Effects of Wealth and Its Distribution on the Moral Hazard Problem

Jin Yong Jung

We analyze how the wealth of an agent and its distribution affect the profit of the principal by considering the simple moral hazard model developed by Baker and Hall (2004). The first result is that a rich agent is preferred over a poor one, which differs from the result of Thiele and Wambach (1999). The distinction comes from our model has only the effect of a change in risk aversion because of an increase in wealth. The second result is that the profit function of the principal is concave in wealth, which presents an implication that the principal prefers a group of agents with low wealth inequality over one with high wealth inequality.

Keywords: Principal–agent problem, Wealth effect, Wealth distribution, Exponential utility, Agency cost

JEL Classification: D31, D82, D86, J33

I. Introduction

In the real world, a principal often faces the situation where he has to select one agent among people with different wealth levels or the situation to select one of groups of agents with different wealth distributions. Under these situations, some questions can be raised to the principal: Which agent or group should be selected to be beneficial to the principal?

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A theoretical answer to the first question is initially found in the paper of Thiele and Wambach (1999). They consider the principal–agent problem in which the utility of an agent has an additively separable form and showed that if the absolute prudence of the agent is not greater than three times his absolute risk aversion degree, then the compensation cost of the principal from a rich agent is greater than that from a poor agent, and that employing the poor agent is beneficial to the principal. Later, Chade and Serio (2014) generalized the result of Thiele and Wambach but did not extend their main result.

In this study, we consider the simple principal–agent problem developed by Baker and Hall (2004). In our model, the agent has a negative exponential utility, the random variable (*e.g.,* signal) correlated stochastically with the hidden action of the agent is normally distributed, and a linear contract exists. Similar to Baker and Hall (2004), we assume that the degree of absolute risk aversion of the agent depends only on his initial wealth, and not on his current income. This assumption enables us to analyze how the wealth or wealth distribution of the agent affects the cost and profit of the principal.

One of our results is that under the assumption that the risk preference of the agent exhibits decreasing absolute risk aversion (DARA), the compensation cost of the principal from a rich agent is cheaper than that from a poor one, and therefore, the rich agent is preferred by the principal. The reason why the compensation cost from the rich agent is cheaper is that according to our assumption, as the wealth of the agent increases and his risk aversion degree decreases, the risk premium that the agent claims against accepting the risky incentive wage designed by the principal will decrease, thereby reducing her compensation cost. Thus, the rich agent is preferred over the poor one because the increase in wealth leads to the decrease in compensation cost.

This result differs from that of Thiele and Wambach (1999). In their result, although the agent has DARA utility, if his utility function satisfies the condition that the degree of absolute prudence is not greater than three times the degree of absolute risk aversion, then the compensation cost from the rich agent is higher than that from the poor one, which means the poor agent is preferred by the principal.

The distinction between our result and their result comes from the fact that when the wealth of the agent increases, a change in the compensation cost of the principal is determined generally by combining the effect of a change in the marginal rate of substitution (MRS) with respect to the income and effort of the agent and the effect of a change in the degree of risk aversion. The MRS indicates how much harder the agent will have to work as his income increases by one unit, and a decrease in the MRS raises the compensation cost of the principal. The risk aversion degree of the agent affects his risk premium claimed against accepting the risky wage provided by the principal, and a decrease in the absolute risk aversion degree reduces the compensation cost of the principal.

In the model of Thiele and Wambach, the MRS decreases in wealth, and the compensation cost of the principal will be increasing in wealth. For the agent with DARA utility, his risk aversion degree decreases as his wealth increases and the compensation cost will decrease. By combining the two effects, Thiele and Wambach (1999) concluded that if the effect of a change in the MRS dominates the effect of a change in risk aversion, then the compensation cost of the principal is increasing in wealth.

