmission. Next, we show that in all equilibria, when players are initially miscoordinated, there must be some information transmission occurring.

Proposition 5. *There is an equilibrium in which, when both players are initially coordinated, no player ever revises their action, irrespective of the state of the world.*

Proof. Consider the initial action profile (X, x) and the strategy in which Player 1 does not revise her action when both players are initially coordinate. If Player 1 does not revise her action the Player 2's belief remain constant at 1/2. Given that, it is optimal for him to remain coordinated with P1 on (X, X).

If Player 1 deviates, we are free to assign any beliefs to Player 2. If we assume that Player 2 believes Player 1 deviates to (Y, X) only when the state is y, then as Player 2 is the strong player he will not revise his action unless less than τ_2 remains until the deadline, making such a deviation for Player 1 not profitable.

Proposition 6. *When both players are initially miscoordinated, there will be some – but not full – information transmission.*

Proof. Assume that the initial prepared profile is (X, Y) and there is no information transmission in equilibrium - that is, Player 1 chooses to switch to Y if given a revision opportunity with probability $\alpha(t) > 0$ that is independent on whether the state of the world is x or y. In that case, the belief of Player 2 remains constant at 1/2, and it is optimal for Player 2 to coordinate with Player 1 at the first revision opportunity. This, in turns, gives incentives to Player 1 not to switch to Y when the state of the world is y.

Note that there cannot be full information transmission: if the deadline is very far, it is not beneficial for P1 to reveal the state of the world, as Player 2 will have enough time to prepared his prepared action.

V. Conclusion

In this paper we introduce one-sided incomplete information in the study of coordination games with a preparation phase. In a common interest game, the informed player always signals her information through her prepared action, unless the deadline is too close, in which case she simply best respond to the other player's prepared action. The uninformed player always prefers to be coordinated with the informed player.

In a coordination game with opposing interest, each player has a different preferred Nash equilibrium which depends on the state of the world. In particular, when the informed player receives less revision opportunities on average, she will be willing to signal the state of the world through her prepared action. When the informed player receives more revision opportunities on average, some information will still be transmitted, although not fully.

There are several ways in which information asymmetries could be extended in this model:

- Increasing asymmetric information: Initially, both players could be uninformed about the state of the world. Player 1 would then receive some binary information according to an independent Poisson process for news arrival. Hwang (2018) studies a dynamic trading game with increasing asymmetric information.
- Uncertainty about the type of game: One or both players could be unsure as to whether they are playing an opposing or common interest game.
- Uncertainty about the arrival rates: The arrival rates play an important role in determining the outcome of the revision game, since the strong player has some commitment power he can use to push forward his preferred action. If one player is uncertain about the arrival rate of the other player, can the other player then pass as a strong player?

Appendix

A. Common Interest Game

a) $V(Y, X, x, t)$ for t Close to 0

For ease of notations, let us use $f(t)$ instead of $V(Y, X, x, t)$. We solve the following first-order differential equation:

$$f'(t) + (\lambda_1 + \lambda_2)f(t) = \lambda_1 + \lambda_2 d$$

subject to the terminal condition $f(0) = 0$.

We first consider the homogeneous equation $f'(t) + (\lambda_1 + \lambda_2)f(t) = 0$, which can be rewritten as $f'(t) / f(t) = -(\lambda_1 + \lambda_2)$, $f(t) \neq 0$. Integrating gives us $\int(f'(t) / f(t))dt = -(\lambda_1 + \lambda_2)t + \beta$, that is $f(t) = \alpha e^{-(\lambda_1 + \lambda_2)t}$, where α is a positive constant.

We now look for a solution to the inhomogeneous equation of the form $f(t) = \alpha(t)e^{-(\lambda_1 + \lambda_2)t}$. Taking the derivative, we get $f'(t) = \alpha'(t)e^{-(\lambda_1 + \lambda_2)t} - (\lambda_1 + \lambda_2)\alpha(t)e^{-(\lambda_1 + \lambda_2)t} = \alpha'(t)e^{-(\lambda_1 + \lambda_2)t} - (\lambda_1 + \lambda_2)f(t)$. Substituting in our original equation, we get $\alpha'(t)e^{-(\lambda_1 + \lambda_2)t} = \lambda_1 + \lambda_2 d$, which gives us $\alpha(t) = \lambda_1 + \lambda_2 de^{(\lambda_1 + \lambda_2)t} / (\lambda_1 + \lambda_2) + K$. Substituting, into f, we get $f(t) = (\lambda_1 + \lambda_2 d) / (\lambda_1 + \lambda_2) + Ke^{-(\lambda_1 + \lambda_2)t}$, and using the terminal condition $f(0) = 0$, we find $K = -(\lambda_1 + \lambda_2 d) / (\lambda_1 + \lambda_2)$, so that

$$f(t) = \frac{\lambda_1}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1 + \lambda_2)t}) + \frac{\lambda_2}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1 + \lambda_2)t})d.$$

b) $U(X, X, p^x(t), t)$ for $t \geq \tau_1$

Again, for ease of notation, let use $h(t)$ and $g(t)$ such that $h(t) = U(X, X, p^x(t), t)$ and $g(t) = U(Y, X, 1, t)$.

