Multi-Firm Mergers with Leaders and Followers

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This paper analyzes mergers involving several leaders and followers in Stackelberg models, with the merged entity acting as a leader. Adding a follower to a merger increases its profitability, and a merger between one leader and any number of followers is always profitable. When a merger involves two leaders, a sufficiently large proportion of followers is required for it to be profitable. A merger is less likely to be profitable when the number of participating leaders is intermediate and the number of participating followers is small. That is, merger profitability is monotonic in the number of followers but not in the number of leaders. All mergers involving leaders and followers are welfare-reducing. Overall, Stackelberg leadership partially alleviates the merger paradox.

Keywords: Mergers, Merger profitability, Merger paradox, Stackelberg, Leaders, Followers

JEL Classification: D43, L13

I. Introduction

From as early as Stigler (1950) to Salant *et al.* (1983), studies have found that mergers in a Cournot model may be unprofitable, unless they involve a substantial proportion (80%) of firms in the industry. Considering that mergers entail output expansion by outsiders, if the

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number of the latter is large relative to the number of insiders, such expansion can be sufficiently large to make the merger unprofitable for the merging firms.

Considerable theoretical literature has investigated this topic, with one branch attempting to enrich the Cournot model by different features (*e.g.*, product differentiation, scarce capital, and so on.). Another branch has looked at firm asymmetries, in particular leader/follower roles. This branch has primarily focused on mergers involving firms of the same type (leaders with leaders, followers with followers) and/or mergers involving only two firms.

This paper contributes to the literature on merger profitability by analyzing mergers involving arbitrary numbers of leaders and followers. In particular, it explicitly considers the asymmetric leader/follower roles in Stackelberg models. It extends the analysis of mergers in Stackelberg models by considering not only bilateral mergers, but also mergers involving any number of leaders and followers.

The present paper sets up a Stackelberg model with leaders and followers. Any merger involves at least one leader and one follower, with the merged entity forming a leader firm. Adding a follower to a merger always increases its profitability. A merger between one leader and any number of followers is always profitable. When a merger involves two leaders, a sufficiently large proportion of followers is required for it to be profitable. In particular, a merger can be profitable even when the number of leaders participating is smaller than the standard threshold, provided that the number of followers participating in the merger is large enough. When all followers participate in a merger, such a merger is more likely to be profitable when the number of followers is large.

Conversely, a merger is less likely to be profitable when the number of participating leaders is intermediate, and the number of participating followers is small. The reason is that with this configuration, the merged entity does not possess sufficient market power (as many leaders remain) to compensate for the lost profits of the participating leaders. In addition, the small number of followers makes their contribution to the profitability of the merger insufficient to compensate for that disadvantage. Pre-merger profits increase linearly with the number of participating leaders, whereas post-merger profits are convex in their number. Given that mergers involving one or all leaders are always profitable, if a range of mergers are unprofitable, this can only mean that an intermediate number of leaders participated in those mergers. These results imply that merger profitability is monotonic in the number of followers, but not in the number of leaders.

Although mergers with followers are often appealing to leaders, the effects on consumers and welfare are less encouraging. Any merger involving at least one leader and one follower, with the merged entity acting as a leader, reduces consumer surplus and welfare, and increases total profits and the profits of each outsider. Overall, the paper shows that Stackelberg leadership partially alleviates the merger paradox.

The paper is organized as follows: the next section provides a review of the literature on merger profitability and the merger paradox, especially in a Stackelberg framework. Section III presents the basic model. Section IV, which constitutes the core of the paper, analyzes mergers involving both leaders and followers. This section is divided into three parts, with the first deriving general results regarding mergers involving leaders and followers. The second part examines specific market structures, which illustrate the general results derived in the first part. The third part analyzes the welfare implications of such mergers. The last section concludes the paper.

II. The literature

In the merger literature, different mechanisms[1] have been proposed by which firms can evade the merger paradox,[2] namely, that "it is ... quite difficult to construct a simple economic model in which there are sizable gains for firms participating in a horizontal merger *that is not a*

[1] Access to scarce capital (Perry, and Porter 1985); product differentiation with Bertrand competition (Deneckere, and Davidson 1985); non-Cournot behavior (Kwoka 1989); capacity constraints (Baik 1995); properties of the demand function (Cheung 1992; Faulí-Oller 1997; Hennessy 2000); choice of product range (Lommerud, and Sϕrgard 1997); the short run *vs*. the long run (Polasky, and Mason 1998); spatial price discrimination (Rothschild *et al.* 2000); dynamic Cournot competition (Dockner, and Gaunersdorfer 2001); technological asymmetry and bargaining (Kabiraj, and Lee 2003); open-loop *vs*. closed-loop strategies (Benchekroun 2003); improved information flows inside the merged entity (Huck *et al.* 2004); setting competition between the internal divisions of the merged firm (Creane, and Davidson 2004); intra-firm coordination (Higl, and Welzel 2005); Research and Development (Atallah 2005); cost-reducing alliance (Sawler 2005); commitment to maintain pre-merger profits (Huck *et al.* 2007); product differentiation with cost asymmetries (Gelves 2014); workers' cooperatives (Delbono, and Lambertini 2014); relative performance rewards (Fan, and Wolfstetter 2015). See Huck *et al.* (2005) for a survey.

[2] The term was first coined by Pepall *et al.* in the first edition of their textbook, but was first formalized by Salant *et al.* (1983).

merger to monopoly" (Pepall *et al.* 2014, p. 388). Of particular relevance to the present paper is the interaction between market leadership and the profitability of mergers. Almost all papers mentioned in Footnote 1 analyze mergers in a Cournot setting. Such analysis can be different in a Stackelberg model, in which some firms act as leaders and others as followers.

For example, Mizutani (2010) shows that following the merger between *Japan Airlines* and *Japan Air System* in 2003, they became an equal rival to *All Nippon Airways*; as such, the competition changed from Stackelberg to Cournot. Romeo (2012) uses Stackelberg merger simulation in the beer industry, and finds that the post-merger price and market shares are very different from those obtained under Bertrand simulations. In the Silicon Valley's semiconductor industry, *Avago Technologies* bought *LSI* in 2014. In May 2015, the former announced its acquisition of *Broadcom*. Both the buyer and the target firms are large players in the industry. The first transaction was valued at $6.6 billion, and the second at $37 billion, resulting in a $77-billion company (Carey 2015). Mergers can occur sequentially over time, even if each time only two firms are involved. Some papers, such as Nilssen, and Sørgard (1998), have studied such sequential mergers.