In our model, the MRS is independent of wealth, and the effect of a change in the MRS due to an increase in wealth is absent. Thus, only the effect of a change in risk aversion exists. If the risk preference of the agent exhibits the DARA property, then the compensation cost of the principal is decreasing in wealth. In the case of increasing absolute risk aversion (IARA), the compensation cost is increasing in wealth.

The other result is related to the second question raised by the principal. The profit function of the principal is concave in the wealth of the agent under a technical but non-restrictive condition. This result, combined with the first result that the profit function of the principal is increasing in wealth, indicates that the profit function is increasing in wealth but at a decreasing rate. This result implies that the principal prefers a group of agents with low wealth inequality over a group with high wealth inequality. The expected profit of the principal from the group of agents wherein wealth is fairly dispersed is not less than that from the group wherein wealth is highly dispersed.

The remainder of this paper is organized as follows. Section II introduces our basic model. Sections III and IV provide our results and the intuitive interpretation of the main results. Section V concludes.

II. Basic Model

A risk-neutral principal and a risk-averse agent exist. The output of principal x is determined by the effort of agent $a \in A = [0, \infty)$ and a random shock θ. The production function for output x is given by $x = a + \theta$, where θ is distributed normally with mean 0 and variance σ^2. Thus, $x \sim N(a, \sigma^2)$.

The effort choice of the agent is unobservable to the principal, but output x is observable to the both without cost. Thus, the principal should design an incentive wage $s(x)$ to depend on output x, which is commonly observable and stochastically dependent on the hidden effort of agent a. The wage contract between the principal and the agent is given by a linear contract, *i.e.*, $s(x) = \alpha + \beta x$, where α is the level of base salary and β is the pay-to-performance sensitivity.[1] Thus, the principal can achieve her objectives, such as cost minimizing and profit maximizing, by selecting optimal values of α and β.

The agent with initial wealth w has an exponential utility, *i.e.*, $U_A = -e^{-\rho(w)(w+s-\upsilon(a))}$, where s is the monetary payoffs from the principal, and $\upsilon(a)$ is the effort cost of the agent measured in monetary units with $\upsilon'(a) > 0$, $\upsilon''(a) > 0$, and $\upsilon'''(a) \geq 0$.[2] The absolute risk aversion degree $\rho(w)$ depends on his initial wealth w, but not on his current income s. The risk attitude of the agent with respect to current income is determined accordingly by his initial wealth w. As in most literature, we assume that $\rho(w)$ is decreasing strictly. Therefore, the initial wealth preference of the agent exhibits DARA.

Following the two-step method of Grossman and Hart (1983), we divide the problem of the principal into profit maximization and cost minimization problems as follows:

$$\pi(w) \equiv \max_{a \in A} a - C(a, w),$$

[1] In the case that the agent has constant absolute risk aversion utility and signal is distributed normally, the reason why a linear contract is considered is based on the result of Holmstrom and Milgrom (1987).

[2] In fact, this utility function was used implicitly in the model developed by Baker and Hall (2004). The certainty equivalence of the expected utility of the agent given linear contract $s(x) = \alpha + \beta x$ is $w + \alpha + \beta E[x|a] - (\sigma^2/2)\,\rho(w) - \upsilon(a)$, which is identical to the expected utility of the agent considered in the model of Baker and Hall (2004).

where

$$C(a, w) \equiv \min_{s(x)} E[s(x) | a],$$

$$\text{s.t. i) } a \in \text{argmax}_{a' \in A}\, EU_A(a') \equiv E[-\exp\{-\rho(w)[s(x) + w - v(a')]\} | a'],$$
$$\text{ii) } EU_A(a) = E[-\exp\{-\rho(w)[s(x) + w - v(a)]\} | a] \geq -e^{-\rho(w)[k+w]},$$
$$\text{iii) } s(x) = \alpha + \beta x.$$

In the above cost minimization problem, the first and second constraints are typically called the incentive compatibility and participation constraints, respectively. In the first participation constraint, $k + w$ denotes the certainty equivalence of the reservation utility of the agent when his initial wealth is equal to w. The cost minimization problem of the principal shows how much it costs to make the agent accept the wage contract under which he selects target effort level $a > 0$.

