Price Rigidity and Means of Payment

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The trade-off between cash and a debit card as a means of payment is incorporated into a search-theoretic model. A buyer incurs the proportional cost of carrying cash into the decentralized goods market, and a seller accepting a debit card bears a fixed record-keeping cost. In an equilibrium, the price of a cash good turns out to be relatively sticky compared with that of a debit-card good. With money supply increasing at a constant rate, the carrying cost of cash proportional to its amount causes the cash balance net of cost to increase at a rate less than the money growth rate. Consumption smoothing also leads to a relatively small decrease in quantity traded in comparison with the increase in cash balance, implying rigid price. Further, the means-of-payment mechanism underlying price rigidity yields an additional distortionary effect of inflation on relative price between cash trade and debit-card trade, which implies higher welfare cost of inflation than that in the standard search-based model.

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I. Introduction

In the real world, we can easily observe that some prices are quite sticky, whereas others change very frequently. Bils and Klenow (2004) examine price changes of 350 categories of goods and services to show that price rigidity varies tremendously across goods. Recently, Boivin *et al.* (2009) have shown that prices of many goods appear sticky in re-

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sponse to monetary policy and macroeconomic disturbances, whereas those same prices respond quickly to sector-specific shocks that affect the relative price of the good compared with other goods.

Price rigidity has been shown to be associated to some extent with means of payment. Using hand-collected data around Ann Arbor, Michigan, in the US, Knotek (2010) shows that *convenient prices*, which simplify and expedite transactions, are set for goods and services typically purchased with cash. Moreover, Knotek (2008) finds that convenience pricing is a source of price rigidity. These two findings imply that the prices of cash goods would be stickier than those purchased with other means of payment. Levy and Young (2004) claim that the unusual price rigidity of the 6.5-ounce Coke in 1886-1959 is associated with monetary denomination, nickel. Finkelstein (2009) also finds that the adoption of electronic toll collection systems, such as E-ZPass, I-Pass, or Fast-Track, has caused an increase in tollway fee. Despite these invocations, an explicit modeling of the relationship between price rigidity and means of payment is rarely found.

The goal of this paper is to provide a micro-foundation for price rigidity in connection with means of payment.[1] Specifically, we consider a standard search-based model of exchange (*e.g.*, Lagos and Wright 2005) augmented with the choice of means of payment, cash or debit card, in response to monetary policy.[2] In the centralized market, each agent holding a given amount of monetary wealth chooses a means of payment together with money balance carried into the decentralized market, where each agent is randomly matched with another agent. In pairwise meetings, agents cannot commit to their future actions and trading histories are private. Hence, all trades should be *quid pro quo* in the sense that

[1] There has been a growing literature on the New Keynesian dynamic stochastic general equilibrium (DSGE) models with a reduced-form approach to price or wage rigidity; see, *e.g.*, Yun (1996), Goodfriend and King (1997), Jung (2003), Smets and Wouters (2003), and Woodford (2003).

[2] Williamson and Wright (2009) incorporate sticky prices into a version of Lagos and Wright (2005) to show that policy implications are similar to those in Woodford (2003), but there are also differences due to microeconomic details. Craig and Rocheteau (2008) have another version of a search-based model of exchange with sticky prices. However, these studies assume price rigidity as in the New Keynesian DSGE models. Head *et al.* (2010) develop a search-based model in which price rigidity arises endogenously. In the presence of search frictions, expected profit from posting a high or low price may be the same for some sellers if aggregate distribution of real prices remains invariant to an increase in money supply.

either cash or debit-card deposit should be transferred in exchange for goods in a single-coincidence meeting in which the terms of trade are determined by a buyer's take-it-or-leave-it offer.

Although either cash or debit-card deposit can be used as a means of payment in pairwise trades, they are not identical assets. Cash holdings entail forgone interest income as well as the risk of loss, theft, and inconvenience of carrying them from the centralized market to the decentralized market. Alternatively, debit-card deposit is free from the risk of loss or theft, and there is negligible inconvenience of carrying a debit card. However, unlike cash transaction, debit-card transaction requires record-keeping cost.[3] This trade-off between cash and a debit card as a means of payment is reflected in the model economy by assuming that a buyer incurs the carrying cost of cash proportional to its amount, and a seller accepting a debit card bears a fixed record-keeping cost. Essentially, this transaction technology is equivalent to the approach taken by Baumol (1952), Tobin (1956), Karni (1973), and more recently by Freeman and Kydland (2000).

We consider equilibria in which the real balance of monetary wealth is constant so that the purchasing power of money in the "frictionless" centralized market depreciates at a given growth rate of money supply. First, we show that, for a given carrying cost of cash per unit, if a fixed record-keeping cost associated with debit transaction is sufficiently large, cash is used as a means of payment in all trades, whereas a debit card is used in all trades if record-keeping cost is sufficiently small. If the record-keeping cost is neither too small nor too large, both cash and debit card are used as means of payment.

Second, the price of cash good is relatively sticky compared with that of debit-card good. If a debit card is used as a means of payment in a pairwise trade, an increase in money supply leads to a proportional increase in money demand. That is, when money supply increases at a constant rate, a fixed record-keeping cost associated with the use of a debit card has no additional distortionary effect in the sense that there is no change in quantity of goods traded in the decentralized market except for a proportional increase in price. However, if cash is used in a pairwise trade, the carrying cost of cash proportional to its amount causes the cash balance net of cost to increase at a rate less than the money growth rate. Moreover, consumption smoothing leads to a relatively

[3] A survey by the Food Marketing Institute shows that the transaction cost of accepting a debit card is much more expensive than cash (Humphrey 2004).

small decrease in quantity traded in comparison with the increase in cash balance, implying rigid price.

Finally, the means-of-payment mechanism underlying price rigidity implies that the welfare differences between debit-only and cash-only trades come from inefficiency due to a higher cost of carrying cash with an increase in money supply. For a zero cost of carrying cash, real balance for a cash trade is equal to that for a debit-card trade net of a fixed record-keeping cost. However, with a positive cost of carrying cash per unit, inflation has a distortionary effect on relative price between cash trade and debit-card trade in the sense that real cash balance and quantity of goods traded are less than those for a debit-card trade. This result implies that welfare cost of inflation is higher than that in the standard search-based model in which the quantity traded in exchange for money decreases with inflation because of its distortionary effect on intertemporal relative price.

These results are quite novel in the sense that sticky price is explicitly derived from the consumer's choice of means of payment in response to monetary policy rather than menu costs as in Mankiw (1985).[4] This result also provides an explanation for the dramatic differences in price rigidities across goods as documented in Bils and Klenow (2004) such that the rigidities of newspaper prices, taxi fares, and vehicle tolls are above average, whereas those of gasoline, airfares, and durable goods are below. The former group of goods is typically purchased by cash, whereas the latter group of goods is typically purchased by electronic forms of payment.

Further, our results imply that price rigidity should decline over time as record-keeping cost decreases with the development of information technology. This finding is consistent with that of Kackmeister (2007), which shows a substantial decrease in price rigidity during 1997-1999 relative to that during 1889-1891. The decline in record-keeping cost expands the use of debit cards, which then makes prices more responsive to monetary policy and macroeconomic disturbances.

Studies on multiple means of payment have been conducted, including He *et al.* (2008) and Li (2008), in which agents can deposit their money in bank accounts and pay for goods using checks or debit cards. In Kim and Lee (2010), heterogeneous preferences are generated endo-

[4] Golosov and Lucas (2007) show that the size of the menu cost required to match the micro-data of price adjustment in an otherwise standard business cycle model is implausibly large to justify the menu-cost argument.

genously by a non-degenerate wealth distribution across agents, and hence the fraction of agents using cash or a debit card is determined as an equilibrium outcome. Williamson (1999), Lagos and Rocheteau (2008), and Telyukova and Wright (2008) also introduce private money, capital, and credit into search-based models as competing means of payment with fiat money. However, these studies abstract from the relationship between means of payment and price dynamics in response to monetary policy.

II. Model

The basic setup comes from Lagos and Wright (2005), and Lagos (2010). Time is discrete. There is a [0, 1] continuum of agents who live forever with discount factor $\beta \in (0, 1)$. In each period, there are two markets, namely, frictionless centralized market (hereinafter CM) and frictional decentralized market (hereinafter DM), which open sequentially. There are three perishable and perfectly divisible consumption goods: *apple*, *banana*, and *general good*. All agents can produce and consume general goods in CM, whereas fruits (apple and banana) are endowed and traded in DM.

Money is the only object in this economy that can be storable across periods. Money is perfectly divisible, and total stock evolves deterministically at the gross rate μ: *i.e.*, $Z_t = \mu Z_{t-1}$, where Z_t denotes the stock of money in CM at period t and $\mu \geq 1$. Each agent receives lump-sum transfer of new money in the beginning of CM.