We solve the following first-order differential equation:

$$h'(t) + \lambda_1 p^y(t)h(t) = \lambda_1 p^y(t)g(t),$$

where (see Equation 18)

$$g(t) = (1 - e^{-\lambda_2(t-\tau_1)})d + e^{-\lambda_2(t-\tau_1)}g(\tau_1),$$

and (see Equation 15)

$$g(\tau_1) = \frac{\lambda_2}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1})d + 1 - \frac{\lambda_2}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1}),$$

along with the terminal condition (see Equation 16)

$$h(\tau_1) = p^x(\tau_1)d + p^y(\tau_1),$$

where

$$p^x(t) = \frac{1}{1 + e^{-\lambda_1(T-t)}}, \qquad p^y(t) = 1 - p^x(t).$$

We first consider the homogeneous equation

$$h'(t) + \lambda_1 p^y(t)h(t) = 0,$$

which has a solution of the type

$$h(t) = \alpha e^{-\int \lambda_1 p^y(t)dt}.$$

To find the indefinite integral $\int \lambda_1 p^y(t)dt$ recall that

$$\frac{\partial p^y(t)}{\partial t} = -\frac{\partial p^x(t)}{\partial t} = \lambda_1 p^y(t)(1 - p^y(t)).$$

Therefore

$$-\lambda_1 p^y(t) = -\frac{1}{1 - p^y(t)}\frac{\partial p^y(t)}{\partial t} = \frac{\partial}{\partial t}[\ln(1 - p^y(t))].$$

so that

$$\int -\lambda_1 p^y(t)dt = \ln(1 - p^y(t)) = \ln p^x(t).$$

We therefore look for a solution of the form

$$h(t) = \alpha(t)p^x(t) = \alpha(t)\frac{1}{1 + e^{-\lambda_1(T-t)}},$$

which has a derivative

$$
\begin{aligned}
h'(t) &= \alpha'(t)p^x(t) + \alpha(t)p'^x(t) \\
&= \alpha'(t)p^x(t) - \alpha(t)\lambda_1 p^x(t)p^y(t) \\
&= \alpha'(t)p^x(t) - \lambda_1 p^y(t)h(t).
\end{aligned}
$$

Plugging into the original differential equation, we get

$$\alpha'(t)p^x(t) = \lambda_1 p^y(t)g(t),$$

so that

$$
\begin{aligned}
\alpha'(t) &= \lambda_1 \frac{p^y(t)}{p^x(t)} g(t) \\
&= \lambda_1 e^{-\lambda_1(T-t)} g(t) \\
&= \lambda_1 e^{-\lambda_1(T-t)}[(1 - e^{-\lambda_2(t-\tau_1)})d + e^{-\lambda_2(t-\tau_1)} g(\tau_1)] \\
&= \lambda_1 e^{-\lambda_1(T-t)} d - \lambda_1 e^{-\lambda_1 T} e^{\lambda_2 \tau_1} e^{(\lambda_1-\lambda_2)t} d + \lambda_1 e^{-\lambda_1 T + \lambda_2 \tau_1} e^{(\lambda_1-\lambda_2)t} g(\tau_1).
\end{aligned}
$$

Integrating gives us

$$
\begin{aligned}
\alpha(t) &= e^{-\lambda_1(T-t)} d - \frac{\lambda_1}{\lambda_1-\lambda_2} e^{-\lambda_1(T-t)} e^{-\lambda_2(t-\tau_1)} d + \frac{\lambda_1}{\lambda_1-\lambda_2} e^{-\lambda_1(T-t)} e^{-\lambda_2(t-\tau_1)} g(\tau_1) + K \\
&= e^{-\lambda_1(T-t)} \left\{ \left[1 - \frac{\lambda_1}{\lambda_1-\lambda_2} e^{-\lambda_2(t-\tau_1)} \right] d + \frac{\lambda_1}{\lambda_1-\lambda_2} e^{-\lambda_2(t-\tau_1)} g(\tau_1) \right\} + K \\
&= \frac{p^y(t)}{p^x(t)} \left\{ \left[1 - \frac{\lambda_1}{\lambda_1-\lambda_2} e^{-\lambda_2(t-\tau_1)} \right] d + \frac{\lambda_1}{\lambda_1-\lambda_2} e^{-\lambda_2(t-\tau_1)} g(\tau_1) \right\} + K.
\end{aligned}
$$