Studies have examined the interaction between Stackelberg leadership and merger profitability. Levy, and Reitzes (1989) model mergers between two firms (who become a leader) in a spatial setting. Levin (1990) analyzes mergers starting from a Cournot setting, but allows the merged firms to have a more general conjectural variation (for instance, become a Stackelberg leader); he finds that when such a merger induces a reduction in output of the merged entity, it is profitable if it involves firms with at least a 50% pre-merger market share. Higgins (1996) analyzes mergers in a price-setting Stackelberg model, and finds that when the merger includes the leading firm, price will increase proportionally to the increase in the capacity of the leader. Huck *et al.* (2001), Feltovich (2001), and Kabiraj, and Mukherjee (2003) find that two leaders (followers) have an incentive to merge if and only if no other leaders (followers) are involved; in addition, a merger between one leader and one follower is always profitable. Feltovich (2001), and Daughety (1990) consider the possibility that two followers merge to form a leader, and find that such a merger is always profitable. Gelves (2010), Takarada, and Hamada (2006) analyze mergers involving leaders and followers, but focus on mergers involving only two firms. Hamada, and Takarada (2007) study mergers in a Stackelberg model. However, they consider only cases in which leaders merge with leaders, or followers with followers. Heywood, and McGinty (2007) and Brito, and Catalão-Lopes (2011) study the effects of cost convexity on the profitability of a merger of two firms. Escrihuela-Villar (2013) studies merger profitability between a leader and a follower, and shows that it depends on the extent of collusion among leaders. Le Pape, and Zhao (2014) study mergers between two firms in a Stackelberg model in the presence of uncertainty on post-merger costs. Cunha, and Vasconcelos (2015) study the profitability of mergers between two firms in a Stackelberg model, where the merger gives rise to efficiency gains.

One common assumption of the studies mentioned above is that only bilateral mergers are considered, whether the merger involves two leaders, two followers, or a follower and a leader. However, just as in a Cournot model, a merger may involve more than two firms. A complete analysis of the profitability of mergers in Stackelberg models requires the consideration of mergers between arbitrary numbers of firms, involving any number of leaders and followers. Few papers have begun to consider mergers involving more than two firms in Stackelberg models. Heywood, and McGinty (2008) study mergers between a leader and a number of followers in a Stackelberg model in the presence of cost convexity, and find that such mergers are always profitable. Escrihuela-Villar, and Faulí-Oller (2008) model mergers involving one leader and several followers, among other types of mergers, and study the role of cost asymmetry between leaders and followers. However, no study considers the general case analyzed here, in which a merger may involve several leaders and several followers.

Moreover, empirical evidence does not indicate a systematic unprofitability of mergers. Scherer (1980) finds that, on average, the private gains from mergers are either negative or almost nil. Bruner (2002) reviews 130 empirical studies on merger profitability between 1971 and 2001, and concludes that shareholders of target firms gain, whereas the returns to the shareholders of the buying firms are close to zero. From reviewing a large number of empirical studies, Budzinski, and Kretschmer (2007) conclude that unprofitable mergers represent between 25% and 50% of mergers. In fact, bilateral mergers are observed in most industries (Office of Fair Trading, 1999); nothing suggests that the bulk of these mergers satisfy the high threshold (required for profitability) imposed by the static Cournot model. Most of the empirical evidence on mergers points in the same direction: mergers are profitable more often than that suggested by the theoretical models.

III. The basic model

L leaders and F followers ($L, F \geq 1$) compete in a Stackelberg market. Market demand is given by $P(Y)=1-Y$, where $Y=Ly_L+Fy_F$ denotes total output, and y denotes the output of an individual firm. For simplicity, no production costs are involved in these mergers. The leaders determine their outputs simultaneously. After observing the leaders' outputs, the followers choose their outputs. The profit of a leader is denoted by $\Pi_L=P(Y)y_L$, while the profit of a follower is $\Pi_F=P(Y)y_F$.

We start with the followers' profit maximization. Follower i solves the following problem:

$$\max_{y_{F,i}} \ \pi_{F,i}=(1-y_{F,i}-(F-1)y_{F,-i}-Ly_L)y_{F,i}, \qquad i=1, \ldots, F, \tag{1}$$

where $-i$ indicates all other firms of the same type. Taking the f.o.c., imposing symmetry, and solving for y_F gives each follower's reaction function:

$$y_F(y_L)=\frac{1-Ly_L}{F+1}. \tag{2}$$

Each leader anticipates this reaction function, and solves the following problem:

$$\max_{y_{L,i}} \ \pi_{L,i} = \left(1 - y_{L,i} - (L-1)y_{L,-i} - F\left[\frac{1 - y_{L,i} - (L-1)y_{L,-i}}{F+1}\right]\right) y_{L,i}, \tag{3}$$

$i=1, \ldots, L.$

By solving this problem, imposing symmetry, and substituting the solution into Equation (2) we obtain outputs in the pre-merger symmetric equilibrium

$$y_L^*(L)=\frac{1}{L+1}, \quad y_F^*(L, F)=\frac{1}{(L+1)(F+1)}. \tag{4}$$

Substituting Equation (4) into profits results in per firm profits as a function of the number of leaders and followers, as expressed below.

$$\pi_L(L, F) = \frac{1}{(L+1)^2 (F+1)}, \quad \pi_F(L, F) = \frac{1}{(L+1)^2 (F+1)^2}. \tag{5}$$

IV. Mergers involving both leaders and followers[3]

A. General market structures

At least one leader and one follower are assumed to participate in a merger, with the merged entity becoming a leader. Thus, with $N_L \in \{1, ..., L\}$ leaders and $N_F \in \{1, ..., F\}$ followers participating in the merger, the number of leaders after the merger is $L - N_L + 1$, whereas the number of followers is $F - N_F$. Such a merger reduces the number of followers more than it reduces the number of leaders, because followers "disappear." From Equation (5), the profits after such a merger are given by

$$\begin{aligned} \pi_L(L-N_L+1, F-N_F) &= \frac{1}{(L-N_L+2)^2 (F-N_F+1)}, \\ \pi_F(L-N_L+1, F-N_F) &= \frac{1}{(L-N_L+2)^2 (F-N_F+1)^2}. \end{aligned} \tag{6}$$

This merger is profitable if and only if

$$\pi_L(L-N_L+1, F-N_F) > N_L \pi_L(L, F) + N_F \pi_F(L, F). \tag{7}$$

A locus of critical values of N_L and N_F determine whether the merger is profitable. Much of the analysis that follows aims at characterizing this locus.

We first consider how the participation of a follower to a merger affects its profitability.