The rest of the model is best described following the sequence of events within a period. At the opening of CM, an idiosyncratic preference shock $\eta_t \in \{a, b\}$ arrives to each agent, which determines utility from consuming fruit in the following DM, where for simplicity $\Pr[\eta_t = a] = \Pr[\eta_t = b] = 1/2$. An agent with $\eta_t = a$ obtains utility only from consuming apples, and an agent with $\eta_t = b$ obtains utility only from consuming bananas in DM. After the realization of a preference shock, agents then produce and consume general goods. An agent obtains utility $v(g)$ from consuming g units of general goods, where $v''(g) < 0 < v'(g)$, $v(0) = 0$, and $v'(0) = \infty$. Each unit of general good is produced using one unit of labor which incurs one unit of disutility. A means of payment is also chosen together with money balance for pairwise trades in the decentralized fruit market by exchanging general goods for money.[5] Agents who choose a debit card

[5] The means-of-payment choice can be interpreted as the portfolio choice of

as a means of payment can freely deposit their money into a debit-card account. The information on debit-card transactions is kept by the government which has a technology for keeping records on transactions associated with debit-card accounts but not on agents' trading histories over time.

Although either cash or a debit card can be used as a means of payment in DM, they are not identical assets in the sense that cash incurs the cost of carrying it into DM, whereas a debit card incurs record-keeping cost. As in the inventory-theoretic approach to money demand, such as in Baumol (1952), Tobin (1956), and more recently Freeman and Kydland (2000), we assume that the carrying cost of cash is proportional to its amount, whereas the record-keeping cost of a debit card is fixed regardless of its amount.[6] Specifically, carrying a unit of cash incurs constant real cost of γ_m in the beginning of DM. This proportional cost includes forgone interest income as the opportunity cost of carrying cash. Alternatively, if interest income on the debit-card deposit is normalized to zero, this opportunity cost can be captured as negative interest income on cash carried around as a means of payment.

In DM, agents with $\eta_t=a$ and $\eta_t=b$ go to the apple market and banana market, respectively. An idiosyncratic endowment shock then arrives to each agent such that in the apple [banana] market, half of the agents are endowed with $\varepsilon_a^h=(1+\varepsilon_a)F$ [$\varepsilon_b^h=(1+\varepsilon_b)F$] units of apples [bananas], and the remaining half with $\varepsilon_a^l=(1-\varepsilon_a)F$ [$\varepsilon_b^l=(1-\varepsilon_b)F$] units of apples [bananas], where $0<\varepsilon_a<\varepsilon_b<1$ and $F>0$. An agent obtains utility $u(q_\eta)$ from consuming q_η units of fruit, where $u''(q_\eta)<0<u'(q_\eta)$, $u(0)=0$, and $u'(\varepsilon_\eta^l)$ is sufficiently large.

After the realization of an endowment shock in DM, a low-endowment agent in each fruit market is randomly matched with a high-endowment agent. Hence, all pairwise meetings are single-coincidence meetings in which a low-endowment agent is a buyer and a high-endowment agent is a seller. Agents cannot make any binding intertemporal commitments, and their trading histories are private. Thus, all trades should be *quid*

money balance between cash and debit-card deposit. This interpretation indicates a somewhat restrictive feature of the model in which a portfolio consisting of either cash only or debit-card deposit only is considered. However, this is not critical in the sense that no one holds a portfolio of both cash and debit-card deposit in the equilibrium (see footnote 11 for details). We are indebted to an anonymous referee for drawing our attention to this issue.

[6] In Freeman and Kydland (2000), cash-holding cost is represented by transaction cost (or shoe leather cost), as in Baumol (1952) and Tobin (1956).

pro quo (see Kocherlakota 1998; Wallace 2001; Corbae *et al.* 2003; and Aliprantis *et al.* 2007). The terms of pairwise trade are determined by Nash bargaining in which a buyer has all the bargaining power.[7] If a buyer uses a debit card in exchange for fruit, the corresponding amount of money is transferred from the buyer's account to the seller's account immediately, and a seller incurs a fixed record-keeping cost γ_d in terms of general good. Note that a flat fee per transaction and immediate clearance capture the features of online (PIN-based) debit cards.[8] Finally, at the end of a period, debit-card deposit is returned to each agent according to the government record, and the next period is started with the end-of-trade wealth.

III. Equilibrium

To facilitate the description of an equilibrium, we first introduce some notations. Let ϕ_t denote the unit price of money in period t in terms of general goods. We will drop the time subscript t hereinafter and index the next (previous) period variable by $+1$ (-1) to prevent confusion. Let $W(z, \eta)$ be the expected discounted utility of an agent entering CM with money balance z and preference shock η, and $V(z, \eta)$ be the expected discounted utility when the agent enters DM. The function $W(z, \eta)$ pertains to after the realization of preference shock $\eta \in \{a, b\}$ in CM, and $V(z, \eta)$ pertains to before the realization of endowment shock $\varepsilon_\eta \in \{\varepsilon_\eta^l, \varepsilon_\eta^h\}$ in DM. In what follows, we will consider the equilibrium in which the real balance of money is constant in CM, $\phi Z = \phi_{+1} Z_{+1}$, which implies $\phi / \phi_{+1} = \mu$.

[7] Although the take-it-or-leave-it offer by a buyer is an extreme case of Nash bargaining, we do so to focus on the consumer side effects on price dynamics, such as the cost of antagonizing customers (Blinder *et al.* 1998), customer costs of price adjustment (Zbaracki *et al.* 2004), or the fear of consumer anger (Rotemberg 2005). In particular, Zbaracki *et al.* (2004) estimate that customer costs of price adjustment comprise 73.40%, whereas that for menu cost remains 3.57%.

[8] The pricing structure of fees for offline (signature-based) debit cards is very similar to that for credit cards. According to the "2005/2006 Study of Consumer Payment Preferences" conducted by the American Bankers Association and Dove Consulting, 16%, 10%, and 55% of debit-card holders use, respectively, online debit cards, offline debit cards, and online-cum-offline debit cards, to make in-store, internet, and bill payments. The remaining 20% of debit-card holders use neither form of debit cards.

A. Centralized Market

In the frictionless CM, agents produce and consume general goods, and choose a means of payment together with money balance carried into DM. Hence, the problem for an agent entering CM with money balance z and preference shock η is

$$W(z,\eta)=\max_{\sigma_\eta}\left\{\begin{array}{l}\sigma_\eta \max_{(g,h,\omega^m)}[\upsilon(g)-h+V(\omega^m,\eta)]+\\(1-\sigma_\eta)\max_{(g,h,\omega^d)}[\upsilon(g)-h+V(\omega^d,\eta)]\end{array}\right\} \tag{1}$$

$$\text{s.t. } h=g-\phi(z+T-\omega^i) \tag{2}$$

$$g\geq 0,\ \omega^i\geq 0,\ h\in[0,\bar{h}]$$

where $\sigma_\eta\in\{\sigma_a, \sigma_b\}$ with $\sigma_a\in[0, 1]$ and $\sigma_b\in[0, 1]$ denote the probability of choosing cash as a means of payment in exchange for apples and bananas in DM, respectively. Moreover, $\omega^i\in\{\omega^m, \omega^d\}$ denotes the money balance carried into DM, where ω^m and ω^d are the cash balance and debit-card deposit, respectively, h is labor supply with $\bar{h}$ denoting its upper bound, and T is a lump-sum transfer of new money, $T=(\mu-1)Z_{-1}+(1/\phi_{-1})(\gamma_m M_{-1}+\gamma_d D_{-1})$ with M_{-1} and D_{-1} denoting aggregate cash holdings and the frequency of debit-card transactions in the previous period, respectively.[9] We simply assume an interior solution for g and h.[10] Substituting h from the budget constraint (2), we have

$$W(z,\ \eta)=\phi(z+T)+\max_g\{\upsilon(g)-g\}+\max_{\sigma_\eta}\{\sigma_\eta(\bar{\upsilon}_{m,\eta}-\bar{\upsilon}_{d,\eta})+\bar{\upsilon}_{d,\eta}\}$$

where $\bar{\upsilon}_{m,\eta}=\max_{\omega^m}[V(\omega^m,\ \eta)-\phi\omega^m]$ and $\bar{\upsilon}_{d,\eta}=\max_{\omega^d}[V(\omega^d,\ \eta)-\phi\omega^d]$.

The first-order conditions with respect to g and $\omega^i\in\{\omega^m,\ \omega^d\}$ are as follows:

$$\upsilon'(g)=1 \tag{3}$$

[9] Although the government cannot observe pairwise trades between agents, it always knows M_{-1} and D_{-1} because it keeps the information on debit-card accounts.