Hence

$$
\begin{aligned}
h(t) &= \alpha(t)p^x(t) \\
&= p^y(t) \left\{ \left[1 - \frac{\lambda_1}{\lambda_1-\lambda_2} e^{-\lambda_2(t-\tau_1)} \right] d + \frac{\lambda_1}{\lambda_1-\lambda_2} e^{-\lambda_2(t-\tau_1)} g(\tau_1) \right\} + p^x(t)K,
\end{aligned}
$$

which gives us, for $t = \tau_1$,

$$h(\tau_1) = p^y(\tau_1) \left\{ \left[1 - \frac{\lambda_1}{\lambda_1-\lambda_2} \right] d + \frac{\lambda_1}{\lambda_1-\lambda_2} g(\tau_1) \right\} + p^x(\tau_1)K.$$

Given the terminal condition $h(\tau_1) = p^x(\tau_1)d + p^y(\tau_1)$, we find that

$$K = d + \frac{p^y(\tau_1)}{p^x(\tau_1)}\left\{1 - \left[1 - \frac{\lambda_1}{\lambda_1 - \lambda_2}\right]d - \frac{\lambda_1}{\lambda_1 - \lambda_2}g(\tau_1)\right\},$$

so that

$$\begin{aligned} h(t) &= p^x(t)\alpha(t) \\ &= p^y(t)\left\{\left[1 - \frac{\lambda_1}{\lambda_1 - \lambda_2}e^{-\lambda_2(t-\tau_1)}\right]d + \frac{\lambda_1}{\lambda_1 - \lambda_2}e^{-\lambda_2(t-\tau_1)}g(\tau_1)\right\} + Kp^x(t) \\ &= p^y(t)\left\{\left[1 - \frac{\lambda_1}{\lambda_1 - \lambda_2}e^{-\lambda_2(t-\tau_1)}\right]d + \frac{\lambda_1}{\lambda_1 - \lambda_2}e^{-\lambda_2(t-\tau_1)}g(\tau_1)\right\} + p^x(t)d \\ &\quad + p^x(t)\frac{p^y(\tau_1)}{p^x(\tau_1)}\left\{1 - \left[1 - \frac{\lambda_1}{\lambda_1 - \lambda_2}\right]d - \frac{\lambda_1}{\lambda_1 - \lambda_2}g(\tau_1)\right\} \\ &= p^x(t)d + p^y(t)\left\{\left[1 - \frac{\lambda_1}{\lambda_1 - \lambda_2}e^{-\lambda_2(t-\tau_1)} + \frac{\lambda_2}{\lambda_1 - \lambda_2}e^{-\lambda_1(t-\tau_1)}\right]d\right. \\ &\quad \left. + \frac{\lambda_1}{\lambda_1 - \lambda_2}(e^{-\lambda_2(t-\tau_1)} - e^{-\lambda_1(t-\tau_1)})g(\tau_1) + e^{-\lambda_1(t-\tau_1)}\right\}, \end{aligned}$$

where we used the fact that

$$p^x(t)\frac{p^y(\tau_1)}{p^x(\tau_1)} = p^y(t)e^{-\lambda_1(t-\tau_1)}.$$

c) $U(X, Y, p^x(t), t)$ for $t \geq \tau_1$

As previously, for ease of notation, let use $h(t)$ and $g(t)$ such that $h(t) = U(X, X, p^x(t), t)$ and $g(t) = U(Y, X, 1, t)$. Let us also use z for $z(t) = U(X, Y, p^x(t), t)$.

We solve the following first-order differential equation

$$z'(t) + (\lambda_1 p^y(t) + \lambda_2)z(t) = \lambda_1 p^y(t)d + \lambda_2 h(t),$$

subject to the terminal condition

$$\begin{aligned} z(\tau_1) &= \frac{\lambda_2}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1})\left[p^x(\tau_1)d + p^y(\tau_1)\right] \\ &\quad + \frac{\lambda_1}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1})\left[p^x(\tau_1) + p^y(\tau_1)d\right] + e^{-(\lambda_1+\lambda_2)\tau_1}p^x(\tau_1), \end{aligned}$$

where

$$h(t) = p^x(t)d + p^y(t)\left\{\left[1 - \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2(t-\tau_1)} + \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1(t-\tau_1)}\right] d + \frac{\lambda_1}{\lambda_1 - \lambda_2}(e^{-\lambda_2(t-\tau_1)} - e^{-\lambda_1(t-\tau_1)})g(\tau_1) + e^{-\lambda_1(t-\tau_1)}\right\}$$

and

$$g(\tau_1) = \frac{\lambda_2}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1})d + 1 - \frac{\lambda_2}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1}).$$

Note that $z(\tau_1)$ can be rewritten as

$$\begin{aligned}
z(\tau_1) &= p^x(\tau_1)\left[\frac{\lambda_2}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1})d + \frac{\lambda_1}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1}) + e^{-(\lambda_1+\lambda_2)\tau_1}\right] \\
&\quad + p^y(\tau_1)\left[\frac{\lambda_2}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1}) + \frac{\lambda_1}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1})d\right] \\
&= p^x(\tau_1)\left[\frac{\lambda_2}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1})d + 1 - \frac{\lambda_2}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1})\right] \\
&\quad + p^y(\tau_1)\left[\frac{\lambda_2}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1}) + \frac{\lambda_1}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1})d\right] \\
&= p^x(\tau_1)g(\tau_1) + p^y(\tau_1)U(X, Y, 0, \tau_1).
\end{aligned}$$

