

An Escape from the “Malthusian Trap”: A Case of the *Chosŏn* Dynasty of Korea from 1701 to 1891 Viewed in Light of the British Industrial Revolution

Donghyu Yang and Shin-Haing Kim *

Population and wage records from the *Chosŏn* Dynasty of Korea from 18th to 19th centuries are compared with Wrigley–Schofield’s wage–population dynamics of Britain during the industrial revolution. A one-sector agrarian Cobb–Douglas economy is proposed for a possible comparison of the two economies. Natural resources such as land are the limiting factors of production that binds an economy in the Malthusian trap. This paper argues that population size is critical for an economy to break through the Malthusian trap. The success of Britain in escaping from the Malthusian trap during the industrial revolution is examined in terms of the productivity of the agricultural sector. A self-generating population equilibrium model for the transitional phase of the post-Malthusian economy is proposed. No comparable productivity improvements in the agricultural sector of *Chosŏn* were observed during the corresponding periods of the British industrial revolution. No substantial changes in the population and wage rates occurred for the *Chosŏn* Dynasty in the 18th to 19th centuries. Essentially, *Chosŏn* remained under the typical Malthusian stationary state.

Keywords: Malthusian trap, *Chosŏn* economy, Britain’s industrial revolution,

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* Professor Emeritus of Economics, School of Economics, Seoul National University, Seoul 151-742, Korea. (E-mail) shk@plaza.snu.ac.kr; Professor of Economics, School of Economics, Seoul National University, Seoul 151-742, Korea. (E-mail) dyang@snu.ac.kr, respectively. We appreciate the comments of the anonymous referee on the original version of this paper. The comments helped improve this paper substantially. We also appreciate the comments made by the members of the Economic History Workshop of Seoul National University. [**Seoul Journal of Economics** 2013, Vol. 26, No. 2]

I. Introduction

The success story of the Korean economy for the past several decades is widely known. The country's rapid growth in terms of exports and per capita income has generated worldwide attention. As demonstrated by the growth experiences of the country, the Korean economy remained under the traditional stationary state from the beginning of the 18th century up to the end of the 19th century when the country was annexed by Japan.

During this stagnating period in the Korean economy, an industrial revolution was spreading across Britain. The two economies during the said period are comparable with respect to the escape from a "Malthusian trap." The British industrial revolution in the 18th and 19th centuries prompted the country's escape from the Malthusian trap. By contrast, the Korean economy failed to escape from the Malthusian population check during the same period.

A recent quantitative approach to the analysis of the *Chosŏn* economy during late 19th century (Rhee *et al.* 2004) provides a way to compare the economies of Korea and Britain from a historical perspective. This paper compares the two economies during the 18th and 19th centuries in terms of population and wage rate dynamics. A comparison of the different features of the two economies is aimed at shedding light on the factors that freed the then British economy from the Malthusian constraint. This attempt is inspired by Wrigley–Schofield's (1993) population–wage dynamics on the British industrial revolution, which serves as basis for discussion in this paper.

As estimated by Kwon and Shin (1977), the *Chosŏn* population was about 2.8 times that of Britain at the beginning of the 18th century. *Chosŏn*'s population growth rate over the period up to the end of the 19th century was almost zero. As a result, the population remained as low as three-quarters of that of Britain by the end of 1871. The population size of the two countries then switched at the middle of the 19th century.

The demographical change in the two economies during the British industrial revolution stands out as a salient feature that distinguishes their economic performances thereafter. What are the contributing factors for the demographic change in Britain? Why was a comparable change absent in the demography of *Chosŏn*? This paper addresses the productivity of the agricultural sector related to the demographical change.

Breaking through the Malthusian trap was possible in the case of Britain because of the high productivity of its agricultural sector. No comparable productivity increase was observed for *Chosŏn* during the same period.

This paper also illustrates that increases in both the productivity of the agricultural sector and in the per capita productivity of agricultural workers during the British industrial revolution triggered a break through from the Malthusian bound. The effect of this phenomenon on the population was obvious through the indirect effects on wage rates.

A unified growth model recently forwarded by Galor-Weil (2000) integrates a demographical change into a neo-classical growth model. It pulls the modern growth regime of the Solow growth model back to the Malthusian stagnation state. A demographical change associated with each developmental stage of an economy transitioning from the Malthusian stationary state to a modern growth regime unifies two regimes of the economy. This unification suggests that population size is critical to the demographic change associated with the formation of human capital.

A household decision on investment for the education of the next generation occurs at the developmental stage in which the population size reaches its critical level. This paper addresses the population equilibrium during the post-Malthusian regime, which refers to the aftermath of the break through of the Malthusian trap. The self-generating and sustainable population equilibrium of the present model is obvious during the developmental stage in which the income effect on a household decision remains a dominant feature.

Section 1 contrasts Wrigley and Schofield's data on the population and real wage rates of Britain with those of *Chosŏn* in reference to the Malthusian bound. In this section, we take the averages of the population and those of the wage rates during the industrial revolution when the two economies were respectively under a Malthusian stationary state. Next, we obtain the standard deviations of both the population and the real wage rates of the two economies from 1701 to 1871. We then compare the ratios of the standard deviations of the wage rates with those of the population of the two economies. The ratio for Britain is three times as high as that for *Chosŏn*. This section further demonstrates the escape of Britain from the Malthusian trap and the simultaneous sinking of *Chosŏn* into the Malthusian stationary state.

Section 2 examines the causes of the escape of the British economy from the Malthusian trap. This breakthrough is largely attributed to the improvement in the productivity of the country's agricultural sector. Evidently, the race between babies and land was won by land over

population during the British industrial revolution, as discussed in this section. In addition, this section presents the conditions of the break through of the Malthusian trap under a one-sector agrarian Cobb–Douglas economy.

Section 3 presents the sustainable population equilibrium. The Granger causality tests are performed among the population, wage rates, and agricultural productivity improvements in the British economy during the industrial revolution. For this purpose, Clark's (2002a, 2002b) time series data on the British agricultural productivity are overlaid on Wrigley–Schofield's population data. As expected, productivity Granger causes population and wage rates. However, the causality test of wage rates for the population fails, whereas population Granger causes productivity, which indirectly causes wage rates and population. These causality tests are supportive of a self-generating circular loophole in demographical change.

Section 4 concludes the paper.