[10] See Lagos and Wright (2005) for the conditions under which solutions of g and h are interior, respectively.

$$\phi \geq \frac{\partial [V(\omega^i, \eta)]}{\partial \omega^i} \tag{4}$$

where equality holds if $\omega^i > 0$. Further, an agent will choose to carry money balance in cash ($\sigma_\eta = 1$) if $\bar{v}_{m,\eta} > \bar{v}_{d,\eta}$, whereas an agent will choose to hold money balance in debit-card deposit ($\sigma_\eta = 0$) if $\bar{v}_{m,\eta} < \bar{v}_{d,\eta}$. We do not consider the case of a perfectly substitutable means of payment, where $\bar{v}_{m,\eta} = \bar{v}_{d,\eta}$.[11] Now, the envelope condition is

$$\frac{\partial W(z, \eta)}{\partial z} = \phi. \tag{5}$$

Condition (3) implies that all agents regardless of z or $\eta \in \{a, b\}$ consume $g^* \in (0, \infty)$ units of general goods such that $v'(g^*) = 1$. Condition (4) determines the money balance carried into DM, which depends only on $\eta \in \{a, b\}$ regardless of z. Condition (5) implies that the value function $W(z, \eta)$ is linear in z. Further, with the assumption of $\mu > \beta$, a buyer's take-it-or-leave-it offer implies a unique solution of ω^i for each $\eta \in \{a, b\}$, as in Lagos and Wright (2005). Hence, wealth distribution across agents entering the decentralized fruit market for apple or banana is always degenerate.

B. Decentralized Market

There are two possible types of single-coincidence meetings in each of the decentralized fruit market depending on the debit card or cash used as a means of payment. The terms of trade (q, p) in a pairwise meeting are determined by the buyer's take-it-or-leave-it offer, where q denotes the quantity of fruit transferred by a seller, and p denotes the amount of money offered by a buyer.

Let $U(q) \equiv u(\varepsilon^l + q) - u(\varepsilon^l)$ and $C(q) \equiv u(\varepsilon^h) - u(\varepsilon^h - q)$. Then, (q_η, p_η) for η

[11] That is, we do not consider a portfolio of money balance that consists of both cash and debit-card deposit. Suppose that $\tilde{z}$ is the money balance carried to the decentralized fruit market. Let $\Theta = (m, d)$ be a portfolio such that $m + d = \tilde{z}$, where m and d denote the cash balance and debit-card deposit, respectively. Now consider the following three portfolios: $\Theta_\varepsilon = (\tilde{z} - \varepsilon, \varepsilon)$, $\Theta_m = (\tilde{z}, 0)$, and $\Theta_d = (0, \tilde{z})$. It is straightforward to show that $V(\Theta_\varepsilon, \eta) < V(\Theta_d, \eta)$ from (11), (16), and the linearity of W. Then, cash only portfolio is optimal if $\max\{V(\Theta_m, \eta), V(\Theta_d, \eta)\} = V(\Theta_m, \eta)$ because $V(\Theta_\varepsilon, \eta) < V(\Theta_d, \eta) < V(\Theta_m, \eta)$, whereas debit-card deposit only portfolio is optimal if $\max\{V(\Theta_m, \eta), V(\Theta_d, \eta)\} = V(\Theta_d, \eta)$ because $V(\Theta_\varepsilon, \eta) < V(\Theta_d, \eta)$.

$\in\{a, b\}$ in the pairwise meeting between a buyer holding money balance ω and a seller holding money balance $\hat{\omega}$ can be expressed as the solution to the following problem:

$$\max_{p_\eta \leq \omega,\, q_\eta \geq 0} [U(q_\eta)+\beta \mathrm{E}_\eta W_{+1}(\omega - p_\eta,\, \eta) - \beta \mathrm{E}_\eta W_{+1}(\omega,\, \eta)]$$

$$\text{s.t. } C(q_\eta)+\gamma_d \mathrm{I}_d \leq \beta\, [\mathrm{E}_\eta W_{+1}(\hat{\omega}+p_\eta,\, \eta) - \mathrm{E}_\eta W_{+1}(\hat{\omega},\, \eta)]$$

where E_η is the expectation with respect to $\eta\in\{a, b\}$, and I_d is an indicator function taking the value of 1 if $\omega=\omega^d$ and 0 otherwise. We assume a tie-breaking rule by which a seller agrees to any offer that makes him/her indifferent between accepting and rejecting. The linear property of W simplifies the above problem as follows:

$$\max_{p_\eta \leq \omega,\, q_\eta \geq 0} [U(q_\eta) - \beta \phi_{+1} p_\eta] \tag{6}$$

subject to $C(q_\eta)=\beta\phi_{+1}p_\eta - \gamma_d \mathrm{I}_d$. The solution to (6) for $\eta\in\{a, b\}$ is

$$q = \begin{cases} \varepsilon_\eta F & \text{if } \beta\phi_{+1}\omega - \gamma_d \mathrm{I}_d \geq C(\varepsilon_\eta F) \\ C^{-1}(\beta\phi_{+1}\omega_\eta - \gamma_d \mathrm{I}_d) & \text{if } \beta\phi_{+1}\omega - \gamma_d \mathrm{I}_d < C(\varepsilon_\eta F) \end{cases} \tag{7}$$

$$p = \begin{cases} \bar{\omega} & \text{if } \beta\phi_{+1}\omega - \gamma_d \mathrm{I}_d \geq C(\varepsilon_\eta F) \\ \omega & \text{if } \beta\phi_{+1}\omega - \gamma_d \mathrm{I}_d < C(\varepsilon_\eta F) \end{cases} \tag{8}$$

where $\bar{\omega}=[C(\varepsilon_\eta F)+\gamma_d \mathrm{I}_d]/\beta\phi_{+1}$.

Notice that $\varepsilon_\eta F$ represents the efficient or first-best quantity of fruit traded, which equates the marginal utility of fruit consumption with its marginal disutility. If a buyer holds a sufficiently large amount of real balance so that $\beta\phi_{+1}\omega - \gamma_d \mathrm{I}_d \geq C(\varepsilon_\eta F)$, he/she gets $\varepsilon_\eta F$ units of fruit in exchange for the real balance of $C(\varepsilon_\eta F)+\gamma_d \mathrm{I}_d$. If $\beta\phi_{+1}\omega - \gamma_d \mathrm{I}_d < C(\varepsilon_\eta F)$, however, a buyer spends all the real balances in exchange for q_η units of fruit, which solves $C(q_\eta)=\beta\phi_{+1}\omega - \gamma_d \mathrm{I}_d$. Note also that, as in other variations of Lagos and Wright (2005), the terms of trade depend only on the buyer's wealth regardless of the seller's wealth.

Now, the Bellman equation for a buyer (low-endowment agent) with ω^m balance of cash ($\sigma_\eta=1$) satisfies

$$V^b(\omega^m,\ \eta)=u(\varepsilon_\eta^l+q_\eta^m)+\beta \mathrm{E}_\eta W_{+1}\,[\tilde{\omega}^m-p_\eta^m,\ \eta] \tag{9}$$

where $\tilde{\omega}^m \equiv [1-(\gamma_m/\phi)]\omega^m$ is the cash balance net of its carrying cost. Here, by the bargaining solutions, $(q_\eta^m, p_\eta^m)=(\varepsilon_\eta F, C(\varepsilon_\eta F)/\beta\phi_{+1})$ if $\beta\phi_{+1}\,\tilde{\omega}^m \geq C(\varepsilon_\eta F)$ and $(q_\eta^m, p_\eta^m)=(C^{-1}(\beta\phi_{+1}\tilde{\omega}^m), \tilde{\omega}^m)$ otherwise. The corresponding Bellman equation for a seller (high-endowment agent) satisfies

$$\begin{aligned} V^s(\omega^m,\eta) = \int \{&[u(\varepsilon_\eta^h - q_\eta^m(\omega)) + \beta \mathrm{E}_\eta W_{+1}(\tilde{\omega}^m + p_\eta^m(\omega),\eta)]\,\mathbf{1}_{\{\omega=\omega^m\}} \\ &+[u(\varepsilon_\eta^h - q_\eta^d(\omega)) + \beta \mathrm{E}_\eta W_{+1}(\tilde{\omega}^m + p_\eta^d(\omega) - \gamma_d/\phi,\eta)]\,\mathbf{1}_{\{\omega=\omega^d\}}\}d\mathbf{F}(\omega) \end{aligned} \tag{10}$$

where $\mathbf{F}(\omega)$ is the wealth distribution across buyers (low-endowment agents), and $\mathbf{1}_{\{\chi\}}$ is an indicator function taking the value of 1 if and only if χ is true. Based on (9) and (10), together with the linearity of W and degenerate distribution of wealth across agents, the expected utility of an agent entering DM with ω^m balance of cash, before knowing the endowment shock, can be written as

$$V(\omega^m,\eta) = \frac{1}{2}[u\,(\varepsilon_\eta^l + q_\eta^m) + u\,(\varepsilon_\eta^h - q_\eta^m)] + \beta \mathrm{E}_\eta W_{+1}(\tilde{\omega}^m,\eta). \tag{11}$$

From (11), the first derivative of $V(\omega^m, \eta)$ with respect to $\omega^m \in [0, \bar{\omega}]$ becomes[12]

$$\frac{\partial V(\omega^m,\eta)}{\partial \omega^m} = \beta\phi_{+1}\left\{1+\frac{1}{2}\left[\frac{u'(\varepsilon_\eta^l + q_\eta^m) - u'(\varepsilon_\eta^h - q_\eta^m)}{C'(q_\eta^m)}\right]\right\} - \left(\frac{\beta}{\mu}\right)\gamma_m. \tag{12}$$

By substituting (12) into (4), we obtain the condition that determines $\omega^m > 0$ for each $\eta \in \{a, b\}$:

$$\frac{\mu}{\beta} + \frac{\gamma_m}{\phi} = \mathcal{L}(q_\eta^m) \tag{13}$$

where $\mathcal{L}(q_\eta^m) = 1+(1/2)[u'(\varepsilon_\eta^l + q_\eta^m) - u'(\varepsilon_\eta^h - q_\eta^m)]/C'(q_\eta^m)$ represents the expected marginal benefit of cash carried into the decentralized fruit market for a pairwise trade.