Our goal is to analyze the effect of the wealth of agent w on the compensation cost $C(a, w)$ and profit function $\pi(w)$ of the principal. We will then discuss who the principal prefers among agents with different wealth levels and which group the principal prefers among groups of agents with different wealth distributions.

III. Analysis

We first deal with the cost minimization problem of the principal, in which the principal induces the agent with initial wealth w to select a given effort $a > 0$. The problem to adopt a linear contract (*i.e.,* the third constraint in the cost minimization problem) is

$$C(a, w) \equiv \min_{\alpha, \beta} E[\alpha + \beta x | a],$$

$$\text{s.t. i) } a \in \text{argmax}_{a' \in A}\, EU_A(a') \equiv E[-\exp\{-\rho(w)[\alpha + \beta x + w - v(a')]\} | a'],$$
$$\text{ii) } EU_A(a) = E[-\exp\{-\rho(w)[\alpha + \beta x + w - v(a)]\} | a] \geq -e^{-\rho(w)[k+w]}.$$

$x \sim N(a, \sigma^2)$, and the certainty equivalence of $EU_A(a)$ is equal to

$$\alpha + w + \beta a - \frac{\rho(w)}{2}\beta^2\sigma^2 - v(a). \tag{1}$$

The effort level of the agent to maximize his expected utility $EU_A(a)$ is equal to effort level to maximize his certainty equivalence, and solving Equation (1) with respect to a yields

$$\beta = v'(a). \tag{2}$$

On the one hand, Equation (2) indicates that when the slope of a linear contract is given by β, the agent will select effort $a = a(\beta)$ to solve $\beta = v'(a)$. On the other hand, this equation tells the principal she must set $\beta = v'(a)$ to induce the given effort level a. Thus, Equation (2) replaces the first constraint.

$\beta = v'(a)$, and the certainty equivalence of $EU_A(a)$ is equal to

$$\alpha + w + v'(a)a - \frac{\rho(w)}{2}[v'(a)]^2\sigma^2 - v(a).$$

The second constraint is then equivalent to

$$\alpha + av'(a) - \frac{\rho(w)}{2}[v'(a)]^2\sigma^2 - v(a) \geq k.$$

This inequality indicates that base salary α must be not less than the value of $k - av'(a) + \rho(w)/2\ [v'(a)]^2\sigma^2 + v(a)$ for the agent to accept the linear contract designed by the principal. Thus, the principal minimizes compensation cost by setting $\alpha = k - av'(a) + \rho(w)/2\ [v'(a)]^2\sigma^2 + v(a)$. The compensation cost of the principal is equal to

$$C(a, w) = k + v(a) + \frac{\rho(w)}{2}[v'(a)]^2\sigma^2 \tag{3}$$

The term $k + v(a)$ in Equation (3) means the cost incurred by the principal when she can observe the effort choice of the agent. The third term $\rho(w)/2\ [v'(a)]^2\sigma^2$ in Equation (3) means the loss incurred by the principal from being unable to observe the effort choice of the agent. If signal uncertainty increases (*i.e.,* σ increases), then the third term also increases. An increase in signal uncertainty causes the incentive problem to become serious.[3]

[3] An increase in signal uncertainty generally makes the incentive problem

We obtain the following propositions directly by using Equation (3).

Proposition 1. Compensation cost $C(a, w)$ is strictly increasing and convex in the target effort level a, *i.e.*, $C_a(a, w) > 0$, and $C_{aa}(a, w) > 0$.