First we solve the homogeneous equation

$$z'(t) + (\lambda_1 p^y(t) + \lambda_2)z(t) = 0,$$

which can be rewritten as

$$\frac{z'(t)}{z(t)} = -(\lambda_1 p^y(t) + \lambda_2).$$

Given that $\partial \ln p^x(t) / \partial t = -\lambda_1 p^y(t)$, we have that

$$\begin{aligned}
\ln z(t) &= -\int (\lambda_1 p^y(t) + \lambda_2)dt + K \\
&= \ln p^x(t) - \lambda_2 t + K,
\end{aligned}$$

so that $z(t) = \alpha p^x(t)e^{-\lambda_2 t}$, with $\alpha > 0$.

We now look for a solution of the type $z(t) = \alpha(t)p^x(t)e^{-\lambda_2 t}$, such that $z'(t) = \alpha'(t)p^x(t)e^{-\lambda_2 t} - (\lambda_1 p^y(t) + \lambda_2)z(t)$. Our differential equation then becomes

$$\alpha'(t)p^x(t)e^{-\lambda_2 t} = \lambda_1 p^y(t)d + \lambda_2 h(t),$$

or

$$\alpha'(t) = \lambda_1 \frac{p^y(t)}{p^x(t)} e^{\lambda_2 t} d + \lambda_2 \frac{1}{p^x(t)} e^{\lambda_2 t} h(t).$$

For the first term of this differential equation, we have

$$\begin{aligned}\int \lambda_1 \frac{p^y(t)}{p^x(t)} e^{\lambda_2 t} d\,dt &= \int \lambda_1 e^{-\lambda_1 (T-t)} e^{\lambda_2 t} d\,dt \\ &= \frac{\lambda_1}{\lambda_1 + \lambda_2} e^{-\lambda_1 (T-t)} e^{\lambda_2 t} d + K_1 \\ &= \frac{\lambda_1}{\lambda_1 + \lambda_2} \frac{p^y(t)}{p^x(t)} e^{\lambda_2 t} d + K_1.\end{aligned}$$

Let us now focus on the second term of the differential equation:

$$\begin{aligned}\lambda_2 \frac{1}{p^x(t)} e^{\lambda_2 t} h(t) &= \lambda_2 e^{\lambda_2 t} d \\ &+ \lambda_2 \frac{p^y(t)}{p^x(t)} e^{\lambda_2 t} \left\{ \left[1 - \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2 (t-\tau_1)} + \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1 (t-\tau_1)} \right] d \right. \\ &\left. + \frac{\lambda_1}{\lambda_1 - \lambda_2} (e^{-\lambda_2 (t-\tau_1)} - e^{-\lambda_1 (t-\tau_1)} g(\tau_1) + e^{-\lambda_1 (t-\tau_1)} \right\} \\ &= \lambda_2 e^{\lambda_2 t} d \\ &+ \lambda_2 e^{-\lambda_1 (T-t)} e^{\lambda_2 t} \left\{ \left[1 - \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2 (t-\tau_1)} + \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1 (t-\tau_1)} \right] d \right. \\ &\left. + \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2 (t-\tau_1)} - e^{-\lambda_1 (t-\tau_1)} g(\tau_1) + e^{-\lambda_1 (t-\tau_1)} \right\} \\ &= \lambda_2 e^{\lambda_2 t} d \\ &+ e^{-\lambda_1 T} \left\{ \left[\lambda_2 e^{(\lambda_1 + \lambda_2)t} - \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} e^{\lambda_1 t} e^{\lambda_2 \tau_1} + \frac{\lambda_2^2}{\lambda_1 - \lambda_2} e^{\lambda_2 t} e^{\lambda_1 \tau_1} \right] d \right. \\ &\left. + \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} (e^{\lambda_1 t} e^{\lambda_2 \tau_1} - e^{\lambda_2 t} e^{\lambda_1 \tau_1}) g(\tau_1) + \lambda_2 e^{\lambda_2 t} e^{\lambda_1 \tau_1} \right\},\end{aligned}$$

so that

$$\int \lambda_2 \frac{1}{p^x(t)} e^{\lambda_2 t} h(t)dt = e^{\lambda_2 t} d$$

$$+ e^{-\lambda_1 T} \left\{ \left[\frac{\lambda_2}{\lambda_1 + \lambda_2} e^{(\lambda_1+\lambda_2)t} - \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{\lambda_1 t} e^{\lambda_2 \tau_1} + \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{\lambda_2 t} e^{\lambda_1 \tau_1} \right] d \right.$$

$$\left. + \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} (\frac{1}{\lambda_1} e^{\lambda_1 t} e^{\lambda_2 \tau_1} - \frac{1}{\lambda_2} e^{\lambda_2 t} e^{\lambda_1 \tau_1}) g(\tau_1) + e^{\lambda_2 t} e^{\lambda_1 \tau_1} \right\} + K_2$$

$$= e^{\lambda_2 t} d$$

$$+ \frac{p^y(t)}{p^x(t)} e^{\lambda_2 t} \left\{ \left[\frac{\lambda_2}{\lambda_1 + \lambda_2} - \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_2 (t-\tau_1)} + \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1 (t-\tau_1)} \right] d \right.$$

$$\left. + \left[\frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_2 (t-\tau_1)} - \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_1 (t-\tau_1)} \right] g(\tau_1) + e^{-\lambda_1 (t-\tau_1)} \right\} + K_2.$$