[12] The slope of $V(\omega^m, \eta)$ at $\omega^m = \bar{\omega}$ is the limiting case from below.

Similarly, the Bellman equation for a buyer with ω^d balance of debit-card deposit ($\sigma_\eta=0$) satisfies

$$V^b(\omega^d,\ \eta)=u(\varepsilon_\eta^l+q_\eta^d)+\beta \mathrm{E}_\eta W_{+1}[\omega^d-p_\eta^d,\ \eta], \tag{14}$$

where $(q_\eta^d, p_\eta^d)=(\varepsilon_\eta F, C(\varepsilon_\eta F)/\beta\phi_{+1})$ if $\beta\phi_{+1}\omega^d-\gamma_d\geq C(\varepsilon_\eta F)$ and $(q_\eta^d, p_\eta^d)=(C^{-1}(\beta\phi_{+1}\omega^d-\gamma_d), \omega^d)$ otherwise. The corresponding Bellman equation for a seller satisfies

$$\begin{aligned} V^s(\omega^d,\eta) = \int \{&[u(\varepsilon_\eta^h - q_\eta^m(\omega)) + \beta \mathrm{E}_\eta W_{+1}(\omega^d + p_\eta^m(\omega),\eta)]\,\mathbf{1}_{\{\omega=\omega^m\}} \\ &+[u(\varepsilon_\eta^h - q_\eta^d(\omega)) + \beta \mathrm{E}_\eta W_{+1}(\breve{\omega}^d + p_\eta^d(\omega),\eta)]\,\mathbf{1}_{\{\omega=\omega^d\}}\}d\mathbf{F}(\omega) \end{aligned} \tag{15}$$

where $\breve{\omega}^d=\omega^d-(\gamma_d/\phi)$ is the money balance net of the record-keeping cost. Then, the linearity of W and degenerate distribution of wealth across agents imply that the expected utility of an agent entering DM with ω^d balance of debit-card deposit, before knowing the endowment shock, can be written as

$$V(\omega^d,\eta) = \frac{1}{2}[u\,(\varepsilon_\eta^l + q_\eta^d) + u\,(\varepsilon_\eta^h - q_\eta^d)] + \beta \mathrm{E}_\eta W_{+1}(\tilde{\omega}^d,\eta) \tag{16}$$

where $\tilde{\omega}^d=\omega^d-(1/2)(\gamma_d/\phi)$.

Substituting the following first derivative of $V(\omega^d,\ \eta)$ with respect to $\omega^d\in[0,\ \bar{\omega}]$ into (4)

$$\frac{\partial V(\omega^d,\eta)}{\partial \omega^d} = \beta\phi_{+1}\left\{1+\frac{1}{2}\left[\frac{u'\,(\varepsilon_\eta^l + q_\eta^d) - u'\,(\varepsilon_\eta^h - q_\eta^d)}{C'(q_\eta^d)}\right]\right\}, \tag{17}$$

we obtain the condition that determines $\omega^d>0$ for each $\eta\in\{a, b\}$:

$$\frac{\mu}{\beta} = \mathcal{L}(q_\eta^d) \tag{18}$$

where $\mathcal{L}(q_\eta^d)=1+(1/2)[u'(\varepsilon_\eta^l+q_\eta^d)-u'(\varepsilon_\eta^h-q_\eta^d)]/C'(q_\eta^d)$ represents the expected marginal benefit of a debit-card deposit held for a pairwise trade in the decentralized fruit market.

Now, for a given cost of carrying cash per unit γ_m and a fixed record-keeping cost γ_d, an equilibrium can be defined as follows.

Definition 1. For a given pair of (γ_m, γ_d), an equilibrium is the sequences of an allocation, price, terms of trade, and means of payment such that for $\eta \in \{a, b\}$

(i) $\{g_{\eta,t}, h_{\eta,t}, \omega_{\eta,t}\}_{t=0}^{\infty}$ satisfies (2), (3), and (13) or (18);

(ii) $\{q_{\eta,t}, p_{\eta,t}\}_{t=0}^{\infty}$ satisfies (7) and (8);

(iii) ϕ_t clears the centralized money market $\Sigma_\eta \omega_{\eta,t}/2 = Z_t$ for all t; and

(iv) $\bar{v}_{m,\eta} > \bar{v}_{d,\eta}$ implies the use of cash as a means of payment ($\sigma_\eta = 1$), and $\bar{v}_{m,\eta} < \bar{v}_{d,\eta}$ implies the use of a debit card as a means of payment ($\sigma_\eta = 0$).

IV. Means of Payment and Price Changes

For a given pair of (γ_m, γ_d), there are potentially four types of equilibria depending on the choice of means of payment $\sigma_\eta \in \{\sigma_a, \sigma_b\}$ in DM: (i) $(\sigma_a, \sigma_b) = (1, 1)$, where cash is used in both apple and banana markets; (ii) $(\sigma_a, \sigma_b) = (0, 0)$, where a debit card is used in both apple and banana markets; (iii) $(\sigma_a, \sigma_b) = (1, 0)$, where cash is used in the apple market, and a debit card is used in the banana market; and (iv) $(\sigma_a, \sigma_b) = (0, 1)$, where a debit card is used in the apple market, and cash is used in the banana market.

In the first two types of equilibria, where $(\sigma_a, \sigma_b) = (1, 1)$ and $(\sigma_a, \sigma_b) = (0, 0)$, respectively, money balances of the banana-market participants are greater than those of the apple-market participants. The given endowment of fruits with $\varepsilon_a < \varepsilon_b$ implies a relatively large quantity traded in the banana market compared with that in the apple market. The banana-market participants are then required to hold a relatively large amount of money balance.

***Lemma* 1.** In the $(\sigma_a, \sigma_b) = (0, 0)$ equilibrium, $\varepsilon_a^l + q_a^d = \varepsilon_b^l + q_b^d$ and $\varepsilon_a^h - q_a^d = \varepsilon_b^h - q_b^d$. In $(\sigma_a, \sigma_b) = (1, 1)$ equilibrium, $\varepsilon_a^l + q_a^m = \varepsilon_b^l + q_b^m$ and $\varepsilon_a^h - q_a^m = \varepsilon_b^h - q_b^m$. In both $(\sigma_a, \sigma_b) = (0, 0)$ and $(\sigma_a, \sigma_b) = (1, 1)$ equilibria, $\omega_a < \omega_b < \bar{\omega}$.

Proof. See Appendix. ■

The following lemma shows that, among the four potential types of equilibria, the $(\sigma_a, \sigma_b) = (0, 1)$ equilibrium does not exist. Hence, if both cash and a debit card are used as the means of payment in the decentralized fruit market, apple-market participants use cash ($\sigma_a = 1$), whereas banana-market participants use a debit card ($\sigma_b=0$). Further, as in Lemma 1, the money balances of the banana-market participants are greater than those of the apple-market participants.

***Lemma* 2.** If cash is used in one of the two decentralized fruit markets, it is the apple market: *i.e.*, $(\sigma_a, \sigma_b)=(1, 0)$. Further, $\tilde{\omega}_a^m < \omega_b^d < \bar{\omega}$.

Proof. See Appendix. ■

This lemma also implies that cash and a debit card become the main means of payment for small and large transactions, respectively, which is in accordance with Prescott (1987), Li (2008), and Kim and Lee (2010). Further, it is consistent with the findings of Bounie and Francois (2009) from the diary data in France that the average sizes for cash and debit-card transaction are 10.8 Euro and 51.3 Euro, respectively.

Finally, the following proposition specifies the equilibrium restrictions on a fixed record-keeping cost γ_d for a given carrying cost of cash per unit γ_m.

***Proposition* 1.** For a given $\gamma_m>0$, $(\sigma_a, \sigma_b)=(1, 1)$ for a sufficiently large γ_d, $(\sigma_a, \sigma_b)=(0, 0)$ for a sufficiently small γ_d, and $(\sigma_a, \sigma_b)=(1, 0)$ for γ_d that is neither too small nor too large.