II. A Malthusian Bound of Wages and Population¹

Malthus's "Essay on Population" suggests that a biological growth of population at an exponential rate is limited by the natural amount of land and its fertility. In the context of the modern economic theory, the law of diminishing returns is applied to agricultural labor. One's living standard in an economy is determined by an endowment of an arable land and by one's productivity. In an economy well-endowed with fertile land, an individual's standard of living is well above that of another who lives in an economy poorly endowed with land. Labor productivity also accounts for a person's living standards and balances the downward effects of the diminishing returns of agricultural labor on land. The final level of one's living standard is a race between productivity and land availability. Productivity exceeding the limit imposed by land availability improves one's living standards. Living standards fall when productivity cannot keep up with the diminishing returns of labor.

In this case, wage rate falls to a Malthusian subsistence rate in which the energy of a person allotted for labor is barely enough for this person to continue to work. Population fluctuates around this subsistence rate. If population size is above the level wherein it can be sustained by the produce of land, wage rate falls below the subsistence level. The mortality

¹ In this paper, wages refer to real wages in terms of agricultural food prices.

rate increases because of malnutrition or poor sanitary and health conditions. Birth rate also falls, given the deliberate choice of families to limit their size or to postpone marriages. This phenomenon is the Malthusian form of a "preventive check" on population. Uncontrollable factors such as wars, famines, and epidemics put a "positive check" on population growth. If the produce of lands can feed more than the existing population, then population increases. Unless counterbalanced by the increase in productivity, wage rate falls, followed by the decrease in population. These decreases in turn lead to the increase in wage rate. At this point, wage rate returns to its original state. Thus, the population size and wage rate of an economy remain within a given boundary, which we hereafter refer to as a "Malthusian bound of wages and population."

We denote the Malthusian subsistence wage rate in terms of agricultural food prices as \underline{w} and the stationary population level corresponding to the subsistence wage rate as \underline{L} . A dot over a variable indicates the rate of change of the variable. Population increases in an economy wherein the current wage rate w_t is above the subsistence wage rate \underline{w} . Conversely, population decreases in an economy wherein the wage rate lies below the subsistence level.

A coefficient $0 < a < 1$ represents a dynamic adjustment coefficient for the change in population. Equation (1) expresses this coefficient in a deviation form of the natural log of the real wage rate from its stationary wage rate:

(1)

where $\ln \underline{w}$ is the natural log of the stationary wage rate. Likewise, the rate of change in wage rate is expressed by the discrepancy between the natural log of the current population and its stationary state level:

(2)

where $\ln \underline{L}$ is a natural log of the stationary population.

A Malthusian stationary state lies on the center S_0 . It is bounded by the intervals of

$$\ln w^L \leq \ln w_t \leq \ln w^U \quad \text{and} \quad \ln L^L \leq \ln L_t \leq \ln L^U.$$

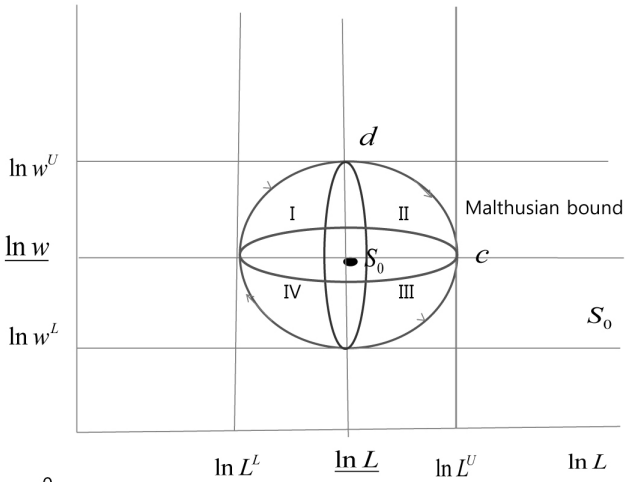


FIGURE 1

A Malthusian bound OF WAGES AND POPULATION

Suppose that a farmer achieves a good harvest year because of good weather conditions. The wage rate must rise above the subsistence level w . Population increases in Equation (1). In Equation (2), wage rate falls because the population is above its stationary level. We denote the upper bound of the wage rate as w^U .² The increase in population continues as long as the wage rate is above its subsistence level. We denote the upper bound of the population as L^U . The decrease in wage rate is followed by the decrease in population because wage rate is below the subsistence rate. The wage rate is further pulled down until the population returns to its stationary level. We denote the lower bound of wage rate as w^L . Upon reaching the lower bound, the wage rate begins to rise again because the population lies below its stationary level L .

As long as the wage rate lies below its subsistence level, the population continues to decrease. Population is reduced to such an extent that it starts to rise again, as the wage rate is above its subsistence level. We denote the lower bound of the population as L^L . Hereafter follows the increases in wage rate and population. The wage rate increases and returns to its upper bound w^U . The same explanation is given for the

² Hereafter, we refer to the variables of population and wage rates in terms of their natural logarithms unless they are confusing not to clarify.

changes in wage rates and population after the former reaches its upper bound.

Various paths of the population and wage dynamics are generated by the size of the coefficients a and b , as expressed in Equation (1) in Appendix 2. A Malthusian stationary state of wage rate \underline{w} and population size \underline{L} exists for the population and wage rate dynamics of Equations (1) and (2). Appendix 1 shows the proof of its existence.¹

Figure 1 shows the phase diagram of the changes in population and wage rate. The horizontal axis represents the natural log of population size, and the vertical axis represents the natural log of wage rate. The upper and lower bounds of the wage rate and population size divide an economy into four quadrants (Figure 1). Quadrant I in Figure 1 characterizes an economy as one wherein the wage rate is above the subsistence level of \underline{w} , and a positive relation exists between wage rate and population. When the wage rate reaches its upper bound w^U , population increases, followed by a decrease in real wage rate. A negative relation exists between the population and wage rate in quadrant II. From its upper bound L^L , population size begins to decline. The real wage rate and population both decline in quadrant III. The real wage rate begins to increase upon reaching its lower bound w^L in quadrant IV.

The upper and lower bounds of wage rate and population (Figure 1) set up a boundary within which both of the two variables vary. A phase diagram of this variation can be represented by a circle-ellipse or the shape in between of the circle and of the ellipse. No definitive pattern is explicit. The time series observations for an economy with a higher variation on wage rate would be situated within an ellipse, the shape of which is biased to the vertical axis. Otherwise, the observations would be found to be biased to the population axis. In this case, representing the scattered historical observations for the population and wage rates over the given period in terms of the ratio of the variances of the two factors is useful.