Hence

$$\alpha(t) = \frac{\lambda_1}{\lambda_1 + \lambda_2} \frac{p^y(t)}{p^x(t)} e^{\lambda_2 t} d + e^{\lambda_2 t} d$$

$$+ \frac{p^y(t)}{p^x(t)} e^{\lambda_2 t} \left\{ \left[\frac{\lambda_2}{\lambda_1 + \lambda_2} - \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_2 (t-\tau_1)} + \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1 (t-\tau_1)} \right] d \right.$$

$$\left. + \left[\frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_2 (t-\tau_1)} - \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_1 (t-\tau_1)} \right] g(\tau_1) + e^{-\lambda_1 (t-\tau_1)} \right\} + K,$$

and, since $z(t) = \alpha(t) p^x(t) e^{-\lambda_2 t}$,

$$z(t) = \frac{\lambda_1}{\lambda_1 + \lambda_2} p^y(t) d + p^x(t) d$$

$$+ p^y(t) \left\{ \left[\frac{\lambda_2}{\lambda_1 + \lambda_2} - \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_2 (t-\tau_1)} + \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1 (t-\tau_1)} \right] d \right.$$

$$\left. + \left[\frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_2 (t-\tau_1)} - \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_1 (t-\tau_1)} \right] g(\tau_1) + e^{-\lambda_1 (t-\tau_1)} \right\} + K p^x(t) e^{-\lambda_2 t}$$

$$= p^x(t) \left[d + K e^{-\lambda_2 t} \right] + p^y(t) \left\{ \left[1 - \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_2 (t-\tau_1)} + \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1 (t-\tau_1)} \right] d \right.$$

$$\left. + \left[\frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_2 (t-\tau_1)} - \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_1 (t-\tau_1)} \right] g(\tau_1) + e^{-\lambda_1 (t-\tau_1)} \right\}$$

$$= p^x(t)Ke^{-\lambda_2 t} + p^x(t)d + p^y(t)\left\{\left[1 - \frac{\lambda_1}{\lambda_1-\lambda_2}e^{-\lambda_2(t-\tau_1)} + \frac{\lambda_2}{\lambda_1-\lambda_2}e^{-\lambda_1(t-\tau_1)}\right]d\right.$$
$$\left. + \frac{\lambda_1}{\lambda_1-\lambda_2}\left[e^{-\lambda_2(t-\tau_1)} - e^{-\lambda_1(t-\tau_1)}\right]g(\tau_1) + e^{-\lambda_1(t-\tau_1)}\right\}$$
$$+ p^y(t)\left\{\left[\frac{\lambda_1}{\lambda_1-\lambda_2}e^{-\lambda_2(t-\tau_1)} - \frac{\lambda_2}{\lambda_1-\lambda_2}e^{-\lambda_2(t-\tau_1)}\right]d\right.$$
$$\left. + \left[\frac{\lambda_2}{\lambda_1-\lambda_2}e^{-\lambda_2(t-\tau_1)} - \frac{\lambda_1}{\lambda_1-\lambda_2}e^{-\lambda_2(t-\tau_1)}\right]g(\tau_1)\right\}$$
$$= p^x(t)Ke^{-\lambda_2 t} + h(t) + p^y(t)e^{-\lambda_2(t-\tau_1)}\left[d - g(\tau_1)\right].$$

Evaluating at $t = \tau_1$, we get $z(\tau_1) = p^x(\tau_1)Ke^{-\lambda_2\tau_1} + h(\tau_1) + p^y(\tau_1)[d - g(\tau_1)]$. We now find the constant term by using the terminal condition, which we rewrote as $z(\tau_1) = p^x(\tau_1)g(\tau_1) + p^y(\tau_1)U(X, Y, 0, \tau_1)$. Combining the two equalities gives us

$$K = \frac{1}{p^x(\tau_1)}e^{\lambda_2\tau_1}\left[g(\tau_1) - h(\tau_1)\right] + \frac{p^y(\tau_1)}{p^x(\tau_1)}e^{\lambda_2\tau_1}\left[U(X,Y,0,\tau_1) - d\right]$$
$$= \frac{1}{p^x(\tau_1)}e^{\lambda_2\tau_1}\left\{g(\tau_1) - h(\tau_1) + p^y(\tau_1)\left[U(X,Y,0,\tau_1) - d\right]\right\}$$