Proof. See Appendix. ■

A. Cash Is Used in All Trades

We are now ready to discuss price dynamics beginning with the $(\sigma_a, \sigma_b)=(1, 1)$ equilibrium in which only cash is used as a means of payment. According to Proposition 1, this type of equilibrium exists if the record-keeping cost is sufficiently large relative to the carrying cost of cash.

***Proposition* 2.** In the $(\sigma_a, \sigma_b)=(1, 1)$ equilibrium, the inflation rate of fruit price is less than the money growth rate μ.

Proof. The inflation rate of fruit price in this equilibrium can be defined as $\pi^m=(\tilde{\omega}_\eta^m/q_\eta^m)/(\tilde{\omega}_{\eta,-1}^m/q_{\eta,-1}^m)$, where $\tilde{\omega}_\eta^m=[1-(\gamma_m/\phi)]\omega_\eta^m$ and q_η^m for $\eta\in\{a, b\}$ satisfies $[(\mu/\beta)+(\gamma_m/\phi)]=\mathcal{L}(q_\eta^m)$ in (14). As $\tilde{\omega}_\eta^m=C(q_\eta^m)/\beta\phi_{+1}$, π^m can be rewritten as

$$\pi^m = \frac{C(q_\eta^m)\,/\,\beta\phi_{+1}q_\eta^m}{C(q_{\eta,-1}^m)\,/\,\beta\phi q_{\eta,-1}^m} = \mu\,\frac{C(q_\eta^m)\,/\,q_\eta^m}{C(q_{\eta,-1}^m)\,/\,q_{\eta,-1}^m}\,.$$

Together with the strict convexity of $C(q)$, $q_{\eta}^{m}<q_{\eta,-1}^{m}$ due to $\phi<\phi_{-1}$ and $\mathcal{L}'(q_{\eta}^{m})<0$ from (13) imply $[C(q_{\eta}^{m})/q_{\eta}^{m}]<[C(q_{\eta,-1}^{m})/q_{\eta,-1}^{m}]$. Therefore, $\pi^{m}<\mu$. ■

Intuitively, price rigidity in the cash-only equilibrium comes from the different adjustment rates between quantity traded and cash balance offered in response to a change in money supply over time. As the stock of money increases at a rate of μ so that the value of money (in terms of general goods) decreases over time ($\phi_{+}/\phi=1/\mu$), the carrying cost of cash per unit, γ_m/ϕ, increases. Therefore, the cash balance net of carrying cost, $\tilde{\omega}_{\eta}^{m}=[1-(\gamma_m/\phi)]\omega_{\eta}^{m}$, increases at a rate less than μ.[13] If the difference between μ and $(\tilde{\omega}^{m}/\tilde{\omega}_{t-1}^{m})$ is covered by a sufficient decrease in the quantity of fruit traded q^{m}, then the inflation rate of fruit price would be identical to the money growth rate ($\pi_{t}^{m}=\mu$).[14] However, the decrease in q^{m} is relatively small, as $C''(q)=-u''(\varepsilon_{\eta}^{h}-q)>0$, which reflects consumption smoothing with concave utility function. That is, as shown in the proof of Proposition 2, the real balance required to compensate for a seller's loss from transferring a unit of fruit to a buyer $(C(q_{\eta}^{m})/q_{\eta}^{m})$ becomes smaller as q^{m} declines; therefore, the inflation rate of fruit price is less than the money growth rate μ.

B. Debit Card is Used in All Trades

We now analyze the debit-only equilibrium that exists for a sufficiently small record-keeping cost relative to the carrying cost of cash.

Proposition 3. In the $(\sigma_a, \sigma_b)=(0, 0)$ equilibrium, the inflation rate of fruit price is equal to the money growth rate μ.

Proof. The inflation rate of fruit price in this equilibrium can be written as

$$\pi^{d}=\frac{\omega_{\eta}^{d}\;/\;q_{\eta}^{d}}{\omega_{\eta,-1}^{d}\;/\;q_{\eta,-1}^{d}}=\frac{[C(q_{\eta}^{d})+\gamma_d]\;/\;\beta\phi_{+1}q_{\eta}^{d}}{[C(q_{\eta,-1}^{d})+\gamma_d]\;/\;\beta\phi q_{\eta,-1}^{d}}=\mu\left\{\frac{[C(q_{\eta}^{d})+\gamma_d]\;/\;q_{\eta}^{d}}{[C(q_{\eta,-1}^{d})+\gamma_d]\;/\;q_{\eta,-1}^{d}}\right\}$$

[13] As $\tilde{\omega}^{m}=[1-(\gamma_m/\phi)]\omega^{m}$ and $\omega_{+1}^{m}=\mu\omega^{m}$ by the market clearing condition, $(\tilde{\omega}_{t}^{m}/\tilde{\omega}_{t-1}^{m})=\{[1-(\gamma_m/\phi)]\mu\omega_{-1}^{m}\}/\{[1-(\gamma_m/\phi_{-1})]\omega_{-1}^{m}\}=\mu\{[1-(\gamma_m/\phi)]/[1-(\gamma_m/\phi_{-1})]\}$. As $(\gamma_m/\phi)>(\gamma_m/\phi_{-1})$, $1-(\gamma_m/\phi)<1-(\gamma_m/\phi_{-1})$. Hence, $(\tilde{\omega}_{t}^{m}/\tilde{\omega}_{t-1}^{m})<\mu$.

[14] For instance, suppose $C(q)=q$. Then, we have $\pi^{m}=(\tilde{\omega}^{m}/\tilde{\omega}_{-1}^{m})(q_{-1}/q)=(\tilde{\omega}^{m}/\tilde{\omega}_{-1}^{m})(\beta\phi\tilde{\omega}_{-1}^{m}/\beta\phi_{+1}\tilde{\omega}^{m})=\mu$.

where q_η^d for each $\eta \in \{a, b\}$ satisfies $(\mu/\beta) = \mathcal{L}(q_\eta^d)$. As μ/β is constant over time, $q_\eta^d = q_{\eta,-1}^d$, which immediately implies $\pi^d = \mu$. ■

When a debit card is used in all trades, there is no change in the quantity of fruit traded over time. Hence, the real balance of money net of the record-keeping cost also remains constant, $\beta\phi\omega_{-1}^d - \gamma_d = \beta\phi_{+1}\omega^d - \gamma_d$. Then, as γ_d is fixed irrespective of the transaction amount and ϕ decreases at a rate of $1/\mu$ (*i.e.*, $\phi = \mu\phi_{+1}$), the money balance in a debit-card account should increase at a rate of μ (*i.e.*, $\omega^d = \mu\omega_{-1}^d$). That is, in this type of equilibrium, monetary expansion is completely absorbed by an increase in nominal money demand.

C. Both Cash and Debit Card are Used in Trades

We finally examine the equilibrium in which both cash and a debit card are used as means of payment. As discussed in Proposition 1, this type of equilibrium exists if record-keeping cost is neither too large nor too small relative to the carrying cost of cash.

***Proposition* 4.** In the $(\sigma_a, \sigma_b) = (1, 0)$ equilibrium, the inflation rate of the apple price is strictly less than that of the banana price.

Proof. As cash is used in the apple market, the inflation rate of apple price, $\pi^a = (\tilde{\omega}_a^m / q_a^m) / (\tilde{\omega}_{a,-1}^m / q_{a,-1}^m)$, is the same as π^m in Proposition 2. Similarly, as a debit card is used in the banana market, the inflation rate of banana price, $\pi^b = (\omega_b^d / q_b^d) / (\omega_{b,-1}^d / q_{b,-1}^d)$, is the same as π^d in Proposition 3. Therefore, we have $\pi^a = \pi^m < \mu = \pi^b = \pi^d$. ■

Propositions 4 and 1 suggest that the overall degree of price rigidity should decline over time, as the record-keeping cost decreases with the development of information technology. The reason is that an increasing fraction of debit-card transactions (relative to cash transactions) facilitates a more flexible price adjustment. This prediction is also consistent with that of Kackmeister (2007) in which price was found considerably more rigid during 1889-1891 than during 1997-1999. Mester (2006) also shows that the fraction of households using debit cards increased from 18% to 60% during 1995-2004.

Further, this result can account for the dramatic differences in price rigidities across goods, as reported in Bils and Klenow (2004). That is, prices of goods usually paid in cash (*e.g.*, newspapers, taxi fares, vehicle tolls, and parking fees) fall at the sticky extreme, whereas prices of goods usually paid by electronic means of payment (*e.g.*, gasoline, airfare, and durable goods) are relatively flexible.