As shown below, we define a Malthusian bound as the minimum ratio of the variances of the wage rates to the variances of the population that is necessary in sustaining population growth:

A break through of the Malthusian bound occurs when the ratio of the variances of the wage rate is above the minimum.

Based on Taylor's approximation of Equations (1) and (2), the following equation measures the Malthusian bound:

where coefficient \bar{b} represents the degree of response of the wage rate to the increase in population under a Malthusian stationary state. Conversely, coefficient \bar{a} represents the degree of response of the population to the increase in wage rate under the stationary state.

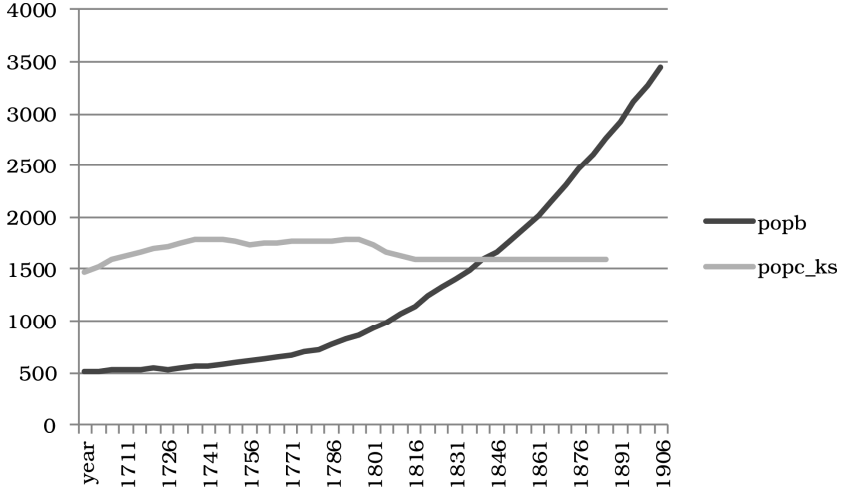
Random events such as wars and epidemics increase population dispersion, whereas events related to productivity and technology affect the variances of wage rates. At a high coefficient ratio (\bar{b}/\bar{a}), the shape of the ellipse of an economy is biased to the wage rate axis. A dotted ellipse in Figure 1 represents a bias toward the wage axis. A bold ellipse represents an economy whose Malthusian bound is less steep.

Next, we compare the performances of the British and *Chosŏn* economies in terms of the changes in their population and wage rates. Figure 1 displays the trends in population and wage rates of Britain and *Chosŏn* from 1701 to 1891. The historical data on population and wage rates by Wrigley-Schofield (1981, p. 408) provide us the historical population trend of Britain from 1701 to 1891.

No comparable time series data on the population and wage rates of *Chosŏn* during the British industrial revolution are available. The sources of data are the King's administrative offices during the Yi dynasty and the clan records in provincial areas.

A study on the population of Korea (*Chosŏn*) was carried out by Lee *et al.* (1975) as far back as reliable data would allow. According to this study, the population remained stationary without drastic deviations from the average annual growth rate of 0.2% for 226 years (1678 to 1904), confirming the Malthusian population theory (Figure 1.1 in Lee *et al.* 1975). The population size of the Yi Dynasty during the closing decade of the 14th century, that is, her founding period, was between 6 to 8 million (Lee *et al.* 1975, p. 3). Kwon and Shin (1977) extrapolated the data backwards by giving appropriate weights to the government household records.

The observed data on the population and real wage rates of *Chosŏn* and Britain are both quinquennial and in natural logarithm. Figure 2 compares the population trend of *Chosŏn* with that of Britain. The sources



Source: British population data are obtained from Wrigley and Schofield (1993, Tables A3.1 and A9.2) and Deane and Cole (1962, Table 3). The population data for *Chosŏn* are from Kwon and Shin (1977).

FIGURE 2

BRITISH AND *Chosŏn*'S POPULATIONS IN TEN THOUSANDS: 1701-1891

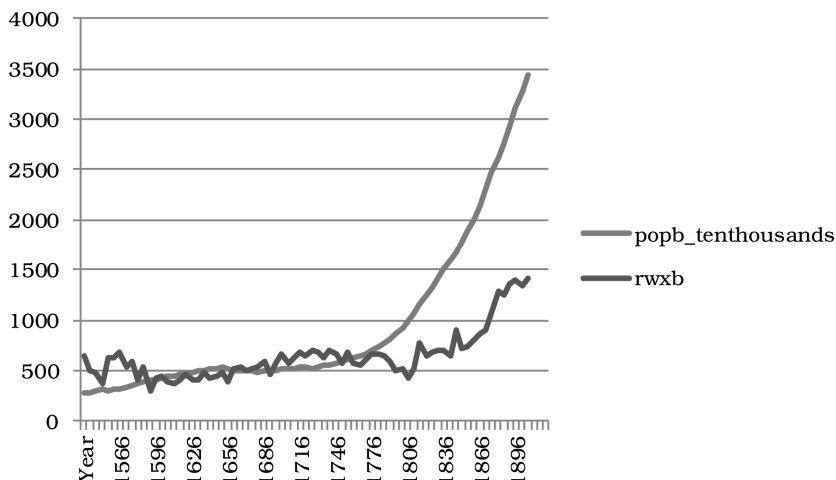
of data include a study on a clan's records in the provincial area of Kyŏngbook (Ahn and Rhee 2001) and the official records of the King's offices (Rhee 2004). The real wage rates for this period are obtained from the data of Park (2004) based on '*Ŭikue*'.³

At the beginning of the 18th century, *Chosŏn*'s population is about three times that of Britain. *Chosŏn*'s population growth rate during this period was almost zero and lagged far behind that of Britain. *Chosŏn*'s population growth was as low as one quarter of that of Britain. This value was then exceeded by that of Britain in the middle of the 19th century. At the turn of the 20th century, the population of *Chosŏn* remained as low as half of that of Britain. The population size of the two countries then switched at the middle of the 19th century.

Figure 3 shows Wrigley-Schofield's (1981, p. 408) historical data on population and real wage of Britain from 1541 to 1911.⁴ The data for the rest of the periods up to 1906 are from Deane and Cole (1962).