Hence we have

$$z(t) = p^x(t)Ke^{-\lambda_2 t} + h(t) + p^y(t)e^{-\lambda_2(t-\tau_1)}\left[d - g(\tau_1)\right]$$
$$= h(t) + p^x(t)e^{-\lambda_2 t}\frac{1}{p^x(\tau_1)}e^{\lambda_2\tau_1}\left\{g(\tau_1) - h(\tau_1) + p^y(\tau_1)\left[U(X,Y,0,\tau_1) - d\right]\right\}$$
$$+ p^y(t)e^{-\lambda_2(t-\tau_1)}\left[d - g(\tau_1)\right]$$
$$= h(t) + e^{-\lambda_2(t-\tau_1)}\left\{p^x(t)\left\{\frac{1}{p^x(\tau_1)}\left[g(\tau_1) - h(\tau_1)\right] + \frac{p^y(\tau_1)}{p^x(\tau_1)}\left[U(X,Y,0,\tau_1) - d\right]\right\}\right.$$
$$\left. + p^y(t)\left[d - g(\tau_1)\right]\right\}.$$

B. Opposing Interest Game, Strong Player 1

a) $U(X, X, p^x(t), t)$ for $t \geq \tau_1$

This calculation is almost identical as the calculation in Apprendix A.b) and will therefore be kept to a minimum. Again, for ease of notation, let use $h(t)$ and $g(t)$ such that $h(t) = U(X, X, p^x(t), t)$ and $g(t) = U(Y, X, 1, t)$.

We solve the following first-order differential equation:

$$h'(t) + \lambda_1 p^y(t)h(t) = \lambda_1 p^y(t)g(t),$$

where

$$g(t) = (1 - e^{-\lambda_2(t-\tau_1)}) + e^{-\lambda_2(t-\tau_1)}g(\tau_1),$$

and

$$g(\tau_1) = \frac{\lambda_2}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1}) + \frac{\lambda_1}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1})d,$$

along with the terminal condition

$$h(\tau_1) = p^x(\tau_1) + p^y(\tau_1)d,$$

where

$$p^x(t) = \frac{1}{1 + e^{-\lambda_1(T-t)}}, \quad p^y(t) = 1 - p^x(t).$$

We first consider the homogeneous equation

$$h'(t) + \lambda_1 p^y(t)h(t) = 0,$$

which has a solution of the form

$$h(t) = \alpha(t)p^x(t) = \alpha(t)\frac{1}{1 + e^{-\lambda_1(T-t)}}.$$

Taking the derivative and plugging back in the original differential equation, we get

$$\alpha'(t)p^x(t) = \lambda_1 p^y(t)g(t),$$

so that

$$\alpha'(t) = \lambda_1 \frac{p^y(t)}{p^x(t)} g(t)$$

$$
\begin{aligned}
&= \lambda_1 e^{-\lambda_1(T-t)} g(t) \\
&= \lambda_1 e^{-\lambda_1(T-t)} [1 - e^{-\lambda_2(t-\tau_1)} + e^{-\lambda_2(t-\tau_1)} g(\tau_1)] \\
&= \lambda_1 e^{-\lambda_1(T-t)} - \lambda_1 e^{-\lambda_1 T} e^{\lambda_2 \tau_1} e^{(\lambda_1-\lambda_2)t} + \lambda_1 e^{-\lambda_1 T + \lambda_2 \tau_1} e^{(\lambda_1-\lambda_2)t} g(\tau_1).
\end{aligned}
$$

Integrating gives us

$$
\alpha(t) = \frac{p^y(t)}{p^x(t)} \left\{ 1 - \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2(t-\tau_1)} + \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2(t-\tau_1)} g(\tau_1) \right\} + K.
$$

Hence

$$
\begin{aligned}
h(t) &= \alpha(t) p^x(t) \\
&= p^y(t) \left\{ 1 - \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2(t-\tau_1)} + \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2(t-\tau_1)} g(\tau_1) \right\} + p^x(t) K,
\end{aligned}
$$

which gives us, for $t = \tau_1$,

$$
h(\tau_1) = p^y(\tau_1) \left\{ 1 - \frac{\lambda_1}{\lambda_1 - \lambda_2} + \frac{\lambda_1}{\lambda_1 - \lambda_2} g(\tau_1) \right\} + p^x(\tau_1) K.
$$

Given the terminal condition $h(\tau_1) = p^x(\tau_1) + p^y(\tau_1)d$, we find that

$$
K = 1 + \frac{p^y(\tau_1)}{p^x(\tau_1)} \left\{ d - 1 + \frac{\lambda_1}{\lambda_1 - \lambda_2} - \frac{\lambda_1}{\lambda_1 - \lambda_2} g(\tau_1) \right\},
$$

so that

$$
\begin{aligned}
h(t) &= p^x(t)\alpha(t) \\
&= p^x(t) + p^y(t) \left\{ 1 - \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2(t-\tau_1)} + \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1(t-\tau_1)} \right. \\
&\quad \left. + \frac{\lambda_1}{\lambda_1 - \lambda_2} \left[e^{-\lambda_2(t-\tau_1)} - e^{-\lambda_1(t-\tau_1)} \right] g(\tau_1) + e^{-\lambda_1(t-\tau_1)} d \right\}.
\end{aligned}
$$

b) $U(X, Y, p^x(t), t)$ for $t \geq \tau_1$

This section is similar to Appendix A.c) and is therefore kept to a minimum. As previously, for ease of notation, let use $h(t)$ and $g(t)$ such that $h(t) = U(X, X, p^x(t), t)$ and $g(t) = U(Y, X, 1, t)$. Let us also use z for $z(t) = U(X, Y, p^x(t), t)$.