Proposition 1. If, for a given N_L, a merger is profitable when N_F followers participate in it, then it is also profitable when N_F+1 followers participate. If, for a given N_L, a merger is unprofitable when N_F followers participate in it, then it is also unprofitable when N_F-1 followers par-

[3] In the appendix we consider mergers involving leaders only or followers only, which were studied by Hamada, and Takarada (2007). In particular, they show that the standard threshold $N^c(N) = (2N+3-\sqrt{(4N+5)})/2$ (where N is the number of firms in the market) applies to mergers of firms of the same type in a Stackelberg market. Thus, a merger of leaders only is profitable if it involves N_L^c leaders, and a merger of followers only is profitable if it involves N_F^c followers.

ticipate.[4]

The gain of adding one follower to the merging entity exceeds the profit made by that follower prior to the merger. Thus, followers benefit from the merging leaders by allowing themselves to be “absorbed” by the merged entity. From Equation (4), we know the output of a leader does not depend on the number of followers. By bringing one more follower into the merger, the output expansion of followers is reduced (because fewer followers are available to expand their output following the merger), whereas the output expansion of each outsider leader is unchanged. Consequently, total expansion of output by outsiders is reduced, the price is higher, and the profits of the post-merger entity are larger. Considering that the profit made by the follower was initially small, the gain its participation provides is always sufficient to compensate for that lost profit.

We now consider the role of leaders. We start by examining mergers involving one leader and any number of followers. This type of merger is common, because a leader can acquire a small follower more easily than a leader of equal size. Huck *et al.* (2001) and Kabiraj, and Mukherjee (2003) find that a merger between a leader and a follower is always profitable. Proposition 2 generalizes this finding.

Proposition 2. A merger between one leader and any number of followers is always profitable.

This finding changes the conventional wisdom regarding merger profitability. With more than one leader present before the merger, the merger involving one leader and any number of followers does not satisfy the standard threshold for profitability. In this case, the participating leader represents less than 80% of leaders. Despite that number, such a merger is profitable because of the participation of the follower(s) to the merger.

We now turn to mergers involving two leaders and any number of followers. Theoretically, the study of such mergers is important, because one of the often cited results of the merger paradox literature is that bilateral mergers are unprofitable whenever outsiders are involved. Such study is important empirically because bilateral mergers between large firms are common, and these firms may want to further monopolize the

[4] All proofs are in the Appendix.

market by predating on smaller firms.

Proposition 3. For a merger involving two leaders to be profitable, it must involve at least N_F^{c2} followers, with

$$N_F^{c2}(L, F) = \frac{(F+1)(\sqrt{5L^2 - 8L - 4} - L)}{2L}. \tag{8}$$

N_F^{c2} increases linearly with F, and increases at a decreasing rate with L. The proportion of followers required for such a merger to be profitable, (N_F^{c2}/F), decreases with F, and increases with L.

If $L=2$, the condition is automatically satisfied because in this case, $N_F^{c2}<0$. Therefore, the merger is profitable. However, even if $L>2$, the merger will still be profitable if enough followers participate. As explained above, the participation of followers does not change the output of leaders, but increases the market price, making the merger more profitable. The critical number of followers required varies with the market structure. This value increases with F because more followers involved results in more outsiders who will expand their output after the merger. Consequently, more of those outsiders must be absorbed by the merger for it to be profitable. However, the proportion of followers required, (N_F^{c2}/F), decreases with F, indicating that when the number of followers is very large, a lower proportion of them is required for profitability. In addition, N_F^{c2} increases with L, as the larger the number of leaders outside the merger, the larger their output expansion is. As such, more followers are needed to participate to compensate for that effect. The proportion of followers increases with L, as N_F^{c2} increases with L. Hence, although the presence of more followers in the market increases the scope of profitable mergers, it also implies that a larger number of them (but a smaller proportion) need to join to make the merger profitable. Followers participating in the merger are a plus for the merging firms, whereas followers remaining outside the merger reduce the profitability of the merger.

Before characterizing how the range of profitable mergers looks in general, we must establish how a profitable Cournot merger (or, equivalently, a merger that would be profitable in a Stackelberg model, but in which no followers participate) is affected by the participation of followers. The following corollary, which follows from the results already derived, answers this question:

Corollary 1. A merger involving $N_L \geq N_L^c$ leaders is profitable irrespective of the number of followers participating in the merger.

The results derived until now indicate that followers have a direct bearing on the profitability of mergers involving leaders and followers. Followers are a desirable target for any solo leader. They increase the likelihood for a merger involving two leaders to be profitable. Moreover, they enhance the profitability of already profitable mergers.

We are now ready to characterize the profitability of mergers with an arbitrary number of leaders and followers. We consider how merger profitability depends explicitly on the number of followers. From this, we have the following result:

Proposition 4. A merger involving N_L leaders is profitable if it involves at least N_F^x followers, with N_F^x given by

$$N_F^x(L, F, N_L) = \frac{(F+1)[L - N_L(L - N_L + 3) + 2 + \sqrt{N_L - 1}\sqrt{L^2(N_L + 3) + N_L(N_L^2 - N_L - 4) - 2L(N_L^2 + N_L - 2)}]}{2(L - N_L + 2)}. \quad (9)$$

N_F^x increases with L and F.

Such a result occurs because more firms in the market result in more outsiders to the merger. Furthermore, to counter the market power and output expansion of outsiders, more followers need to join the merger to make it profitable.

We next consider how merger profitability depends on the number of participating leaders.

Corollary 2. If a merger is unprofitable, it must involve an intermediate number of leaders. In particular, it must involve $N_L \in \{N_L^-, \ldots, N_L^+\}$ leaders, with $N_L^-(L, F, N_F) > 1$ and $N_L^+(L, F, N_F) < N_L^c$.

As shown in the proof of Corollary 2 in the Appendix, pre-merger profits increase linearly with N_L, whereas post-merger profits are convex in N_L. Post-merger profits are convex in N_L, because a higher N_L indicates enhanced market power, fewer outsiders to expand output, and a higher price (as will be shown later). Considering that post-merger profits

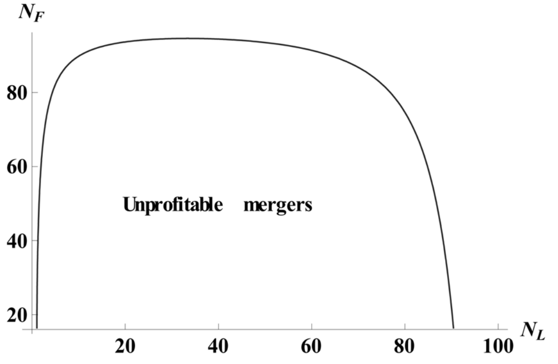


FIGURE 1
N_F^x AS A FUNCTION OF N_L (WITH $L = F = 100$)

are higher at $N_L = 1$ and at $N_L = L$, when the two curves (pre-merger and post-merger profits) meet, they must meet for two values of N_L, N_L^- and N_L^+. Thus, mergers are profitable for very low and very high N_L, but unprofitable for intermediate N_L (these relationships are also visible in Figure 5 (in Section IV.B below). The intuition for this result is that the profits of leaders are initially high. Therefore, a substantial degree of market power of the post-merger firm is required to cover their lost profits when several leaders join. When the number of leaders involved is intermediate and the number of followers is small, the market power of the post-merger firm is insufficient to compensate for the lost profits, and the merger is not profitable.