³ '*Ŭikue*' refers to the archives at the administrative offices of the King during the Yi Dynasty.

⁴ It is a reproduction of Wrigley-Schofield's (1981, p. 408) Figure 10.3.



Source: British population and real wage index data are from Wrigley and Schofield (1993 Tables A3.1 and A9.2) and Deane and Cole (1962 Table 3).

FIGURE 1.3

BRITISH POPULATION AND REAL WAGE INDEX: 1541-1911

Figure 3 displays a negative relation between the real wage rate and population size for the periods leading to 1600. The increase in population pulled down the wage rate, which then rose as the population size decreased, thereby supporting the Malthusian hypothesis. Thereafter, the relationship weakened, and no definitive pattern could be observed for about two centuries. The year of 1781 marked a reversal in the relationship, which transitioned from a negative to a positive one. The wage rate did not fall despite the increase in population, thus opposing the Malthusian expectation.

The population size of *Chosŏn* in 1701 was 14.6 million. This value steadily increased at an annual growth rate of 0.5% until reaching its peak level of 17.9 million in 1741. The population size remained on that level for about a half a century up to 1801. By 1891, it fell back to 16 million. As for the wage rates, no significant variations were observed during this period.

Table 1 compares the two economies in terms of variations in the population and real wage rates from 1701 to 1891.

The dispersion of Britain's wage rates relative to that of the population from 1701 to 1891 is three times as high as that of *Chosŏn*. This phe-

TABLE 1
THE POPULATION AND REAL WAGE RATES OF BRITAIN AND CHOSŎN FROM
1701 TO 1891*

	Britain	<i>Chosŏn</i>
	mean	mean
lnpop S_p	16.08 (0.56)	12.14 (0.72)
lnrwx S_w	6.53 (0.23)	6.21 (0.09)
S_w/S_p	0.41	0.13

Notes: Standard deviations are in parentheses. Notations of S_p and S_w are respectively the standard deviations of population and wage rates. In this table, lnpop refers to a natural log of the population size, and lnrxw refers to a natural log of the real wage index.

nomenon can be explained by the events that occurred during this period, which favored the changes in wage rate of Britain more than those of *Chosŏn*. The effects of these events were more evident in the changes in population than in the changes in wage rates in the case of *Chosŏn* for this period. No significant differences on the means of the natural log of the wage rates of the two nations were observed.

How can we explain a break through of the Malthusian bound? An economy within the Malthusian bound cannot escape as long as the real wage rate decreases because of the increase in population. In this regard, we present Result 1 as follows:

Result 1:

An economy within a Malthusian bound, given by the population and wage rate dynamic equations of (1) and (2), respectively, degenerates into the stationary state if $0 < a < 1$. This economy escapes from the trap provided that $1 \leq a$.

Appendix 1 provides the proof of this result.

One may consider a case in which an economy stretches out of the Malthusian bound up to a critical level from which the escape becomes plausible. Upon reaching this critical bound, the population increases because of the increase in the real wage rate. This result suggests that the sign of coefficient b on the wage dynamic equation in (2) needs to be reversed to a positive one. An economy with a positive coefficient of

TABLE 2
POPULATION AND WAGE RATE ADJUSTMENT COEFFICIENTS FOR BRITAIN
AND CHOSŎN: 1701-1891

	Britain	Chosŏn
\hat{a}	0.97** (0.14) [0.00]	-0.00 (0.01) [0.95]
\hat{b}	0.01 (0.01) [0.23]	0.04 (0.06) [0.47]

Notes: Standard errors are in parentheses.

$p > |t|$ values are in brackets under standard errors.

**Significant at the 1% level.

$0 < b < 1$ reaches a high level of population equilibrium. We present Result 2 below.

Result 2:

An economy out of the Malthusian bound exhibits high population equilibrium for the coefficient of $0 < b < 1$.

Appendix 2 provides the proof of this result.

In the following, we estimate coefficients a and b for both Britain and *Chosŏn* from 1701 to 1891 and compare the coefficients in relation to Results 1 and 2.

Table 2 presents the regression results of the population changes on wage rates and those of the wage rate changes on population for both Britain and *Chosŏn*.

A regression of the British population growth is run on the natural log deviation of the wage rate generated from the average of the natural log of the wage rate from 1701 to 1891. The regression yields a coefficient size of 0.97, suggesting that the population increased by as much as 0.97% as the real wage rate increased by 1%. The regression coefficient of 0.97 is an estimate of coefficient \hat{a} in the dynamic equation of (1). The fact that the coefficient size of 0.97 is not significantly different from 1 implies that the British population did not roll back to the Malthusian trap in view of Equation .

Coefficient \hat{b} is estimated by a regression of the real wage rate growth on the natural log deviation of the population generated from the average

of the natural log of the population size. We find that the coefficient is as low as 0.01, the significance of which is not confirmed. Nonetheless, a negative coefficient is not observed, as expected in Equation (2).

The doubling of the British population growth in 1786, which represented a one-digit percentage growth, leads me to infer that the country must be situated at point *d* on top of quadrant I (Figure 1). This situation represents the country's break through of the Malthusian bound.

The fact that the signs of the two estimated coefficients *a* and *b* are not opposite each other indicates that the British economy during the industrial revolution is situated in quadrant I (Figure 1). Otherwise, the signs of the two coefficients are opposite each other.

The results for the regression coefficients of *Chosŏn* are different. The population adjustment coefficient with respect to the real wage rate is almost zero, whereas the wage rates are found to respond to changes in population at a positive rate of 0.04. Coefficient \hat{a} for *Chosŏn* carries a negative sign at -0.00. Although its significance is doubtful, the sign of the coefficient suggests that the wage rate is so low, well below the subsistence rate, that its increase has no effects on population.

A positive sign of the estimated coefficient \hat{b} indicates that an economy within a Malthusian bound is located either in quadrant I or in quadrant III. A negative sign of coefficient \hat{a} indicates that the economy is located either in quadrant III or in quadrant IV. Consistent with the signs of coefficients *a* and *b*, *Chosŏn* is situated at point *c* (Figure 1). The population of *Chosŏn* was approximately 17.9 million for a period of sixty years (1741 to 1801) without any substantial changes. Thereafter, it fell back to 16 million. This observation on population change suggests that *Chosŏn* was situated on the edge of the Malthusian bound at point *c* for the latter half of the 18th century and then fell back to quadrant III during the 19th century.