Proof. Differentiating Equation (3) with respect to a gives

$$C_a(a, w) = \rho(w)\sigma^2 \times v'(a)v''(a) + v'(a) > 0, \tag{4}$$

where the strict inequality holds given $v'(a) > 0$ and $v''(a) > 0$. Differentiating Equation (4) with respect to a gives

$$C_{aa}(a, w) = \rho(w)\sigma^2 \times \{[v''(a)]^2 + v'(a)v'''(a)\} + v''(a) > 0, \tag{5}$$

where strict inequality holds considering $v'(a) > 0$, $v''(a) > 0$, and $v'''(a) \geq 0$ by assumption. Q.E.D.

The above proposition shows that under the assumption that the marginal effort cost of the agent is convex, compensation cost $C(a, w)$ of the principal is increasing in the target effort a at an increasing rate. Equation (2) indicates that a continuous increase in target effort a leads to a monotonic increase in pay-to-performance sensitivity β. Thus, as a increases continuously, the amount of incentive with which the principal should provide the agent to induce target effort a rises at an increasing rate. A monotonic increase is induced in his risk premium that the risk-averse agent claims against accepting the risky incentive wage provided by the principal, which in turn causes a monotonic increase in the compensation cost $C(a, w)$ of the principal.

Proposition 2. $C_w(a, w) < 0$, and $C_{aw}(a, w) < 0$. The convexity of $C(a, w)$ in w is equivalent to the convexity of $\rho(w)$.

Proof. Differentiating Equation (3) with respect to w yields

serious (see Grossman, and Hart (1983); Kim (1995) for theoretical results). This theoretical result has been applied to various fields. For example, it is known that a stock-holding manager is reluctant to undertake R&D projects due to risk-reduction incentive. For empirical works about it, see cho (1992) among others.

$$C_w(a,\, w) = \frac{\rho'(w)}{2}\,[v'(a)]^2\sigma^2 < 0, \tag{6}$$

where the strict inequality holds given $\rho'(w) < 0$ by assumption. Differentiating Equation (6) with respect to a gives

$$C_{aw}(a,\, w) = \rho'(w)\sigma^2 \times v'(a)v''(a) < 0, \tag{7}$$

where the strict inequality holds considering $\rho'(w) < 0$, $v'(a) > 0$, and $v''(a) > 0$. Differentiating Equation (6) with respect to w yields

$$C_{ww}(a,\, w) = \frac{\rho''(w)}{2}\,[v'(a)]^2\sigma^2. \tag{8}$$

This equation shows that the sign of $C_{ww}(a, w)$ is the same as that of $\rho''(w)$. Q.E.D.

The above proposition contains three results. The first is that compensation cost $C(a, w)$ is decreasing in the wealth of agent w. The proof of Proposition 2 shows that the sign of $C_w(a, w)$ is the same as that of $\rho'(w)$. Thus, $C_w(a, w) < 0$ comes from $\rho'(w) < 0$ (see Equation (6)).[4] The intuitive reason is that as the initial wealth of agent w increases and his absolute risk aversion degree decreases, his risk premium claimed against accepting the risky incentive wage will decrease, which reduces the compensation cost of the principal $C(a, w)$. The compensation cost of the principal is decreasing in the initial wealth of the agent due to the effect of risk aversion.

The second is that the cross derivative of compensation cost $C(a, w)$ has a negative sign. The proof of Proposition 2 also indicates that the cross derivative and the first derivative of $\rho(w)$ have the same sign. $\rho'(w) < 0$, and thus, the cross derivative of $C(a, w)$ has a negative sign. The second result means the target effort level a and wealth w of the agent are substitutes to the principal only when the degree of absolute risk aversion of the agent is decreasing in his initial wealth w.

The third is that the sign of $C_{ww}(a, w)$ is the same as that of $\rho''(w)$.

[4] However, if $\rho'(w) > 0$ or $\rho'(w) = 0$, then compensation cost $C(a, w)$ will be increasing or constant in w, respectively.