The relationship between N_F^x and N_L is illustrated in Figure 1. Profitable mergers are above the curve, whereas unprofitable mergers are below it. If N_L is very small, the merger is profitable with any strictly positive number of followers. If N_L is very large, the merger is profitable even without any followers. In between, N_F^x first increases, then decreases with N_L. The figure illustrates that unprofitable mergers occur for intermediate values of N_L.

As L or F increase, N_F^x increases (from Proposition 4 above). As this happens, N_L^- shifts left, and N_L^+ shifts right. Consequently, the range of unprofitable mergers increases. The observation from Figure 1 that N_L^- is non-decreasing and N_L^+ is non-increasing in N_F means that the participation of more followers to the merger reduces the number of leaders required for profitability. Moreover, the participation of followers reduces the number of leaders required for profitability, as opposed to the case where only leaders merge. Similarly, the participation of leaders reduces the number of followers required for profitability, as opposed to the case where only followers merge.

Taken together, Proposition 4 and Corollary 2 imply that unprofitable mergers occur for low values of N_F, combined with intermediate values of N_L. This result reverses one of the main findings of the merger paradox literature, which states that "if a merger by a specified number of firms causes losses (respectively, gains), a merger by a smaller (larger) number of firms will cause losses (gains)" (Salant *et al.* 1983, p. 193). As such, the solution to merger unprofitability was to enlarge the pool of participating firms. In a Stackelberg model, a merger may become unprofitable by adding more leaders, and may become profitable by removing leaders.

However, this monotonicity of merger profitability has been reversed with cases only concerning leaders. That is, our results imply that more leaders joining a merger does not necessarily enhance merger profitability. However, for followers, the traditional result holds. It is still true that more followers joining the merger always increases merger profitability.

The results derived clearly show that followers facilitate and enhance profitability. Now, we take a closer look at the market side of followers. In fact, for a certain number of followers to join a merger and make it profitable, the number of followers must be sufficient in the first place. At the same time, as explained in Proposition 3, the number of followers required to make a merger involving two leaders (this will also be true for any N_L) profitable increases with F.

The following two propositions clarify the role of F in merger profitability. We consider the profitability of mergers involving all followers, and investigate whether this is sufficient to ensure profitability. The following proposition gives a negative answer to this question.

Proposition 5. A critical number of followers must be present prior to the merger, F^c, such that, for a merger involving N_L leaders and all followers ($N_F=F$), the merger is profitable if and only if $F>F^c$.

Proposition 5 takes the extreme case of all followers joining the merger, and asks whether this is sufficient to ensure profitability. The answer depends on L and N_L. The critical threshold F^c provides the minimal number of followers such that it will become profitable if all followers join the merger.

Numerical simulations suggest that F^c increases with L, and increases,

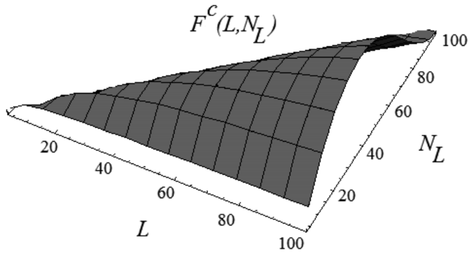
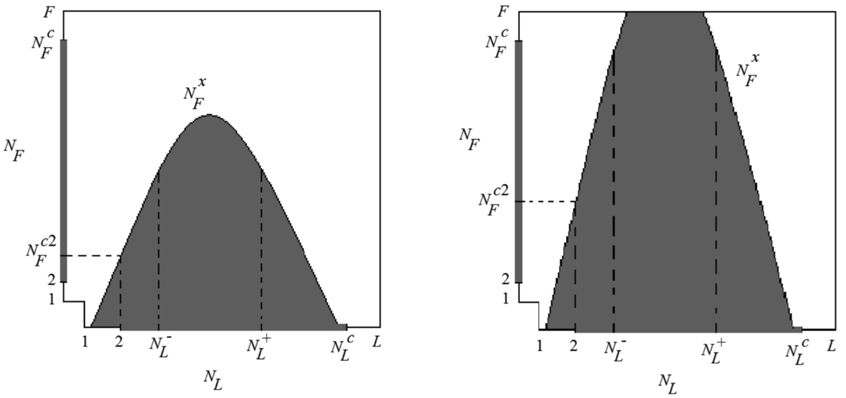


FIGURE 2
NUMBER OF FOLLOWERS REQUIRED IN THE MARKET WHEN $N_F = F$ ($L = 100$)

then decreases with N_L. The threshold increases with L, as the number of leaders increases relative to the number of followers. The likelihood increases such that even if all followers join, the merger will not be profitable (for a given N_L). Thus, a small number of followers reduces the probability of benefitting from them to ensure merger profitability. In addition, F^c increases, then decreases with N_L. As explained above, mergers are more likely to be profitable when a small or large number of leaders join in. When N_L is small, the merger is more likely to be profitable. As such, fewer followers are needed (assuming they all participate in the merger) to make the merger profitable. However, as N_L increases, the merger becomes unprofitable. As such, more followers must join the merger. Beyond a certain point, the joining leaders have less and less market power prior to the merger, and require a lesser increase in profits by the merged entity. Hence, the need for followers declines, and F^c declines with N_L. Figure 2 illustrates F^c as a function of L and N_L.

The inverted-U relationship between F^c and N_L is another facet of the result. When mergers are unprofitable, they are for intermediate numbers of leaders participating in the merger (from Proposition 4). F^c reaches its maximum for intermediate values of N_L, which means that for those values of N_L, the most followers are needed to make the merger profitable. In contrast, F^c is smaller at small and large values of N_L, meaning that for those values, only few followers are needed to make the merger profitable.

The following corollary takes this argument one step further, and asks the question: supposing all followers join the merger, when are enough of them around for a merger to become profitable, irrespective of the number of leaders joining in (at least one, to induce leadership)?



(a) $F>F^{cc}(L)$ (b) $F<F^{cc}(L)$

Note: Grey Areas Represent Unprofitable Mergers

FIGURE 3
PROFITABLE AND UNPROFITABLE MERGERS

Corollary 3. If $F>F^{cc}=\max_{N_L} F^c$, then a merger involving all followers is profitable for any number of leaders participating in the merger.