In view of Equation (1) in Appendix 1, coefficient \hat{a} is as low as -0.00, which suggests that the *Chosŏn* economy degenerated into the Malthusian stationary state. Furthermore, the real wage rates of *Chosŏn* did not improve so as to escape from the Malthusian bound.

An increase in the British population in the latter half of the 18th century is significant. The population size of the country was 5.8 million in 1751 and 7.3 million in 1786, indicating an increase of 1.5 million in thirty-five years. For the years prior to 1786, the rate of increase in population rarely exceeded one percentage point. An increase by one-digit percentage point was only observed after 1786. As shown in Figure 3, the real wage rate of Britain did not fall despite the increase in popu-

lation in 1786.

III. A Break through of the Malthusian bound: The Agriculture of Britain during the Industrial Revolution

One of the most significant effects of the British industrial revolution is the country's break through of the Malthusian bound. This observation can be best described by the remark of Mokyr (2009) in the introduction to "Enlightened Economy": "The significance of the Industrial Revolution was that the race between babies and resources was won, resoundingly, by resources."

In a Malthusian economy, household income is equated with the subsistence of the household. An income below the subsistence level obliges the household to reduce its family size. This phenomenon refers to Malthus's "preventive check." In the context of the neoclassical family decision theory, the allocation of time for raising a child as a substitute for the quality of the next generation is not accounted for; only the income effect is considered. In this respect, the increase in wage rate under a Malthusian economy leads to an increase in population size. Conversely, the decrease in wage rate below the subsistence level results in the decrease in population, as depicted by the Malthusian "preventive check."

On a transitional phase from a traditional Malthusian state to a modern growth regime, the increase in population size precedes the technical progresses. In a unified growth theory, the increase in population size of a Malthusian economy transitioning to a modern growth regime is treated exogenously. This section focuses on how the increase in population size was achieved in the transition to the modern growth regime for the case of the British industrial revolution.

Population change and technical progress interact with each other. The factor that primarily causes the chain of interactions remains controversial today. For instance, Boserup (1981) argues that population size initiates the development of technical progresses, whereas technical progress induces a formation of human capital in a unified growth theory (Galor 2000). The increase in population size provides the economies of scale, thereby making the application of the technical progresses conducive.

To explain a sustained population growth requires a population model with a self-generating equilibrium, which could be achieved by the fol-

lowing two channels. An increase in the population favors labor productivity, which in turn increases the population. A virtuous circle exists for the increase in population, thus providing a momentum from which an economy can break through the Malthusian limit circle.

Another channel of a self-generating population scheme is the lecture of Fogel (1994) on his physiological wage rate hypothesis. The lecture calls our attention to a need for an integration of the physiological nature of wages related to a Malthusian population theory. An intake of calories obtained from food is converted as work output. A sufficient amount of nutrients is therefore required for workers to have enough energy. An adequate nutrient intake increases the intensity of work, and at the same time, laborers with a low income level can participate actively in the labor force, given their improved health conditions. An increased labor productivity due to a worker's good health results in higher wages, which again lead to the increase in population. Poor health causes a laborer's low productivity and low wage, which results in a low population size.

One-sector model: an agricultural economy

Section 1 illustrates that a momentum leading to the break through of Britain of the Malthusian bound is based on the fact that a dispersion of the wage rates relative to that of the population is above its critical minimum. Another observation is that the population adjustment coefficient with respect to wage rate is almost unity. These two observations explain the sustainability of population growth.

A possible supposition is that the improved productivity of an agricultural worker is a contributing factor for the break through of the Malthusian bound and sustains population growth. This section examines this possibility.

To discuss this problem in terms of a model, we introduce a one-sector agrarian Cobb-Douglas economy. This model intends to capture the role of arable lands and agricultural productivity in breaking through the Malthusian bound. Agricultural outputs for time period t denoted by $Y_A(t)$ are produced by land T and labor L_t in a Cobb-Douglas production function, as shown below:

(3)

where $0 < \alpha < 1$ denotes the distributive share of a land owner, and $1 - \alpha$ denotes a laborer's share of the total agricultural produce.

A Fogel's physiological factor as represented by the efficiency term, $B(L)$, is incorporated into the per capita form of production function where $B(L) > 0$; $B'(L) < 0$. Let β be a coefficient that represents the degree to which labor productivity increases because of a worker's physiological improvement. This representation intends to capture Fogel's physiological wage rate hypothesis. For simplicity, let $B(L) = AL^\beta$ for $0 < \beta < 1$, where A represents the technological level of the agricultural sector.

This representation demonstrates the rate at which the marginal productivity rate of an agricultural worker decreases by $-\alpha(-\alpha)$ without an increase in the productivity of the agricultural worker. Comprising the agricultural worker's productivity is the rate of the decrease in his/her marginal productivity represented by $-(\alpha - \beta)[1 - (\alpha - \beta)]$. This term must be positive for a considerably high productivity coefficient β . A positive sign of this coefficient suggests that the productivity improvement effect outweighs the agricultural worker's diminishing marginal productivity effect. Rearranging these terms yields an inequality condition:

The Left Hand Side (LHS) of the equation is a distributive share of an agricultural worker to that of a landowner, whereas the RHS is an agricultural worker's rate of productivity improvement divided by the square of the share of the landowner's rental income. The result is the minimum required rate of an agricultural worker's productivity improvement to break through the Malthusian trap. Upon fulfillment of this condition, the wage rate falls into quadrant II (Figure 1). The above inequality is rearranged to yield inequality (4).

(4)

Inequality (4) implies that the product of the distributive parameter of a landowner and that of an agricultural worker does not exceed the rate of the productivity improvement of the agricultural worker. The product of the two distributive parameters increases with respect to the landowner's distributive share for the case wherein this distributive share is smaller than 1/2. The result suggests that breaking through the Malthusian trap is related to the distributive share of a landowner and that of an agricultural worker for a given productivity level of an agri-

cultural worker. An income share in favor of a landowner delays the break through of the trap.

We present Result 3 below.

Result 3:

To break through a Malthusian trap, an agricultural worker's productivity exponent β must be no less than the product of the distributive shares of the landowner and the agricultural worker $\alpha(1-\alpha)$ for a Cobb-Douglas economy.