In other words, the concavity or convexity of $\rho(w)$ is equivalent to the concavity or convexity of $C(a, w)$ in w, respectively. We consider the case where $\rho(w)$ is convex. In this case, $\rho(w)$ is decreasing at a decreasing rate as w increases. The reason why $C(a, w)$ is (decreasing and) convex in w in this case is that given that the absolute risk aversion degree of the agent is decreasing at a decreasing rate (*i.e.,* $\rho'(w) < 0$ and $\rho''(w) < 0$), as his initial wealth w increases, his risk premium decreases at a decreasing rate, from which the compensation cost of the principal $C(a, w)$ decreases at a decreasing rate. In the case with concave $\rho(w)$, considering that $\rho(w)$ is decreasing at an increasing rate, as w increases, the risk premium of the agent decreases at an increasing rate, from which the compensation cost $C(a, w)$ decreases at an increasing rate.

We then deal with the profit maximization problem of the principal to determine the optimal effort level. Her problem is

$$\pi(w) = \max_{a \in A} a - C(a, w).$$

The following lemma provides the sufficient conditions for the existence and uniqueness of the profit-maximizing effort level.

Lemma 1. Suppose that $v'(0) = 0$ and $v'(\infty) = \infty$. The effort level of the agent to maximize the profit of the principal uniquely exists.

Proof. The first-order condition is

$$1 - C_a(a, w) = 0.$$

Let $h(a) = 1 - C_a(a, w)$, where $C_a(a, w) = v'(a)[\rho(w)\sigma^2 v''(a) + 1]$. $h(0) = 1 > 0$, and $h(\infty) = -\infty$, at least one solution $a \in A = (0, \infty)$ exists to satisfy $h(a) = 0$. $C_{aa}(a, w) > 0$ by Proposition 1, and $h'(a) = -C_{aa}(a, w) < 0$ for all $a \in A$, which implies that the solution is unique. Q.E.D.

Let $a = a(w)$ solve $1 - C_a(a, w) = 0$. $a(w)$ indicates the optimal effort level of the agent with initial wealth w. Let $\beta(w) = v'(a(w))$. Equation (2) shows that $\beta(w)$ is the optimal pay-to-performance sensitivity. By definition, the profit function of the principal is

$$\pi(w) \equiv a(w) - C(a(w), w).$$

Proposition 3. $\pi'(w) > 0$, $a'(w) > 0$, and $\beta'(w) > 0$.

Proof. The envelope theorem indicates that $\pi'(w) = -C_w(a(w), w) > 0$, where strict inequality holds given $C_w(a, w) < 0$ by Proposition 2. By the definition of $a(w)$, $C_a(a(w), w) \equiv 1$, and its differentiating gives

$$a'(w) = -\frac{C_{aw}}{C_{aa}} > 0,$$

where strict inequality is satisfied given $C_{aa} > 0$ by Proposition 1 and $C_{aw} < 0$ by Proposition 2. Differentiating identity $\beta(w) = v'(a(w))$ gives

$$\beta'(w) = v''(a(w))a'(w) > 0,$$

where strict inequality holds because $v'' > 0$ and $a'(w) > 0$. Q.E.D.

Proposition 3 provides three results: the profit function of the principal is strictly increasing in the initial wealth of agent w, and optimal effort level $a(w)$ and pay-to-performance sensitivity $\beta(w)$ are increasing in initial wealth w. The first result means the principal prefers a rich agent over a poor one. As his initial wealth w increases, the absolute risk aversion degree of the agent decreases (*i.e.*, $\rho'(w) < 0$), and the compensation cost of the principal $C(a, w)$ decreases based on Proposition 2, which implies that the profit function of the principal $\pi(w)$ is increasing in w. As shown in the proof of Proposition 3, the second result that the optimal effort of the agent $a(w)$ is increasing in his initial wealth w depends on the signs of the second derivative $C_{aa}(a, w)$ and the cross derivative $C_{aw}(a, w)$. $C(a, w)$ is convex in a by Proposition 1, and the sign of $C_{aw}(a, w)$ is negative by Proposition 2. Accordingly, optimal effort $a(w)$ is increasing in w. The third result that the optimal pay-to-performance sensitivity $\beta(w)$ is increasing in w is derived from the effort cost $v(a)$ being convex and the optimal effort $a(w)$ increasing in w.