This corollary says that if F is sufficiently large relative to L, then a merger where all followers join will be profitable, irrespective of N_L. The critical value of F needed for this to occur is the maximum number of followers required by a specific number of leading firms, for a given L. If F is large enough for the merger to be profitable with this value of N_L, then it will also be large enough for the merger to be profitable for any other value of N_L. Thus, F^{cc} is obtained by maximizing F^c over N_L. If the merger is profitable under harsh conditions (N_L is such that a very large number of followers are needed), then it will be profitable under easier conditions (N_L requires fewer followers to join the merger to make it profitable). Moreover, numerical simulations indicate that F^{cc} is increasing in L.

Followers provide a double benefit with respect to merger profitability for leaders. On one hand, the standard threshold is unchanged from a Cournot model. Hence, even if they do not join a merger, they do not reduce the likelihood of its profitability. On the other hand, they can increase the likelihood of merger profitability by participating in it, and acting as “substitutes” for other leaders who have not joined.

Using the different thresholds derived above, Figure 3 illustrates the regions of profitable and unprofitable mergers in two cases: the case where $F > F^{cc}$, such that a merger involving all followers is profitable for any number of participating leaders (Figure 3a), and the case $F < F^{cc}$, such that even participation by all followers does not guarantee profitability (Figure 3b). These two figures incorporate mergers involving only leaders or only followers (on the axes), which must satisfy the Cournot threshold to be profitable.

B. Specific market structures

Now that we have analyzed some general properties of mergers in Stackelberg models, we turn to particular market structures to obtain results that are more specific. Three symmetric and two asymmetric market structures are considered: $(L=2, F=2)$, $(L=3, F=3)$, $(L=10, F=10)$, $(L=10, F=3)$, and $(L=3, F=10)$. These choices represent sufficient variety in terms of degree of concentration and asymmetry between leaders and followers to illustrate the results derived above.

We first consider the market structures $(L=2, F=2)$ and $(L=3, F=3)$. In the $(L=2, F=2)$ market, four mergers (involving at least one leader and one follower) are possible: $(N_L=1, N_F=1)$, $(N_L=1, N_F=2)$, $(N_L=2, N_F=1)$, and $(N_L=2, N_F=2)$. In the $(L=3, F=3)$ market, 9 mergers are possible.

Proposition 6. With $(L=2, F=2)$ or $(L=3, F=3)$, all mergers involving at least one leader and one follower, with the merged entity acting as a leader, are profitable.

In the $(L=2, F=2)$ market, all possible mergers are profitable. Except for the merger $(N_L=2, N_F=2)$, all other three mergers entail only 50% of leaders and/or 50% of followers are engaged in the merger. This result is well below the 80% threshold required when only leaders or only followers merge together (see Proposition 8 in the Appendix). Yet, these mergers are profitable, because they entail the participation of firms with different market positions. For example, the merger $(N_L=1, N_F=1)$ is profitable even though it involves only one follower.

The other market structure considered in Proposition 6 is $(L=3, F=3)$. Nine possible mergers are found, and all of them are profitable. For instance, the merger $(N_L=2, N_F=1)$ is between firms that have a market share well below what is required when only leaders are involved. Yet, it is profitable because one follower joined. This joining of the follower

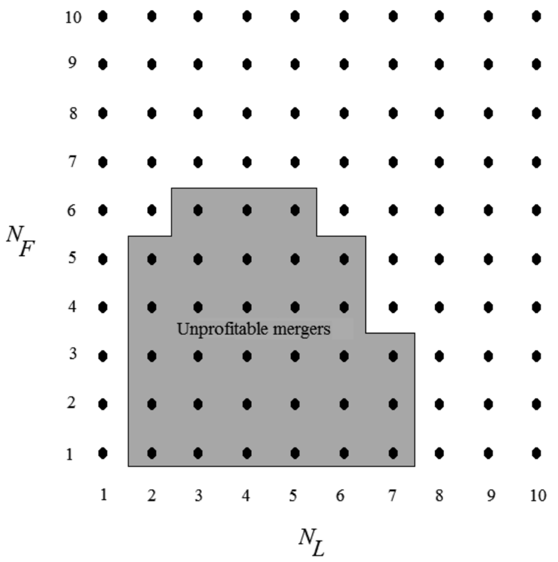


FIGURE 4
MERGER PROFITABILITY WITH $L=F=10$

does not change the output of the merging entity (see Expression 4), but sufficiently increases the price (since that follower disappears) to make the merger profitable in spite of the output expansion by outsiders. In this regard, the follower acts as a "substitute" for the third leader who is not part of this merger.

We then consider the market structure ($L=10, F=10$). Based on Equation (7), a merger of N_L and N_F firms is profitable if and only if

$$\frac{1}{(11-N_F)(12-N_L)^2}-\frac{11N_L+N_F}{14641}>0. \tag{10}$$

A hundred possible mergers are found in this market. Based on Equation (10), Figure 4 illustrates the possible mergers and their profitability. When $N_F \le 6$, a merger is profitable if and only if N_L is very small or very large. When $N_F>6$, all mergers involving at least one leader are profitable. This is consistent with Proposition 4, which establishes that mergers can be unprofitable for intermediate numbers of participating leaders.

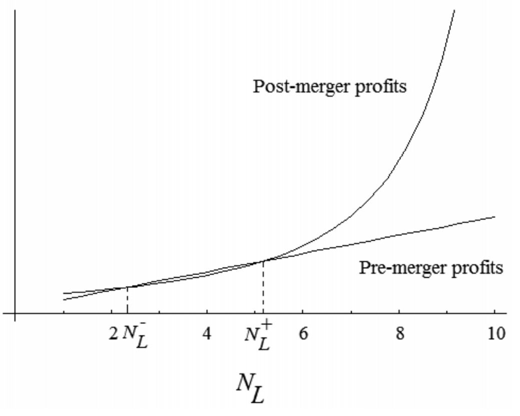


FIGURE 5
PRE AND POST-MERGER PROFITS ($L=F=10$, $N_F=6$)

To make the profit comparisons behind Figure 4 more precise, we consider the row of Figure 4 with $N_F=6$. This row indicates that a merger involving 6 followers is profitable if and only if $N_L \in \{1, 2, 6, 7, 8, 9, 10\}$. With $L=F=10$ and $N_F=6$, a merger involving N_L leaders is profitable if and only if

$$\pi_L(10-N_L+1,10-6)-N_L\pi_L(10,10)-6\pi_F(10, 10)=\frac{1}{5(12-N_L)^2}-\frac{11N_L+6}{14641}>0. \quad (11)$$

This expression has two roots: $N_L=2.3$, and $N_L=5.2$. Hence, the merger is not profitable for $N_L \in \{3, 4, 5\}$, but is profitable otherwise.