V. Welfare Implications

In an inflationary economy, the distortionary effect of an increasing cost of carrying cash per unit (γ_m/ϕ_t) on the quantity of goods traded and the consequent price rigidity imply extra wedge in the welfare cost of inflation. To see this effect, we consider the $(\sigma_a, \sigma_b)=(1, 0)$ equilibrium and let $\mathrm{W}(\Phi)$ denote the expected discounted utility for an economy with the real balance of $\Phi=\phi Z$ in CM. As cash and a debit card are used in the decentralized market for apple and banana, respectively, $\mathrm{W}(\Phi)$ can be expressed as $\mathrm{W}(\Phi)=\bar{g}+(1/4)[\mathrm{U}(q_a^m)+\mathrm{U}(q_b^d)]+\beta \mathrm{W}(\Phi_{+1})$ where $\bar{g}=v(g^*)-g^*$, $\mathrm{U}(q_a^m)=u(\varepsilon_a^l+q_a^m)+u(\varepsilon_a^h-q_a^m)$, $\mathrm{U}(q_b^d)=u(\varepsilon_b^l+q_b^d)+u(\varepsilon_b^h-q_b^d)$, and $\Phi=\Phi_{+1}$ in the equilibrium of our interest. By repeated substitution, we obtain

$$\mathrm{W}(\Phi)=\frac{\bar{g}}{1-\beta}+\frac{1}{4(1-\beta)}\mathrm{U}(q_b^d)+\frac{1}{4}\sum_{\tau}\beta^{\tau}\mathrm{U}(q_{a,t+\tau}^m)$$

where $q_b^d=q_{b,t}^d=q_{b,t+1}^d=q_{b,t+2}^d=\cdots$ from (18) and $q_{a,t}^m>q_{a,t+1}^m>q_{a,t+2}^m>\cdots$ from (13) because of a decreasing $\phi_{t+\tau}$ at the rate of μ.

Based on (13) and (18), if $\gamma_m=0$, the real balance for cash trade in the apple market is equivalent to that net of a fixed record-keeping cost γ_d for a debit-card trade. As the real balance for cash trade with $\gamma_m>0$ is less than that for a debit-card trade, the quantity of apple traded accordingly is less than that with $\gamma_m=0$. Therefore, following Bailey (1956), the welfare cost of cash trade due to γ_m/ϕ_t (net of γ_d) for a given μ can be written as

$$\mathrm{C}_{\mu}\equiv \mathrm{W}_d-\mathrm{W}=\frac{1}{4}\sum_{\tau}\beta^{\tau}[\mathrm{U}(q_a^d)-\mathrm{U}(q_{a,t+\tau}^m)]>0 \tag{19}$$

where $\mathrm{W}_d=[\bar{g}/(1-\beta)]+[4(1-\beta)]^{-1}[\mathrm{U}(q_a^d)+\mathrm{U}(q_b^d)]$.[15]

Note that $\mathrm{C}_{\mu}>0$ implies that the welfare cost of inflation is larger than that in the existing search-based models of exchange. In Lagos and Wright (2005), additional welfare cost of inflation comes from sharing the bargaining power between a buyer and a seller, whereas, here, it comes from the carrying cost of cash per unit (γ_m/ϕ_t). For example, suppose $\mu=1$ in an economy $\mathcal{A}$ and $\mu'>1$ in an economy $\mathcal{B}$. Thus, the welfare

[15] As $(1/2)\{[u'(\varepsilon^l+q^m)/u'(\varepsilon^h-q^m)]+1\}=\mathcal{L}(q^m)>\mathcal{L}(q^d)=(1/2)\{[u'(\varepsilon^l+q^d)/u'(\varepsilon^h-q^d)]+1\}$, $\Sigma_{\tau}\beta^{\tau}\{[u(\varepsilon_a^l+q_a^d)+u(\varepsilon_a^h-q_a^d)]-[u(\varepsilon_a^l+q_{a,t+\tau}^m)+u(\varepsilon_a^h-q_{a,t+\tau}^m)]\}>0$.

cost of inflation is defined as follows:

$$\begin{aligned} W^{\mathcal{A}}-W^{\mathcal{B}} &= [W_d^{\mathcal{A}}-W_d^{\mathcal{B}}]+[(W_d^{\mathcal{B}}-W^{\mathcal{B}})-(W_d^{\mathcal{A}}-W^{\mathcal{A}})] \\ &= [W_d^{\mathcal{A}}-W_d^{\mathcal{B}}]+[C_{\mu'}-C_{\mu}]. \end{aligned} \tag{20}$$

Now, suppose $\Pr[\eta_t=a]=\alpha$ and $\Pr[\eta_t=b]=(1-\alpha)$ denote the fraction of cash trade and debit-card trade, respectively, in the decentralized fruit market. The first term on the right-hand side in (20) can be written as

$$\begin{aligned} W_d^{\mathcal{A}} - W_d^{\mathcal{B}} &= \frac{1}{1-\beta}\left\{\left[\frac{\alpha}{2}U(\hat{q}_a^d)+\frac{(1-\alpha)}{2}U(\hat{q}_b^d)\right]-\left[\frac{\alpha}{2}U(\breve{q}_a^d)+\frac{(1-\alpha)}{2}U(\breve{q}_b^d)\right]\right\} \\ &= \frac{1}{2(1-\beta)}[U(\hat{q}_a^d)-U(\breve{q}_b^d)] \end{aligned} \tag{21}$$

where $\hat{q}^d$ satisfies (18) with $\mu=1$ (economy $\mathcal{A}$) and $\breve{q}^d$ satisfies (18) with $\mu'>1$ (economy $\mathcal{B}$). Further, $u(\varepsilon_a^l+q_a^d)+u(\varepsilon_a^h-q_a^d)=u(\varepsilon_b^l+q_b^d)+u(\varepsilon_b^h-q_b^d)$ from Lemma 1 implies $U(q_a^d)=U(q_b^d)$ for a given $\mu\geq 1$. Notice that (21) captures the distortionary effect of inflation on the intertemporal relative price, which is the typical welfare cost of inflation in a search-based model of exchange.

The second term on the right-hand side in (20) can be expressed as

$$C_{\mu'} - C_{\mu} = \frac{\alpha}{2}\left\{\sum_{\tau}\beta^{\tau}[U(\breve{q}_a^d)-U(\breve{q}_{a,t+\tau}^m)]-\sum_{\tau}\beta^{\tau}[U(\hat{q}_a^d)-U(\hat{q}_{a,t+\tau}^m)]\right\} \tag{22}$$

where $\hat{q}^m$ and $\breve{q}^m$ satisfy (13) with $\mu=1$ (economy $\mathcal{A}$) and $\mu'>1$ (economy $\mathcal{B}$), respectively. Notice that the carrying cost of cash per unit (γ_m/ϕ_t) increases with μ as ϕ_t decreases over time, implying a fall in cash balance and quantity traded. Hence, (22) is positive as long as β is not sufficiently small. This equation captures the additional welfare cost of inflation due to an increase in (γ_m/ϕ_t). In short, inflation has an additional distortionary effect on relative price between cash trade and debit-card trade.

This welfare implication also suggests that welfare cost of inflation should decrease over time because of a decline in cash trade. Indeed, as record-keeping cost decreases with the development of information technology, debit-card trade has increased substantially, whereas cash trade has decreased steadily. According to Humphrey (2004), the share of cash in legal consumer payments decreased from 31% to 20% during 1974-2000, whereas the share of debit cards increased rapidly to 7% during the 1990s. Moreover, the "Study of Consumer Payment Preferences" conducted by the American Bankers Association and Dove Consulting show that during 1999-2008, in-store purchases paid by cash decreased from 39% to 29%, whereas those paid by online debit cards increased from 11% to 20%. Notice that $(W_d^{\mathcal{A}}-W_d^{\mathcal{B}})$ in (21) does not depend on the fraction of cash trade $\alpha \in (0, 1)$, whereas $(C_{\mu'}-C_{\mu})$ in (22) decreases as α declines. Therefore, the welfare cost of inflation $(W^{\mathcal{A}}-W^{\mathcal{B}})$ in (20) should decrease as the fraction of cash transactions (α) falls with the development of information technology.

VI. Concluding Remarks

This paper investigates the means-of-payment mechanism to provide one of the possible micro-foundations for price rigidity in response to monetary policy. We incorporate a trade-off between cash and a debit card as means of payment in the sense that a buyer incurs the carrying cost of cash proportional to its amount, whereas a seller accepting a debit card bears a fixed record-keeping cost regardless of transaction amount. In an inflationary economy where money supply increases at a constant rate, cash balance increases over time at a rate less than the money growth rate because of a higher cost of carrying cash with an increase in money supply. Consumption smoothing implies a relatively small decrease in quantity of goods traded compared with an increase in cash balance, implying price rigidity.

So far, we have discussed the relationship between price rigidity and means of payment for the purchase of a *single* good or service, such as newspaper, taxi fare, vehicle toll, gasoline, airfare, and durable goods, as noted in Knotek (2010). However, if a certain good is purchased together with other goods, a buyer would consider the total amount of transaction rather than the price of individual goods in the choice of means of payment. Our model suggests that a buyer should use a debit card to pay a relatively large amount for those goods that tend to be purchased altogether such as fruits, vegetables, and dairy products, which implies the relatively flexible prices of the agricultural products, as shown in Bils and Klenow (2004).

Finally, empirical studies with extensive data on the payment patterns should be indispensable to understand the importance of the mechanism for nominal price rigidity associated with means of payment. We leave this to future work.