Our results presented in Proposition 3 depend crucially on the assumption that the risk preference of the agent shows DARA (*i.e.*, $\rho'(w) < 0$)). As shown in the proof of Proposition 3, they are derived basically from the result that $C_w(a, w) < 0$ and $C_{aw}(a, w) < 0$, which come from the assumption that $\rho'(w) < 0$, as shown in the proof of Proposition 2. As a result, considering $\rho'(w) < 0$, we have $C_w(a, w) < 0$ and $C_{aw}(a, w) < 0$, which imply that $\pi'(w) > 0$, $a'(w) > 0$, and $\beta'(w) > 0$. If the risk preference of the agent indicates constant absolute risk aversion (*i.e.*, $\rho'(w) = 0$), then we have $C_w(a, w) = 0$ and $C_{aw}(a, w) = 0$, which indicate that $\pi'(w) = 0$, $a'(w) = 0$, and $\beta'(w) = 0$. Thus, in the case that the degree of absolute risk aversion of the agent is independent of his initial wealth w (*i.e.*, $\rho'(w) = 0$), no wealth effect occurs. If the risk preference of the agent shows IARA (*i.e.*, $\rho'(w) > 0$), then we prove that $C_w(a, w) > 0$ and $C_{aw}(a, w) > 0$, from which we finally have $\pi'(w) < 0$, $a'(w) < 0$, and $\beta'(w) < 0$. In other words, the wealth effect is negative in this case.

Proposition 4. If $\rho(w)\rho''(w) \geq 2(\rho'(w))^2$, then $\pi''(w) < 0$.

Proof. $\pi'(w) = -C_w(a(w), w)$, and

$$a'(w) = -\frac{C_{aw}}{C_{aa}}.$$

We then have

$$\pi''(w) = -C_{wa}a'(w) - C_{ww} = -\frac{1}{C_{aa}}[C_{aa}C_{ww} - (C_{aw})^2].$$

The use of Equations (5), (7), and (8) gives

$$C_{aa}C_{ww} - (C_{aw})^2 = \frac{\sigma^4}{2}[v']^2\{(v'')^2[\rho\rho'' - 2(\rho')^2] + \rho''\rho v'v'''\} + \frac{\sigma^2}{2}\rho''(v')^2 v''.$$

The condition that $\rho(w)\rho''(w) \geq 2(\rho'(w))^2$ requires $\rho''(w)$ be positive. $v'''(a) \geq 0$, and $\rho''(w) > 0$. Accordingly, we have $C_{aa}C_{ww} - (C_{aw})^2 > 0$, which implies together with $C_{aa} > 0$ that $\pi''(w) < 0$. Q.E.D.

This proposition demonstrates that the profit function of the principal $\pi(w)$ is increasing in the wealth of the agent, but at a decreasing rate, if the inequality that $\rho(w)\rho''(w) \geq 2(\rho'(w))^2$ is satisfied. The condition in Proposition 4 is satisfied in the case where $\rho(w)$ is a hyperbolic function of w. Let

$$\rho(w) = \frac{\delta}{w + \gamma} > 0$$

to verify this case.

$$\rho(w)\rho''(w) = 2\frac{\delta^2}{(w+\gamma)^4},$$

and

$$2[\rho'(w)]^2 = 2\frac{\delta^2}{(w+\gamma)^4}.$$

Therefore, the condition that $\rho(w)\rho''(w) \geq 2[\rho'(w)]^2$ is always satisfied. On the basis of Proposition 4, if the absolute risk aversion degree of the agent is a hyperbolic function of w, then the profit function of the principal is concave in the initial wealth of the agent w.