To better understand this result, Figure 5 illustrates the pre-merger and post-merger profits as a function of N_L with $N_F=6$. When $N_L=1$, the merger is highly profitable. When $N_L=2$, the merger is still profitable, but only marginally. When the second leader joins in, its contribution to the profits of the merged firms is less than the profits sacrificed when that firm was not merged (this is reflected by the slope of the curve, which is lower than the slope of the straight line at that point). However, because the merger profits were so high at $N_L=1$, the resulting merger profits at $N_L=2$ are still higher than without the merger. Nevertheless, when the third and subsequent leaders join, the merger is not profitable anymore. Only when $N_L \geq 6$ does the merger become profitable again. Thus, a discontinuity exists in the relationship between the number of leaders joining the merger and its profitability.

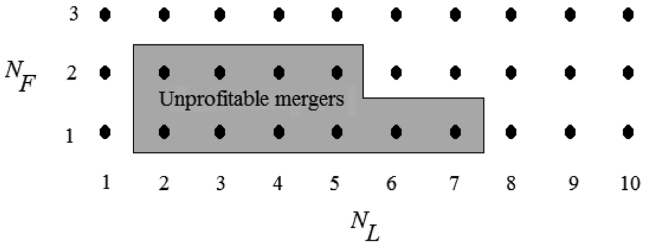


FIGURE 6
MERGER PROFITABILITY WITH $L=10$, $F=3$

We then consider the asymmetric market structure ($L=10, F=3$). Figure 6 illustrates the results. The range of profitable mergers is qualitatively similar to the range obtained from the previous market structure ($L=10, F=10$). With ($L=10, F=3$), if all followers participate, then the merger is profitable for any number of participating leaders. If at least one follower does not participate, then the merger is profitable if and only if N_L is very small ($N_L=1$) or relatively large, with a substitution between leaders and followers. The more followers participate, the fewer leaders are required to do so.

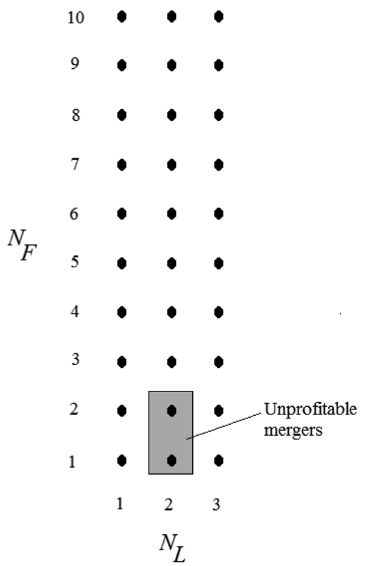


FIGURE 7
MERGER PROFITABILITY WITH $L=3$, $F=10$

The last market structure considered, $(L=3, F=10)$, yields a similar result. As seen in Figure 7, the only unprofitable mergers are those with $(N_L=2, N_F\leq 2)$. Again, an intermediate number of leaders and a small number of followers yield an unprofitable merger. Given the small number of leaders, the scope for substitution between leaders and followers is not present.

The comparison between Figures 6 and 7 reveals that the range of unprofitable mergers is larger when L is large relative to F. This is because a large L expands the range of "intermediate" values of N_L, for which mergers are less likely to be profitable. Therefore, more merger unprofitability can be found in markets where the number of leaders is large relative to the number of followers.

The study of these specific market structures depicts the results derived above for more general market structures. For example, Figures 4, 6, and 7 are illustrations of the case depicted in Figure 3a, where the participation of all followers guarantees merger profitability irrespective of N_L.

C. Welfare

What is the effect of mergers involving at least one leader and one follower on welfare? Such mergers increase the degree of concentration, and, not surprisingly, are welfare-reducing. Any merger increases price. To see this, we substitute equilibrium outputs from Equation (4) into the demand function to get the equilibrium price:

$$\begin{aligned} P^*(L,F) &= 1 - Ly_L^*(L) - Fy_F^*(L,F) \\ &= 1 - L\frac{1}{L+1} - F\frac{1}{(L+1)(F+1)} \\ &= \frac{1}{(L+1)(F+1)}. \end{aligned} \tag{12}$$

This equation clearly indicates that price decreases with the number of firms. Hence, any merger will hurt consumers.

We define consumer surplus as

$$CS(L,F) = \frac{Y(L,F)^2}{2}. \tag{13}$$

We then define welfare as a function of the number of leaders and followers as

$$W(L, F)=CS+L\pi_L+F\pi_F. \tag{14}$$

The next proposition formalizes the effect of mergers on consumers, outsiders, and total welfare.

Proposition 7. Any merger involving at least one leader and one follower, with the merged entity acting as a leader, reduces consumer surplus and welfare, and increases total profits and the profits of each outsider.

Considering that the merger entails an increase in concentration, adding to the fact that firms are symmetric, such a merger always benefits outsiders, even though it may or may not benefit the merging firms. In addition, it reduces consumer surplus and welfare.[5] Thus, the vigilance of antitrust authorities need not be relaxed because a merger involves a mixture of leader and follower firms.

V. Conclusions

This paper has extended the analysis of the profitability of mergers to Stackelberg models. Followers alleviate the merger paradox in that they increase the range of profitable mergers for leaders. Unprofitable mergers are typically those involving an intermediate number of leaders and a small number of followers. The reason is that with this configuration, the merged entity does not possess enough market power (because many leaders remain) to compensate for the lost profits of the participating leaders. In addition, the small number of followers makes their contribution to the profitability of the merger insufficient to compensate for that disadvantage. Pre-merger profits increase linearly with the number of participating leaders, whereas post-merger profits are convex in their number. Given that mergers involving one or all leaders are always profitable, this creates a situation in which if a merger is unprofitable, this can only be the case of an intermediate number of participating leaders. This implies that merger profitability is monotonic in the number

[5] Experimental evidence suggests mergers in Stackelberg models benefit outsiders and reduce consumer surplus, as predicted by theory, buy they do not change the profits of the merging firms, contrary to theoretical predictions. See Huck (2009).

of followers, but not in the number of leaders. Notably, the participation of followers in the merger, rather than their presence per se, makes mergers profitable more often. Hence, it is not really Stackelberg leadership, but the absorption of followers that enhances merger profitability. All mergers involving at least one leader and one follower are price-increasing and welfare-reducing.

When a merger between a number of leaders is unprofitable, it may be easier to bring in one or more followers than to bring in additional leaders, as followers are smaller in size. Yet, despite their small size, they may be sufficient to make the merger profitable. The model can also be interpreted as saying that despite their size, small firms can make mergers between larger firms profitable.