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Appendix

***Proof of Lemma* 1:** Suppose a debit card is used as a means of payment in all trades. As $C'(q_\eta)=u'(\varepsilon_\eta^h-q_\eta^d)$, from (18), we have

$$\frac{\mu}{\beta}=\frac{1}{2}\left[\frac{u'(\varepsilon_\eta^l+q_\eta^d)+u'(\varepsilon_\eta^h-q_\eta^d)}{u'(\varepsilon_\eta^h-q_\eta^d)}\right].$$

The constant μ/β, regardless of η, implies $[u'(\varepsilon_a^l+q_a^d)/u'(\varepsilon_a^h-q_a^d)]=[u'(\varepsilon_b^l+q_b^d)/u'(\varepsilon_b^h-q_b^d)]$. Then, as $\varepsilon_a^l>\varepsilon_b^l$, $\varepsilon_a^h<\varepsilon_b^h$ and $u''(q)<0<u'(q)$, $\varepsilon_a^l+q_a^d=\varepsilon_b^l+q_b^d$ and $\varepsilon_a^h-q_a^d=\varepsilon_b^h-q_b^d$. The exact same arguments in the case in which cash is used in all trades give the results $\varepsilon_a^l+q_a^m=\varepsilon_b^l+q_b^m$ and $\varepsilon_a^h-q_a^m=\varepsilon_b^h-q_b^m$. If a debit card is used in all trades, $q_a^d<q_b^d$ because $\varepsilon_a^l+q_a^d=\varepsilon_b^l+q_b^d$ and $\varepsilon_a^l>\varepsilon_b^l$. Similarly, if cash is used in all trades, $q_a^m<q_b^m$ because $\varepsilon_a^l+q_a^m=\varepsilon_b^l+q_b^m$ and $\varepsilon_a^l>\varepsilon_b^l$. Therefore, in either case, $\omega_a<\omega_b$ because $C'(q_\eta)=u'(\varepsilon_\eta^h-q_\eta)>0$. Finally, we need to show that $\omega_b<\bar{\omega}$. As $\bar{\omega}$ is the money balance corresponding to the first-best quantity of fruit traded, it suffices to show that $q_b^i<\varepsilon_b F$ for $i\in\{m, d\}$. Consider the $(\sigma_a, \sigma_b)=(0, 0)$ equilibrium. Then, q_b^d satisfies

$$\frac{\mu}{\beta}=\mathcal{L}(q_b^d)=\frac{1}{2}\left[\frac{u'(\varepsilon_b^l+q_b^d)+u'(\varepsilon_b^h-q_b^d)}{u'(\varepsilon_b^h-q_b^d)}\right].$$

Thus, $\mathcal{L}(0)>(\mu/\beta)$ because $u'(\varepsilon_b^l)$ is sufficiently large, and $\mathcal{L}(\varepsilon_b F)<(\mu/\beta)$ because $u'(\varepsilon_b^l+q_b^d)=u'(\varepsilon_b^h-q_b^d)$ at $q_b^d=\varepsilon_b F$ and $\mu>\beta$. Further, $\mathcal{L}'(q_b^d)$ for $q_b^d\in[0, \varepsilon_b F]$ is

$$\mathcal{L}'(q_b^d)=\frac{[u''(\varepsilon_b^l+q_b^d)][u'(\varepsilon_b^h-q_b^d)]+[u''(\varepsilon_b^h-q_b^d)][u'(\varepsilon_b^l+q_b^d)]}{[u'(\varepsilon_b^h-q_b^d)]^2}<0$$

where $\mathcal{L}'(\varepsilon_b F)$ is the limiting case from below. Therefore, q_b^d satisfying (18) is strictly less than $\varepsilon_b F$. Similar arguments for the equilibrium $(\sigma_a,$

$\sigma_b)=(1, 1)$ give the result $q_b^m < \varepsilon_b F$.

***Proof of Lemma* 2:** As $(\mu/\beta)=\mathcal{L}(q_\eta^d)$ from (18), (13) can be rewritten as $(\gamma_m/\phi)=\mathcal{L}(q_\eta^m)-\mathcal{L}(q_\eta^d)$. Through a simple manipulation of this equation using $C'(q_\eta)=u'(\varepsilon_\eta^h - q_\eta)$, we have

$$\frac{\gamma_m}{\phi}=\frac{1}{2}\left\{\left[\frac{u'(\varepsilon_\eta^l + q_\eta^m)}{u'(\varepsilon_\eta^h - q_\eta^m)}\right]-\left[\frac{u'(\varepsilon_\eta^l + q_\eta^d)}{u'(\varepsilon_\eta^h - q_\eta^d)}\right]\right\}.$$

Hence, $q_\eta^m < q_\eta^d$ because $u'(\varepsilon_\eta^l + q_\eta)/u'(\varepsilon_\eta^h - q_\eta)$ decreases with q_η and $(\gamma_m/\phi)>0$. As $\varepsilon_a < \varepsilon_b$ implies $q_a < q_b$ in equilibrium, the smaller outcome q_η^m should be the quantity for apples traded (q_a^m), and the larger outcome q_η^d should be the quantity for bananas traded (q_b^d). $\tilde{\omega}_a^m < \omega_b^d$ is again an obvious consequence of $C'(q_\eta)=u'(\varepsilon_\eta^h - q_\eta)>0$. Finally, the proof of $\omega_b^d < \bar{\omega}$ is the same as the last part of the proof of Lemma 1.

***Proof of Proposition* 1:** Let us first define $\bar{U}_\eta^d=(1/2)[\mathrm{U}(q_\eta^d)]-(\phi-\beta\phi_{+1})\omega_\eta^d-(1/2)(\beta/\mu)\gamma_d$ and $\bar{U}_\eta^m=(1/2)[\mathrm{U}(q_\eta^m)]-(\phi-\beta\phi_{+1})\omega_\eta^m-(\beta/\mu)\omega_\eta^m\gamma_m$, where $\mathrm{U}(q_\eta^d)=u(\varepsilon_\eta^l + q_\eta^d)+u(\varepsilon_\eta^h - q_\eta^d)$, $\mathrm{U}(q_\eta^m)=u(\varepsilon_\eta^l + q_\eta^m)+u(\varepsilon_\eta^h - q_\eta^m)$, and q_η^m and q_η^d satisfy (13) and (18), respectively, for a given (γ_d, γ_m). As $\bar{v}_{d,\eta}=\bar{U}_\eta^d + \beta \mathrm{E}_\eta W_{+1}(0, \eta)$ and $\bar{v}_{m,\eta}=\bar{U}_\eta^m + \beta \mathrm{E}_\eta W_{+1}(0, \eta)$, it suffices to compare $\bar{U}_\eta^d$ and $\bar{U}_\eta^m$ to choose the means of payment. (i) By Lemma 2, if cash is used in the banana market, then it is also the case in the apple market. As $\partial\phi/\partial\gamma_d=(1/\beta\omega_b^d)>0$, γ_d gives an effect on $\bar{U}_b^m$ through ϕ. Specifically, $\partial\bar{U}_b^m/\partial\gamma_d>0$ because

$$\begin{aligned}\frac{\partial \bar{U}_b^m}{\partial \gamma_d} &= \frac{1}{2}[u'(\varepsilon_b^l + q_b^m) - u'(\varepsilon_b^h - q_b^m)]\frac{\partial q}{\partial \phi}\frac{\partial \phi}{\partial \gamma_d} - \omega_b^m \frac{\partial \phi}{\partial \gamma_d} + \left(\frac{\beta}{\mu}\right)\omega_b^m \frac{\partial \phi}{\partial \gamma_d} \\ &= \left[\frac{\mu-\beta}{\beta}+\frac{\gamma_m}{\phi}\right]\left(\frac{\beta}{\mu}\right)\omega_b^m \frac{\partial \phi}{\partial \gamma_d} - \omega_b^m \frac{\partial \phi}{\partial \gamma_d} + \left(\frac{\beta}{\mu}\right)\omega_b^m \frac{\partial \phi}{\partial \gamma_d} \\ &= \left(\frac{\gamma_m}{\phi}\right)\left(\frac{\beta}{\mu}\right)\omega_b^m \frac{\partial \phi}{\partial \gamma_d} > 0.\end{aligned}$$