An economy breaks through of the Malthusian trap provided that the difference between an agricultural worker's productivity and the product of the distributive shares of the rents and wages (*dagprws*) is positive.

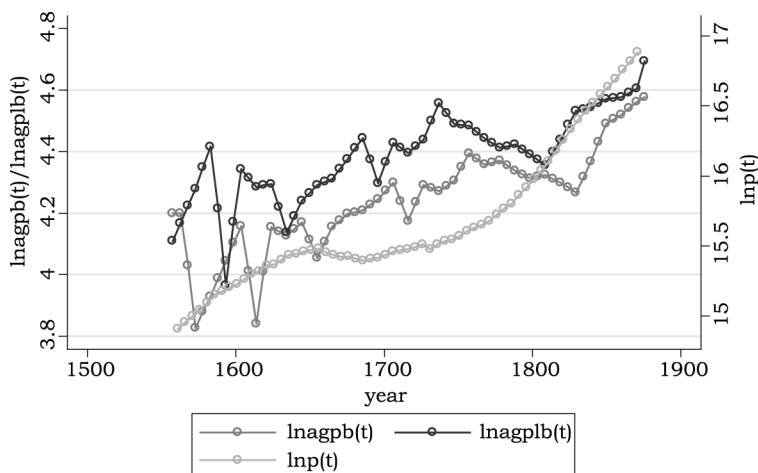
How would an income distribution between a landowner and a wage earner and the improvement of an agricultural worker during the British industrial revolution be reflected in inequality (4)? Clark's time series data on the British agricultural productivity and per unit labor productivity (Table 5 in Clark 2002b) as well as his data on the rental-wage ratio (Table 9 in Clark 2002a) provide us a way to compare the parameters for inequality (4).

Figure 4 shows an overlap of Clark's time series data on agricultural and per capita agricultural productivity and Wrigley-Schofield's population data.⁵ The horizontal axis represents the time scale in years. The natural log of Clark's time series data on productivity is on the vertical scale on the left, whereas the natural log of Wrigley-Schofield's population data is on the vertical scale on the right.

A sharp increase in the population occurs when productivity improvement increases greatly in both measures. This scenario occurred at the beginning of the 17th and 18th centuries. In between the two centuries, a steady increase in three of these variables could be observed (Figure 4).

Clark's data on wage-rent ratio stand for the distributive coefficient shares on the LHS of Equation (4). We obtain a per capita agricultural productivity improvement rate from Clark's productivity index.

⁵To cope with Wrigley-Schofield's population and wage rate quinquennial time series data, a mid-point of Clark's decennial time series data is established for every first year of a decade.



Source: Population data are taken from Wrigley and Schofield (1993), and the agricultural productivity data are taken from Clark (2002b).

FIGURE 4

BRITAIN'S POPULATION AND AGRICULTURAL PRODUCTIVITY: 1541-1871

TABLE 3*

MALTHUSIAN BREAKTHROUGHS DURING THE BRITISH INDUSTRIAL REVOLUTION

year	1556	1561	1566	1571	1576	1581	1586	1591	1596	1601
<i>dagprws</i>	-1.2	-1.073	-0.946	-0.271	0.399	0.435	0.467	-0.128	-0.718	-0.468
	1606	1611	1616	1621	1626	1631	1636	1641	1646	1651
	-0.219	0.158	0.532	0.166	-0.194	-0.326	-0.458	-0.372	-0.286	-0.042
	1661	1666	1671	1676	1681	1686	1691	1696	1701	1706
	0.07	-0.063	-0.071	-0.079	-0.002	0.077	-0.198	-0.474	-0.369	-0.265
	1716	1721	1726	1731	1736	1741	1746	1751	1756	1761
	0.089	-0.052	-0.193	0.014	0.222	0.09	-0.041	-0.186	-0.332	-0.329
	1771	1776	1781	1786	1791	1796	1801	1806	1811	1816
	-0.397	-0.468	-0.383	-0.298	-0.3	-0.303	-0.381	-0.459	-0.225	0.01
	1821	1826	1831	1836	1841	1846	1851	1856	1861	1866
	0.325	0.64	0.52	0.4	0.24	0.08	0.085	0.09	0.028	-0.035
	1871									
										0.195

*The first rows of Table 3 are the years, and the second rows are *dagprws*, which represent the differences between the growth rate of the agricultural worker's productivities and the products of the shares of rents and wages of the agricultural sector.

Table 3 presents the periods during which the improvement rate of the per capita agricultural productivity of Britain was above the minimum required value for the economy to break through the Malthusian bound. These periods are referred to as the shaded years, in which $dagprws > 0$ in Table 3. The shaded years occurred during the latter half of the 16th century (1576, 1581, and 1586) and the first half of the 17th century (1611, 1616, 1621, 1656, and 1661). The economy slumped backed to the Malthusian trap until the revamp during the first half of the 18th century (1716, 1731, 1736, and 1741). It remained stagnant under the Malthusian trap during the latter half of the 18th century. Only after the beginning of the 19th century (1816) did the economy sustain its break through of the Malthusian trap.

IV. A Self-Generating Population Equilibrium

The break through of the Malthusian bound at the beginning of the 17th century was coincidental with a revisionist's episode of the first wave of the British agricultural revolution (Allen 1999; Clark *et al.* 1995). After 1816, no negative sign can be observed for $dagprws$ in Table 4, except for the period of 1866, indicating that the population growth was sustainable after 1816. This section presents a sustainable population equilibrium in terms of the productivity of the per capita agricultural worker.

To precisely describe the inter-relations among the variables of population, agricultural productivity, and per capita productivity of an agricultural worker (Figure 4), Granger causalities are tested. A total of 63 observations cover the periods from 1541 to 1781 based on quinquennial data. In this test, we take quinquennial data four as a time lag, which covers a period of twenty years.

Table 4 presents the results of the *F-tests* for the regressions of $\ln pop$ on the per capita unit productivity of an agricultural worker $\ln agplb$. To determine whether the causality runs in both directions, $\ln aglb$ is regressed on $\ln pop$.

The Granger causality runs in both directions on a time lag of four. A change in the natural log of the productivity of the per capita agricultural worker causes the change in the British population over the period of the industrial revolution. This change is significant at the 5% level. Likewise, the change in the natural log of the population causes the change in the natural log of the productivity of the per capita British agricultural worker. This change is significant at the 1% level.