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Appendix

Mergers of leaders only or followers only

To better situate our results relative to the literature, we consider mergers involving leaders only or followers only in this section. Such mergers have been studied by Hamada, and Takarada (2007), and thus the results of this section are not new to the literature. Considering a merger involving only leaders, we let $N_L \in \{2, \ldots, L\}$ leaders merge and form one firm. The number of leaders becomes $L - N_L + 1$, whereas the number of followers is unchanged at F. Substituting these values into Equation (5) yields per firm post-merger profits:

$$\begin{aligned} \pi_L(L-N_L+1, F) &= \frac{1}{(L-N_L+2)^2(F+1)}, \\ \pi_F(L-N_L+1, F) &= \frac{1}{(L-N_L+2)^2(F+1)^2}. \end{aligned} \tag{15}$$

This merger is profitable if and only if the profits made by the merged firms after the merger exceed the profits they made before the merger:

$$\pi_L(L-N_L+1, F) > N_L \pi_L(L, F). \tag{16}$$

Substituting Equation (5) and Equation (15) into Equation (16) yields

$$\frac{1}{(L-N_L+2)^2(F+1)} > N_L \frac{1}{(L+1)^2(F+1)}. \tag{17}$$

Solving Equation (17) for N_L shows Equation (17) is satisfied if and only if[6]

$$N_L > N_L^c(L) = \frac{2L+3-\sqrt{4L+5}}{2}, \tag{18}$$

where N_L^c denotes the critical threshold for profitability. N_L^c does not depend on F. Moreover, this threshold is the same critical threshold for the profitability of a merger in a Cournot model. Therefore, the presence of followers does not alleviate the merger paradox. A substantial proportion (at least 80%) of leaders still needs to merge to ensure profitability. Given that the merger entails an expansion of the output of the outsider leaders, the number of merging leaders must be sufficiently high for this expansion not to hurt the merged entity considerably. Moreover, even though followers also expand their output following the merger by leaders (this is clear from expression 4), this does not change N_L^c relative to a Cournot model.

We next analyze the profitability of a merger involving only followers. Let $N_F \in \{2, \ldots, F\}$ be the number of merging followers. After the merger L leaders and $F-N_F+1$ followers exist. Note that contrary to Daughety (1990), we do not assume that the merged entity becomes a leader. From Equation (5) the profits following such a merger are given by

$$\begin{aligned} \pi_L(L, F-N_F+1) &= \frac{1}{(L+1)^2(F-N_F+2)}, \\ \pi_F(L, F-N_F+1) &= \frac{1}{(L+1)^2(F-N_F+2)^2}. \end{aligned} \tag{19}$$

This merger is profitable if and only if

$$\pi_F(L, F-N_F+1) > N_F\, \pi_F(L, F). \tag{20}$$

[6] Equating the two sides of Equation (17) yields three roots, only one of which is valid, *i.e.* is comprised between 2 and L.

By using Equation (5) and Equation (15) and the same method as above, this condition is satisfied if and only if

$$N_F > N_F^c(F) = \frac{2F + 3 - \sqrt{4F + 5}}{2}. \tag{21}$$

From Equation (18) and Equation (21), we see a perfect symmetry between the profitability (or not) of a merger involving only leaders and a merger involving only followers. The same proportion of firms is required in each case. From Equation (4) we know that the output of leaders is independent of F, hence the only effect of the merger by followers is to induce an expansion of the output of the outsiders.[7]

The result of this subsection can be summarized as follows.

Proposition 8. In a Stackelberg market, a merger between N_L leaders is profitable if and only if $N_L > N_L^c$. A merger between N_F followers is profitable if and only if $N_F > N_F^c$. Moreover, $N_L^c = f(L)$, $N_F^c = f(F)$, $N^c = f(N)$, where N^c is the critical threshold for profitability of a merger in a Cournot market, given by

$$N^c(N) = f(N) = \frac{2N + 3 - \sqrt{4N + 5}}{2}. \tag{22}$$

N is the number of firms in the Cournot market, and f is the common functional form defining the critical number of firms.[8]

This result indicates that if the followers (leaders) do not participate in a merger, they have no effect on the profitability of a merger involving only leaders (followers). A similar result was derived by Hamada, and Takarada (2007). This result constitutes a generalization of the results of Huck *et al.* (2001) and Kabiraj, and Mukherjee (2003) who show that the two leaders (followers) have an incentive to merge if and only if no other leaders (followers) are present.

Interestingly, even though the critical number of firms is similar to the standard Cournot model, the results do not necessarily go in the direction of the merger paradox. For example, in a market composed of two leaders and a large number of followers, a merger of the two lead-

[7] Here, "outsiders" refers to followers not included in the merger.

[8] See Salant *et al.* (1983), footnote 6.

ers is always profitable, yet their market share may be quite small.[9]

Proof of proposition **1.**

$$\frac{\partial \pi_L(L-N_L+1, F-N_F)}{\partial N_F} = \frac{1}{(L-N_L+2)^2(F-N_F+1)^2} > \frac{\partial [N_L\, \pi_L(L, F) + N_F\, \pi_F(L, F)]}{\partial N_F} = \frac{1}{(L+1)^2(F+1)^2}. \quad (23)$$

Proof of proposition **2.** Evaluating Equation (7) at $N_L=1$ yields

$$\pi_L(L-1+1, F-N_F) - \pi_L(L, F) - N_F\, \pi_F(L, F) = \frac{{N_F}^2}{(L+1)^2(F+1)^2(F+1-N_F)} > 0. \quad (24)$$

Proof of proposition **3.** Substituting $N_L=2$ into Equation (7), setting it equal to zero, and solving for N_F yields Equation (8).

$$\frac{\partial N_F^{c2}}{\partial F} = \frac{\sqrt{5L^2-8L-4}-L}{2L} > 0. \quad (25)$$

This last derivative is independent of F, implying that N_F^{c2} increases linearly with F.

$$\frac{\partial N_F^{c2}}{\partial L} = \frac{2(L+1)(F+1)}{\sqrt{5L^2-8L-4}L^2} > 0. \quad (26)$$

$$\frac{\partial^2 N_F^{c2}}{\partial L^2} = -\frac{2(F+1)(10L^3+3L^2-24L-8)}{(\sqrt{5L^2-8L-4}L)^3} < 0. \quad (27)$$

$$\frac{\partial \frac{N_F^{c2}}{F}}{\partial F} = \frac{L-\sqrt{5L^2-8L-4}}{2LF^2} < 0. \quad (28)$$

Given that N_F^{c2} is increasing in L, the proportion $(N_F^{c2})/F$ is also increasing in L.

[9] I thank an anonymous referee for pointing out this aspect.

***Proof of corollary* 1.** From proposition 8 we know that a merger involving leaders only is profitable if and only if $N_L \geq N_L^c$. Moreover, from proposition 1 we know that adding a follower to a merger increases its profitability.