We solve the following first-order differential equation

$$z'(t) + (\lambda_1 p^y(t) + \lambda_2)z(t) = \lambda_1 p^y(t)d + \lambda_2 h(t),$$

subject to the terminal condition

$$z(\tau_1) = \frac{\lambda_2}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1})[p^x(\tau_1) + p^y(\tau_1)d] + \frac{\lambda_1}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1})[p^x(\tau_1)d + p^y(\tau_1)d],$$

where

$$h(t) = p^x(t) + p^y(t)\left\{1 - \frac{\lambda_1}{\lambda_1 - \lambda_2}e^{-\lambda_2(t-\tau_1)} + \frac{\lambda_2}{\lambda_1 - \lambda_2}e^{-\lambda_1(t-\tau_1)} + \frac{\lambda_1}{\lambda_1 - \lambda_2}[e^{-\lambda_2(t-\tau_1)} - e^{-\lambda_1(t-\tau_1)}]g(\tau_1) + e^{-\lambda_1(t-\tau_1)}d\right\}$$

and

$$g(\tau_1) = \frac{\lambda_1}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1})d + \frac{\lambda_2}{\lambda_1 + \lambda_2}(1 - e^{-(\lambda_1+\lambda_2)\tau_1}).$$

Note that $z(\tau_1)$ can be rewritten as (see (12))

$$z(\tau_1) = p^x(\tau_1)g(\tau_1) + p^y(\tau_1).$$

First we solve the homogeneous equation $z'(t) + (\lambda_1 p^y(t) + \lambda_2)z(t) = 0$ and get a solution of the type $z(t) = \alpha(t)p^x(t)e^{-\lambda_2 t}$. Our original differential equation then becomes

$$\alpha'(t)p^x(t)e^{-\lambda_2 t} = \lambda_1 p^y(t) + \lambda_2 h(t),$$

or

$$\alpha'(t) = \lambda_1 \frac{p^y(t)}{p^x(t)}e^{\lambda_2 t} + \lambda_2 \frac{1}{p^x(t)}e^{\lambda_2 t}h(t).$$

For the first term of this differential equation, we have

$$\int \lambda_1 \frac{p^y(t)}{p^x(t)} e^{\lambda_2 t} dt = \int \lambda_1 e^{-\lambda_1 (T-t)} e^{\lambda_2 t} dt$$
$$= \frac{\lambda_1}{\lambda_1 + \lambda_2} e^{-\lambda_1 (T-t)} e^{\lambda_2 t} + K_1$$
$$= \frac{\lambda_1}{\lambda_1 + \lambda_2} \frac{p^y(t)}{p^x(t)} e^{\lambda_2 t} + K_1$$

Let us now focus on the second term of the differential equation:

$$\begin{aligned}\lambda_2 \frac{1}{p^x(t)} e^{\lambda_2 t} h(t) &= \lambda_2 e^{\lambda_2 t} \\ &+\lambda_2 \frac{p^y(t)}{p^x(t)} e^{\lambda_2 t} \left\{ 1 - \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2 (t-\tau_1)} + \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1 (t-\tau_1)} \right. \\ &\left. + \frac{\lambda_1}{\lambda_1 - \lambda_2} [e^{-\lambda_2 (t-\tau_1)} - e^{-\lambda_1 (t-\tau_1)}] g(\tau_1) + e^{-\lambda_1 (t-\tau_1)} d \right\} \\ &= \lambda_2 e^{\lambda_2 t} \\ &+ e^{-\lambda_1 T} \left\{ \lambda_2 e^{(\lambda_1 + \lambda_2) t} - \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} e^{\lambda_1 t} e^{\lambda_2 \tau_1} + \frac{\lambda_2^2}{\lambda_1 - \lambda_2} e^{\lambda_2 t} e^{\lambda_1 \tau_1} \right. \\ &\left. + \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} [e^{\lambda_1 t} e^{\lambda_2 \tau_1} - e^{\lambda_2 t} e^{\lambda_1 \tau_1}] g(\tau_1) + \lambda_2 e^{\lambda_2 t} e^{\lambda_1 \tau_1} d \right\},\end{aligned}$$

so that

$$\begin{aligned}\int \lambda_2 \frac{1}{p^x(t)} e^{\lambda_2 t} h(t) dt &= e^{\lambda_2 t} \\ &+ \frac{p^y(t)}{p^x(t)} e^{\lambda_2 t} \left\{ \frac{\lambda_2}{\lambda_1 + \lambda_2} - \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_2 (t-\tau_1)} + \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1 (t-\tau_1)} \right. \\ &\left. + \left[\frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_2 (t-\tau_1)} - \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_1 (t-\tau_1)} \right] g(\tau_1) + e^{-\lambda_1 (t-\tau_1)} d \right\} \\ &+ K_2.\end{aligned}$$