TABLE 4*
F(4, 54)-tests FOR REGRESSIONS OF *lnpopb* AND *lnagplb*

regression equations	F-test

Note: A numerator of an *F*-test is a degree of freedom represented by a time period lag of four; the degree of freedom on the denominator is 54, which is obtained by deducting the number 9 from the 63 time periods covered by the regressions. The number 9 comes out of two multiplied by the number of variables 4 to which a number 1 is added.

Surprisingly enough, the change in the natural log of the wage rates caused by the change in population is not significant. The productivity of a per capita agricultural worker, however, Granger causes the wage rate at a 1% level of significance on a time lag of four. The result suggests that the change in the wage rate resulting from the productivity of the per capita agricultural worker affects the population. Changes in wage rate could also be the result of other random factors such as good or bad harvests, which are indiscernible from productivity improvements. We doubt that the other random factors associated with changes in wage rate affect the population more greatly than productivity.

These causality tests do not change much with the variation of the time lags. The Granger causality of the per capita agricultural worker productivity on the change of the population fails on a time lag of one. This result indicates that a period of five years, which is equal to one unit of time lag, may not be long enough for an increase in an agricultural worker's productivity to indirectly affect the population through the changes in wage rates.

Granger causality tests between the productivity of the agricultural sector and the population share similar results with those of the per capita productivity of the agricultural worker. They are significant in both directions, with their significance increasing up to the 1% level.

In sum, the increase in wage rate resulting from the increase in either the per capita productivity of the agricultural worker or the agricultural productivity affects the population. A positive feedback effect of the increase in population on productivity does not pull the population back

into the Malthusian stationary state. Both the wage rate and the population increase. Random events other than productivity effects, such as good or bad harvests, would not be effective in sustaining the population. The results of the Granger causality tests for Britain during the industrial revolution (Table 3) are supportive of a self-generating population change.

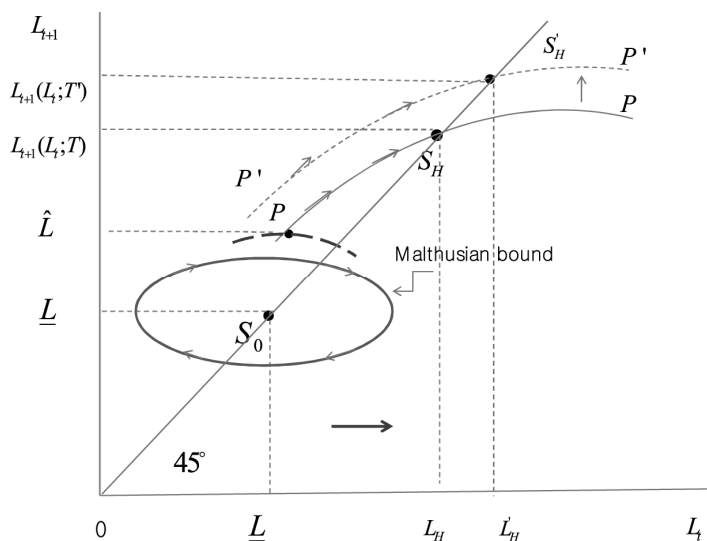
This sustainable population equilibrium may be viewed from the perspective of Fogel's physiological wage rate hypothesis. The nutritional and health conditions of an agricultural worker, which are improved by the increases in wage rates of previous periods, result in the worker's productivity improvement. The population of the next period is therefore nutritionally improved, which then contributes to the increase in productivity of the agricultural workers of the next period. This process suggests the sequential changes in population, which is given by Equation (5) as follows:

(5)

where L_t and L_{t-1} represent the population of a period t and $t-1$, respectively. A positive $\phi'(L_{t-1}) > 0$ reflects the Granger causality of the population on the productivity of an agricultural worker.

However, a demographic change at a certain point in time is replaced by the substitution effects. Associated with the substitution effects is the formation of human capital. Note that the changes in wage rate mediate the rate of population changes between the two periods. This mediation lowers the rate of the increase in population. This possibility is accounted for by a negative sign of the second derivative of $\phi''(L_{t-1}) < 0$.

Figure 5 displays two possible equilibria of an economy. One is a stationary Malthusian equilibrium, and the other is a high population equilibrium. The horizontal axis is the population size of period t , and the vertical axis is the population size of the succeeding period $t+1$. A population schedule PP represents the growth path of the population. Point S_0 in 3.1 denotes a Malthusian stationary state. A high population equilibrium is at point S_H . It is reached by an economy wherein the productivity of an agricultural worker is high enough that the population size is above its critical level \hat{L} . A break through of the Malthusian bound occurs, followed by a co-movement of the wage rates and the population along the population schedule PP until the crossover point



Point S_0 is a stationary Malthusian state, point \hat{L} is a critical population level, and L_H is a high equilibrium. An increase in the endowment of land T or an increase in agricultural productivity A shifts the population schedule PP upwards to $P'P'$. A new population equilibrium is reached at point L'_H .

FIGURE 5

A BREAK THROUGH OF THE Malthusian bound

S_H is reached with a 45° line.

An increase in agricultural productivity and in the use of coals, which replaced wood for energy and heating purposes during the industrial revolution, results in more land for farming. In addition, an institutional change, such as the introduction of an enclosure and the fallow system, contributes to land productivity.

Wrigley (1988, pp. 56-7) also emphasizes the importance of the use of coals as an energy source of an economy in agricultural productivity, as shown in the following passage:

“Or, to put it in a different way, tapping coal reserves on a steadily increasing scale produced much the same effect as would have resulted from the addition of millions of acres of cultivable land to the landscape of England, making it capable of yielding far more of the fruits of the earth than previously”

The expansion of arable land by irrigation or by any other means, such

as the use of coal as a substitute for wood, would favor more the distributive share of income for the agricultural worker than that for the landowner. The expansion of land contributes to the break through of the Malthusian bound in fulfillment of the inequality of Equation (4).

The population schedule PP moves upwards to PP in Figure 4 because of the expansion of arable lands. Population increases from L_H to L'_H . Contributing factors to the increase in productivity of agricultural food supply, such as the fallow system and enclosures, instead of the increase in productivity of the per capita agricultural worker would raise the population schedule upwards.