***Proof of proposition* 4**. The l.h.s. of Equation (7) is given by the first part of Equation (6). The r.h.s. of Equation (7) can be calculated as

$$N_L\, \pi_L(L, F) + N_F\, \pi_F(L, F) = \frac{N_L + N_F + FN_L}{[(L+1)(F+1)]^2}. \tag{29}$$

Equating the first part of Equation (6) with Equation (29) and solving for N_F yields two roots, only one of which is positive. This positive root we call N_F^x as given by Equation (9). To see how N_F^x varies with L, we differentiate Equation (9) w.r.t. L, which yields

$$\frac{(\partial N_F^x)}{\partial L} = \frac{2(L+1)(F+1)\sqrt{N_L - 1}}{(L - N_L + 2)^2 \sqrt{L^2(N_L + 3) + N_L(N_L^2 - N_L - 4) - 2L(N_L^2 + N_L - 2)}} > 0\ . \tag{30}$$

The effect of F on N_F^x is given by

$$\frac{(\partial N_F^x)}{\partial F} = \frac{L - N_L(L - N_L + 3) + 2 + \sqrt{N_L - 1}\sqrt{L^2(N_L + 3) + N_L(N_L^2 - N_L - 4) - 2L(N_L^2 + N_L - 2)}}{2(L - N_L + 2)} > 0. \tag{31}$$

Equation (31) is positive given that its numerator is the second term of the numerator of N_F^x, which is positive.

***Proof of corollary* 2.** We know that a merger (with a strictly positive number of participating followers) is profitable when exactly one leader participates (proposition 2) or when all leaders participate (Corollary 1) in it. Thus, if a merger is unprofitable, for a given N_F, it must involve an intermediate number of participating leaders. From Equation (6) and Equation (7) we know that the partial derivative of pre-merger profits w.r.t. N_L is equal to $\pi_L(L, F)$. This derivate does not depend on N_L, hence pre-merger profits linearly increase with N_L. By contrast, the partial derivative of post-merger profits w.r.t. N_L is given by $(\partial\pi_L(L-N_L+1, F-N_F))/(\partial N_L)=2/((L-N_L+2)^3(F-N_F+1))$; this expression is positive and increasing in N_L, hence post-merger profits are convex in N_L. Thus, if the linear (pre-merger) and convex (post-merger) curves are to meet, given that the convex curve is higher for $N_L=1$ and $N_L=L$, they will have to meet at two points. We let N_L^- denote the smaller of these values, and N_L^+ denote the larger value. These critical values delimit the range of unprofitable mergers.[10] Proposition 2 establishes that $N_L^->1$, whereas Corollary 1 establishes that $N_L^+<N_L^c$.

***Proof of proposition* 5.** Setting $N_F=F$ in Equation (7) we can solve for F^c:

$$F^c(L, N_L)=\frac{N_L^3-N_L^2(2L+3)+N_LL(L+2)-L^2+2+(L-N_L+2)\sqrt{(N_L-1)[L^2(N_L+3)-2L(N_L^2+N_L-2)+N_L(N_L^2-N_L-4)]}}{2(L+1)^2}. \tag{32}$$

***Proof of corollary* 3.** This follows from proposition 5.

***Proof of proposition* 6.** Substituting $L=F=2$ into Equation (7) yields

$$\frac{1}{(3-N_F)(4-N_L)^2}-\frac{3N_L+N_F}{81}>0. \tag{33}$$

This expression is positive for all N_L, $N_F\in\{1, 2\}$.
Substituting $L=F=3$ into Equation (7) yields

$$\frac{1}{(4-N_F)(5-N_L)^2}-\frac{4N_L+N_F}{256}>0. \tag{34}$$

This expression is positive for all N_L, $N_F\in\{1, 2, 3\}$.

[10] When, for a given N_F, the two curves do not meet, no unprofitable mergers exist. In addition, a specific value of N_F, which makes post-merger profits tangent to pre-merger profits, exists; in this case the curves meet only once, and the merger is unprofitable for a unique value of N_L, that is, $N_L^-=N_L^+$. However, in most cases this unique value of N_L is unlikely to be an integer; hence this tangency is of little interest.

***Proof of proposition* 7.** The change in consumer surplus is given by

$$CS(L-N_L+1, F-N_F)-CS(L, F)= \frac{1}{2}\left[\left(\frac{L-N_L+1+(F-N_F)(L-N_L+2)}{(L-N_L+2)(F-N_F+1)}\right)^2-\left(\frac{L+F+LF}{(L+1)(F+1)}\right)^2\right]<0. \tag{35}$$

This expression is negative for all L, $F>0$, $N_L \in \{1, \ldots, L\}$, $N_F \in \{1, \ldots, F\}$. The change in the profit of an outsider leader (if any) is given by

$$\pi_L(L-N_L+1, F-N_F)-\pi_L(L, F)=\frac{1}{(L-N_L+2)^2(F-N_F+1)} -\frac{1}{(L+1)^2(F+1)}>0. \tag{36}$$

This expression is positive for all L, $F>0$, $N_L \in \{1, \ldots, L-1\}$, $N_F \in \{1, \ldots, F\}$.

The change in the profits of a follower is

$$\pi_F(L-N_L+1, F-N_F)-\pi_F(L, F)=\frac{1}{(L-N_L+2)^2(F-N_F+1)^2} -\frac{1}{(L+1)^2(F+1)^2}>0. \tag{37}$$

This expression is positive for all L, $F>0$, $N_L \in \{1, \ldots, L\}$, $N_F \in \{1, \ldots, F-1\}$.

The change in total profits is given by

$$(L-N_L+1)\pi_L(L-N_L+1, F-N_F)+(F-N_F)\pi_F(L-N_L+1, F-N_F)-L\pi_L(L, F)-F\pi_F(L, F)= \frac{L-N_L+1}{(L-N_L+2)^2(F-N_F+1)}+\frac{F-N_F}{(L-N_L+2)^2(F-N_F+1)^2}-\frac{L}{(L+1)^2(F+1)} -\frac{F}{(L+1)^2(F+1)^2}>0. \tag{38}$$

This expression is positive for all L, $F>0$, $N_L \in \{1, \ldots, L\}$, $N_F \in \{1, \ldots, F\}$. The change in welfare is given by

$$W(L-N_L+1, F-N_F)-W(L, F)=CS(L-N_L+1, F-N_F)-CS(L, F) \\ +(L-N_L+1)\pi_L(L-N_L+1, F-N_F)-L\pi_L(L, F) \\ +(F-N_F)\pi_F(L-N_L+1, F-N_F)-F\pi_F(L, F)<0. \quad (39)$$

Substituting consumer surplus and profits, verifying that Equation (39) is negative for all $N_L \in \{1, ..., L\}$, $N_F \in \{1, ..., F\}$ is easy.

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