The result of Proposition 4 presents an implication that the principal prefers a group of agents with low wealth inequality over a group with high wealth inequality. We consider two groups of agents with different wealth distributions: $F(w)$ and $G(w)$. The expected profits of the principal under wealth distributions $F(w)$ and $G(w)$ are equal to $E_F[\pi(w)] \equiv \int\pi(w)dF(w)$ and $E_G[\pi(w)] \equiv \int\pi(w)dG(w)$, respectively. The condition that $G(w)$ is a mean preserving spread of $F(w)$ means the wealth inequality of the group with distribution $G(w)$ is higher than that of the group with distribution $F(w)$. Thus, if $G(w)$ is a mean preserving spread of $F(w)$, given that $\pi(w)$ is a concave function, then the expected profit of the principal under distribution $F(w)$ is not less than that under wealth distribution $G(w)$, (*i.e.*, $E_F[\pi(w)] \geq E_G[\pi(w)]$). Therefore, the principal prefers the group of agents with low wealth inequality over the one with high wealth inequality.

IV. Comparison with Previous Results

Section 3 shows that under the assumption that the risk preference of the agent exhibits DARA, his wealth w exerts a negative effect on the compensation cost of the principal $C(a, w)$, and a positive effect on the profit function $\pi(w)$. The principal then prefers the agent with high wealth. However, this result differs from previous results. The reason is discussed in this section.

The previous result on the wealth effect on the agency problem was initially found in the paper of Thiele and Wambach (1999).[5] They

[5] Chade and Serio (2014) extended the result of Thiele and Wambach by

showed that in the principal–agent problem, in which the utility of the agent is additively separable (*i.e.*, $u(w + s) - v(a)$, where $u(w + s)$ denotes the utility from final wealth $w + s$, and $v(a)$ denotes the disutility from effort a), if the degree of absolute prudence of the agent is not greater than three times his absolute risk aversion degree (*i.e.*, $P(w) \leq 3A(w)$, where $P(w) \equiv - u'''(w)/u''(w)$, and $A(w) \equiv - u''(w)/u'(w)$), then the compensation cost of the principal is increasing in wealth, which implies that the profit of the principal is decreasing in wealth. In other words, when the agent has a utility function with $P(w) \leq 3A(w)$, the poor agent is preferred over the rich agent, considering that the compensation cost from the agent is increasing in wealth.

We consider the case that $A(w) < P(w) \leq 3A(w)$ for all w to determine how the result of Thiele and Wambach (1999) differs from our result. Thiele and Wambach (1999) stated that a poor agent is preferred. However, their result is contrary to our result in this case. The equivalent condition for the utility of the agent to have DARA property is $A(w) < P(w)$. Thus, in this case, our result indicates that a rich agent is preferred because the compensation cost of the principal is decreasing in wealth. If the agent has DARA utility with $P(w) \leq 3A(w)$, then our result is opposite to the results obtained by Thiele and Wambach (1999).

The distinction between our result and that of Thiele and Wambach (1999) comes from the fact that an increase in the wealth of an agent leads to a decrease in the MRS with respect to income s and effort a in their model. In our model, the MRS is independent of the wealth of the agent. The utility of the agent is additively separable, *i.e.*, $u(s) - v(a)$, in their model. We let $s = s(w, a)$ solve the equation $u(s + w) - v(a) = u(k + w)$. The MRS with respect to income s and effort a is

$$MRS \equiv \frac{da}{ds}\bigg|_{u(s+w)-v(a)=u(k+w)} = \frac{1}{s_a(w, a)} = \frac{u'(s(a, w) + w)}{v'(a)}. \tag{9}$$

The MRS indicates how much harder the agent will work as his income increases by one unit. By differentiating identity $u(s(a, w) + w) - v(a) \equiv u(k + w)$ with respect to w, we have

proving its necessary part.