Chosŏn's population and agricultural productivity (1701-1891)

No comparable time series productivity data are available for the *Chosŏn* and British agriculture. Cha and Rhee (2004) infer the agricultural productivity of *Chosŏn* based on paddy land price. They observe a simultaneous movement of the population and land price. The population of *Chosŏn* peaked at 17.9 million in 1741. Five years later, the real land price reached its highest level at 6905 sŏk per durak.⁶ With the fall of land price to 4314 sŏk per durak in 1761, the population fell to 17,307,000. A similar co-movement between population size and real paddy land price occurred in 1801. The population size was 17,090,000, and the real land price was 7313 sŏk per durak during the peak year. Within a decade (1811), the population fell sharply to 16,680,000, and the real land price fell by 42% from 7313 to 4252 sŏk per durak.

The causes of the fall of land prices include the mismanagement of irrigation, the failure of the government to promote the productivity of the agricultural sector, and the flooding caused by natural disasters (Jun *et al.* 2008). Among these factors, the most important is the unavailability of substitute energy sources for heating and warming. The use of coal was yet to be introduced to *Chosŏn* during the British industrial revolution. Wood served as the main energy source, and forests were consequently damaged because of the high need for heating. The destruction of forests reduced the number of lands available for farming or animal husbandry. Lee (2004) shows evidences of price increases of wooden products during this period, which are indicative of the insufficiency of arable lands. The price of a coffin made of wood increased three to four times from the middle of the 18th century to the middle of the 19th cen-

⁶ A durak is equivalent to about 495 m². A sŏk is a unit of measurement for the weight of rice; 1 sŏk is equivalent to 107 kilograms (Park and Yang 2007).

tury (Table 1 in Lee 2004).

The productivity argument of this paper is consistent with the *Chosŏn* model of Jun, Lewis and Kang (2008). The argument distinguishes the economic performances of *Chosŏn* by period (17th, 18th and 19th centuries). The expansion periods of *Chosŏn* in terms of population during the 17th century were followed by the stable periods of the 18th century and by the shrinking of the population during the latter periods of the 19th century. In light of the Smithian physiocratic model, Jun, Lewis and Kang attribute the decline in economic performances to the inefficient management of the *Chosŏn* government at the end of the 19th century and to bad climate conditions.

The decrease in the number of arable lands resulting from the decrease in the agricultural productivity of *Chosŏn* suggests that the population schedule (Figure 5) shifted downward. The shift resulted in the decrease in population size, which led to the return of the *Chosŏn* economy to the Malthusian trap in the 19th century.

In conclusion, evidence proving that the *Chosŏn* economy was free from the Malthusian constraint during the 18th and 19th centuries is difficult to find. During the same periods, no comparable productivity improvements to the agricultural sector of the British economy could be observed. Information on *Chosŏn*'s mineral energy sources as well as incentive systems for enclosures and fallow systems were unavailable.

The calculation by Park and Yang (2007) of the required amount of food for a hypothetical household to subsist indicates that *Chosŏn*'s owner-tenants or tenants did not have enough land for their subsistence. They were forced to look for other sources of income other than agricultural farming to survive. Picturing *Chosŏn* breaking through the Malthusian bound is difficult, as the region is characterized to be under the typical Malthusian stationary state, wherein the land is a limiting factor for the survival of the population.

IV. Conclusion

The British industrial revolution is not simply a historic economic event, and its pervasiveness across geographical areas and over historical time periods is not ephemeral. This event ignited a permanent transformation in the industrial structures of economies. The transformation spread across other countries and across generations, penetrating still the countries that are currently undergoing industrialization.

This paper envisages a *Chosŏn* economy in a historical perspective of the British industrial revolution. A one-sector agrarian Cobb-Douglas economy is introduced based on Fogel's physiological wage rate hypothesis. Increases in productivity of the British agricultural sector contributed to the country's break through of the Malthusian trap, whereas no such productivity effects occurred for *Chosŏn*.

Granger causality tests indicate that agricultural productivity Granger causes population, whereas wage rates do not pass the causality test. Conversely, population Granger causes productivity, whereas productivity Granger causes wage rates. These causality tests are supportive of Fogel's physiological wage rate hypothesis. Along this line, a sustainable population equilibrium is established for a post-Malthusian regime.

Corresponding evidence of agricultural productivity improvements in *Chosŏn* could not be found for the 17th and 18th centuries. The poor control of the water system for paddy lands, a lack of incentives for tenants, deforestation resulting from the lack of energy substitutes, mismanagement of the agrarian storage system by the government, and flooding due to bad climate conditions were the attributes that led to the failure of *Chosŏn* to break through the Malthusian bound.

Economic historians attribute the success of the industrial revolution to technologically innovative economic environments. On the economic landscape of the British economy during the industrial revolution, an interaction could be observed between the population and agricultural productivity. This paper emphasizes the importance of agricultural productivity in the break through of the Malthusian trap during the British industrial revolution.

Compared with Britain, *Chosŏn* remained under the Malthusian stationary state during the industrial revolution. She remained as a dormant agrarian economy until she faced the global challenge of opening of her port.

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Appendix 1

For the Malthusian dynamic equations of (1) and (2), a stationary population size of \underline{L} exists for $0 < a < 1$; $0 < b < 1$.

Proof. Dynamic equations (1) and (2) can be presented in the following matrix form:

which yields characteristic roots satisfying

Characteristic roots λ_1, λ_2 are in the form of

for which $\lambda_1 = a; \lambda_2 = -b$ and

(1)

where c_1 and c_2 are constants.

Appendix A: Data for Britain.

Data on the population and real wage rate of Britain are obtained from Wrigley and Schofield's Tables A3.1 and A9.2 (1993). Population data on the years after 1871 are filled in by applying decennial rates calculated from Deane and Cole's Table 3 (1962) on the population growth of the United Kingdom. A quinquennial natural log on the time series data of the English population and real wage rate are respectively represented as $\ln popb$ and $\ln rwx b$.

Appendix 2

For the Malthusian dynamic equations of (1) and (2), a positive coefficient b indicates the existence of a high equilibrium population size of \bar{L} for $0 < a < 1$; $0 < b < 1$.

Proof: A matrix form of dynamic equations (1) in Appendix 1 yields characteristic roots $\tilde{\lambda}_1, \tilde{\lambda}_2$, which are expressed in the form of

for which

$$\tilde{\lambda}_1 = a; \tilde{\lambda}_2 = b, \text{ and}$$

(1)

Hence,

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