Hence

$$\begin{aligned}\alpha(t) = &\frac{\lambda_1}{\lambda_1 + \lambda_2} \frac{p^y(t)}{p^x(t)} e^{\lambda_2 t} + e^{\lambda_2 t} \\ &+ \frac{p^y(t)}{p^x(t)} e^{\lambda_2 t} \left\{ \frac{\lambda_2}{\lambda_1 + \lambda_2} - \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_2 (t-\tau_1)} + \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1 (t-\tau_1)} \right.\end{aligned}$$

$$+\left[\frac{\lambda_2}{\lambda_1-\lambda_2}e^{-\lambda_2(t-\tau_1)}-\frac{\lambda_1}{\lambda_1-\lambda_2}e^{-\lambda_1(t-\tau_1)}\right]g(\tau_1)+e^{-\lambda_1(t-\tau_1)}d\Bigg\}+K,$$

and, since $z(t)=\alpha(t)p^x(t)e^{-\lambda_2 t}$,

$$\begin{aligned}
z(t) &= \frac{\lambda_1}{\lambda_1+\lambda_2}p^y(t)d+p^x(t)\\
&+p^y(t)\left\{\frac{\lambda_2}{\lambda_1+\lambda_2}-\frac{\lambda_2}{\lambda_1-\lambda_2}e^{-\lambda_2(t-\tau_1)}+\frac{\lambda_2}{\lambda_1-\lambda_2}e^{-\lambda_1(t-\tau_1)}\right.\\
&\left.+\left[\frac{\lambda_2}{\lambda_1-\lambda_2}e^{-\lambda_2(t-\tau_1)}-\frac{\lambda_1}{\lambda_1-\lambda_2}e^{-\lambda_1(t-\tau_1)}\right]g(\tau_1)+e^{-\lambda_1(t-\tau_1)}d\right\}+Kp^x(t)e^{-\lambda_2 t}\\
&= p^x(t)Ke^{-\lambda_2 t}+p^x(t)+p^y(t)\left\{1-\frac{\lambda_1}{\lambda_1-\lambda_2}e^{-\lambda_2(t-\tau_1)}+\frac{\lambda_2}{\lambda_1-\lambda_2}e^{-\lambda_1(t-\tau_1)}\right.\\
&\left.+\frac{\lambda_1}{\lambda_1-\lambda_2}\left[e^{-\lambda_2(t-\tau_1)}-e^{-\lambda_1(t-\tau_1)}\right]g(\tau_1)+e^{-\lambda_1(t-\tau_1)}d\right\}\\
&+p^y(t)\left\{\left[\frac{\lambda_1}{\lambda_1-\lambda_2}e^{-\lambda_2(t-\tau_1)}-\frac{\lambda_2}{\lambda_1-\lambda_2}e^{-\lambda_2(t-\tau_1)}\right]\right.\\
&\left.+\left[\frac{\lambda_2}{\lambda_1-\lambda_2}e^{-\lambda_2(t-\tau_1)}-\frac{\lambda_1}{\lambda_1-\lambda_2}e^{-\lambda_2(t-\tau_1)}\right]g(\tau_1)\right\}\\
&= p^x(t)Ke^{-\lambda_2 t}+h(t)+p^y(t)e^{-\lambda_2(t-\tau_1)}\left[1-g(\tau_1)\right].
\end{aligned}$$

Evaluating at $t=\tau_1$, we get $z(\tau_1)=p^x(\tau_1)Ke^{-\lambda_2\tau_1}+h(\tau_1)+p^y(\tau_1)[1-g(\tau_1)]$. We now find the constant term by using the terminal condition, which we rewrote as $z(\tau_1)=p^x(\tau_1)g(\tau_1)+p^y(\tau_1)$. Combining the two equalities gives us

$$K=\frac{1}{p^x(\tau_1)}e^{\lambda_2\tau_1}\left\{g(\tau_1)-h(\tau_1)\right\}$$

Hence we have

$$\begin{aligned}
z(t) &= p^x(t)Ke^{-\lambda_2 t}+h(t)+p^y(t)e^{-\lambda_2(t-\tau_1)}\left[1-g(\tau_1)\right]\\
&= h(t)+e^{-\lambda_2(t-\tau_1)}\left\{\frac{p^x(t)}{p^x(\tau_1)}\left[g(\tau_1)-h(\tau_1)\right]\right.\\
&\left.+p^y(t)\left[1-g(\tau_1)\right]\right\}.
\end{aligned}$$

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