$$s_w(a, w) + 1 = \frac{u'(k + w)}{u'(s(a, w) + w)} > 0, \tag{10}$$

where the strict inequality holds from $u'(s) > 0$ for all s. Thus, differentiating the MRS in Equation (9) with respect to w yields

$$\frac{dMRS}{dw} = \frac{u''(s(a, w) + w)}{v'(a)} \times [s_w(a, w) + 1] < 0,$$

where the strict inequality holds from $u'' < 0$ and $v' > 0$ and that $s_w(a, w) + 1 > 0$, as shown in Equation (10), which indicates that the MRS of the agent with wealth w is decreasing in w. When the wealth of agent w increases, he will work less hard than ever. Thus, the principal should design a wage contract to offer a strong incentive for the agent for her to induce the same effort, which leads to an increase in her compensation cost. In summary, an increase in wealth leads to a decrease in MRS, which increases the compensation cost of the principal in the model of Thiele and Wambach (1999).

When the utility of the agent exhibits DARA (*i.e.*, $A(w) < P(w)$), an increase in wealth w leads to a decrease in the degree of risk aversion, which reduces the compensation cost of the principal. Therefore, as the wealth of the agent increases, if the effect of a decrease in the MRS dominates the effect of a decrease in the degree of absolute risk aversion, then the compensation cost will increase. Thiele and Wambach (1999) explained that inequality $P(w) \leq 3A(w)$ is the condition under which the former effect dominates the latter effect.[6] Consequently, although the agent has DARA utility, the poor agent may be preferred by the principal.

In our model, the utility of the agent is $u(w + s - v(a)) = -e^{-\rho(w)[w+s-v(a)]}$, and the MRS is independent of wealth. We let $s = \hat{s}(a, w)$ solve $u(s + w - v(a)) = u(k + w)$. In our case, the MRS for the agent with wealth w is

$$MRS = \left.\frac{da}{ds}\right|_{u(s+w-v(a))=u(k+w)} = \frac{1}{\hat{s}_a(a, w)} = \frac{1}{v'(a)},$$

which shows that the MRS is independent of w. As the wealth of

[6] For more detailed explanation, see Thiele and Wambach (1999).

the agent w changes, the MRS remains unchanged, resulting in the compensation cost of the principal being unchanged. In other words, the effect of a change in MRS is zero. Thus, in our model, only the effect of a change in risk aversion exists. When his risk preference shows DARA, an increase in wealth decreases the compensation cost.[7] Under the situation that the risk preference of the agent exhibits DARA, the principal prefers the rich agent who has low compensation cost.

V. Conclusion

We consider the moral hazard model in which an agent has a negatively exponential utility, with the degree of absolute risk aversion decreasing in his initial wealth, his effort cost is measured by monetary units, and a linear contract exists.

In this model, we show that the compensation cost of the principal is increasing and convex in the target effort and decreasing in the wealth level of the agent. The convexity of the compensation cost in his wealth comes from the convexity of his degree of absolute risk aversion. We present the profit function, optimal effort level, and optimal sensitivity of the principal as all increasing with the wealth of the agent based on the properties of compensation cost. The profit function of the principal is also concave in the wealth of the agent, which implies that a group of agents with low wealth inequality are preferred by the principal over a group with high wealth inequality.

Our result that a rich agent is preferred by the principal differs from the results obtained by Thiele and Wambach (1999). Their model considers the positive effect of decreasing MRS on compensation cost and the negative effect of decreasing the degree of risk aversion. Therefore, if the condition that the positive effect dominates the negative effect is satisfied, then the principal will prefer a poor agent. In our model, no positive effect of a change in MRS is considered. If the risk preference of the agent exhibits DARA, then the principal prefers a rich agent because he is minimally risk averse.

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[7] If the risk preference of the agent shows IARA, then an increase in his wealth will increase the compensation cost in our model.

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