Clearly, $\partial\bar{U}_b^d/\partial\gamma_d<0$ and $\bar{U}_b^d>\bar{U}_b^m$ for $\gamma_d=0$ with $\gamma_m>0$. Therefore, for a given $\gamma_m>0$, $\tilde{\gamma}_d^b>0$ such that $\bar{U}_b^d(\tilde{\gamma}_d^b)=\bar{U}_b^m(\tilde{\gamma}_d^b)$ is well defined. Then, if γ_d is greater than $\tilde{\gamma}_d^b$, $\bar{U}_b^d(\gamma_d)<\bar{U}_b^m(\gamma_d)$, and hence cash is superior to a debit card as a means of payment. (ii) By Lemma 2 again, it suffices to show that a debit card is used in the apple market. The exact same argument in (i) implies that in the apple market, $\tilde{\gamma}_d^a > 0$ such that $\bar{U}_a^d(\tilde{\gamma}_d^a) = \bar{U}_a^m(\tilde{\gamma}_d^a)$ is well defined for a given $\gamma_m > 0$. Then, if γ_d is less than $\tilde{\gamma}_d^a$, $\bar{U}_a^d(\gamma_d) > \bar{U}_a^m(\gamma_d)$, and hence a debit card is superior to cash as a means of payment. (iii) As claimed in (i) and (ii), for a given $\gamma_m > 0$, $\tilde{\gamma}_d^a$ and $\tilde{\gamma}_d^b$ such that

$$\frac{1}{2}[\mathrm{U}(q_a^d) - \mathrm{U}(q_a^m)] - (\phi - \beta\phi_{+1})(\omega_a^d - \omega_a^m) + \left(\frac{\beta}{\mu}\right)\gamma_m\omega_a^m = \frac{1}{2}\left(\frac{\beta}{\mu}\right)\tilde{\gamma}_d^a$$

$$\frac{1}{2}[\mathrm{U}(q_b^d) - \mathrm{U}(q_b^m)] - (\phi - \beta\phi_{+1})(\omega_b^d - \omega_b^m) + \left(\frac{\beta}{\mu}\right)\gamma_m\omega_b^m = \frac{1}{2}\left(\frac{\beta}{\mu}\right)\tilde{\gamma}_d^b$$

are well defined. From the above, we have

$$(\tilde{\gamma}_d^b - \tilde{\gamma}_d^a) = 2\left(\frac{\mu}{\beta}\right)(\phi - \beta\phi_{+1})[(\omega_a^d - \omega_a^m) - (\omega_b^d - \omega_b^m)] + 2\gamma_m(\omega_b^m - \omega_a^m)$$

where we use the result in Lemma 1, $\mathrm{U}(q_a^d) = \mathrm{U}(q_b^d)$ and $\mathrm{U}(q_a^m) = \mathrm{U}(q_b^m)$. As $\omega_b^m > \omega_a^m$ by Lemma 1, if $[(\omega_a^d - \omega_a^m) - (\omega_b^d - \omega_b^m)] = [(\omega_b^m - \omega_a^m) - (\omega_b^d - \omega_a^d)] > 0$, then $\tilde{\gamma}_d^b > \tilde{\gamma}_d^a$. As $\beta\phi_{+1}\,\omega_b^m[1 - (\gamma_m/\phi)] = u(\varepsilon_b^h) - u(\varepsilon_b^h - q_b^m)$, $\beta\phi_{+1}\,\omega_a^m[1 - (\gamma_m/\phi)] = u(\varepsilon_a^h) - u(\varepsilon_a^h - q_a^m)$, and $\varepsilon_b^h - q_b^m = \varepsilon_a^h - q_a^m$, we have

$$\omega_b^m - \omega_a^m = \frac{1}{\beta\phi_{+1}}\frac{\phi}{(\phi - \gamma_m)}[u(\varepsilon_b^h) - u(\varepsilon_a^h)].$$

Similarly, as $\beta\phi_{+1}\,\omega_b^d = u(\varepsilon_b^h) - u(\varepsilon_b^h - q_b^d) + \gamma_d$, $\beta\phi_{+1}\,\omega_a^d = u(\varepsilon_a^h) - u(\varepsilon_a^h - q_a^d) + \gamma_d$, and $\varepsilon_b^h - q_b^d = \varepsilon_a^h - q_a^d$, we have

$$\omega_b^d - \omega_a^d = \frac{1}{\beta\phi_{+1}}[u(\varepsilon_b^h) - u(\varepsilon_a^h)].$$

Therefore, $[(\omega_b^m - \omega_a^m) - (\omega_b^d - \omega_a^d)] = (1/\beta\phi_{+1})[\gamma_m/(\phi - \gamma_m)]\,[u(\varepsilon_b^h) - u(\varepsilon_a^h)] > 0$ for $\gamma_m > 0$. Now, if $\gamma_d \in (\tilde{\gamma}_d^a, \tilde{\gamma}_d^b)$, $\bar{U}_a^d(\gamma_d) < \bar{U}_a^m(\gamma_d)$ and $\bar{U}_b^d(\gamma_d) > \bar{U}_b^m(\gamma_d)$, and hence cash is used in the apple market, and a debit card is used in the banana market.

References

Aliprantis, D., Camera, G., and Puzzello, D. "Anonymous Markets and Monetary Trading." *Journal of Monetary Economics* 54 (No. 7 2007): 1905-28.

Bailey, M. "The Welfare Cost of Inflationary Finance." *Journal of Political Economy* 64 (No. 5 1956): 93-110.

Baumol, W. "The Transactions Demand for Cash: An Inventory Theoretic Approach." *Quarterly Journal of Economics* 66 (No. 4 1952): 545-56.

Bils, M., and Klenow, P. "Some Evidence on the Importance of Sticky Prices." *Journal of Political Economy* 112 (No. 5 2004): 947-85.

Blinder, A., Canetti, E., Lebow, D., and Rudd, J. *Asking About Prices: A New Approach to Understanding Price Stickiness*. New York: Russell Sage Foundation, 1998.

Boivin, J., Giannoni, M., and Mihov, I. "Sticky Prices and Monetary Policy: Evidence from Disaggregated US Data." *American Economic Review* 99 (No. 1 2009): 350-84.

Bounie, D., and Francois, A. "Cash, Check or Debit Card? The Effects of Transaction Characteristics on the Use of Payment Instruments." Telecom ParisTech Working Paper, 2009.

Corbae, D., Temzelides, T., and Wright, R. "Directed Search and Monetary Exchange." *Econometrica* 71 (No. 3 2003): 731-56.

Craig, B., and Rocheteau, G. "State-Dependent Pricing, Inflation, and Welfare in Search Economies." *European Economic Review* 52 (No. 3 2008): 441-68.

Finkelstein, A. "E-Ztax: Tax Salience and Tax Rates." *Quarterly Journal of Economics* 124 (No. 3 2009): 969-1010.

Freeman, S., and Kydland, F. "Monetary Aggregates and Output." *American Economic Review* 90 (No. 5 2000): 1125-35.

Golosov, M., and Lucas, R. "Menu Costs and Phillips Curves." *Journal of Political Economy* 115 (No. 1 2007): 171-99.

Goodfriend, M., and King, R. "The New Neoclassical Synthesis and the Role of Monetary Policy." *NBER Macroeconomics Annual* 12 (1997): 231-96.

He, P., Huang, L., and Wright, R. "Money, Banking, and Monetary Policy." *Journal of Monetary Economics* 55 (No. 6 2008): 1013-24.

Head, A., Liu, L., Menzio, G., and Wright, R. Sticky Prices? Manuscript, Queen's University, 2010.

Humphrey, D. "Replacement of Cash by Cards in US Consumer Payments." *Journal of Economics and Business* 56 (No. 3 2004): 211-25.

Jung, Y. "Staggered Price Settings, Exchange Rates, and Monetary Policy." *Seoul Journal of Economics* 16 (No. 4 2003): 423-59.

Kackmeister, A. "Yesterday's Bad Times are Today's Good Old Times: Retail Price Changes Are More Frequent Today than in the 1890s." *Journal of Money, Credit and Banking* 39 (No. 8 2007): 1988-2020.

Karni, E. "The Transactions Demand for Cash: Incorporation of the Value of Time into the Inventory Approach." *Journal of Political Economy* 81 (No. 5 1973): 1216-25.

Kim, Y. S., and Lee, M. "A Model of Debit Card as Means of Payment." *Journal of Economic Dynamics and Control* 34 (No. 8 2010): 1359-68.

Knotek, E. "Convenient Prices and Price Rigidity: Cross-Sectional Evidence." *Review of Economics and Statistics*, Forthcoming, 2010.

________. "Convenient Prices, Currency, and Nominal Rigidity: Theory with Evidence from Newspaper Prices." *Journal of Monetary Economics* 55 (No. 7 2008): 1303-16.

Kocherlakota, N. "Money is Memory." *Journal of Economic Theory* 81 (No. 2 1998): 232-51.

Lagos, R. "Asset Prices and Liquidity in an Exchange Economy." *Journal of Monetary Economics* 57 (No. 8 2010): 913-30.

Lagos, R., and Rocheteau, G. "Money and Capital as Competing Media of Exchange." *Journal of Economic Theory* 142 (No. 1 2008): 247-58.

Lagos, R., and Wright, R. "A Unified Framework for Monetary Theory and Policy Analysis." *Journal of Political Economy* 113 (No. 3 2005): 463-84.

Levy, D., and Young, A. "The Real Thing: Nominal Price Rigidity of the Nickel Coke, 1886-1959." *Journal of Money, Credit and Banking* 36 (No. 4 2004): 765-99.

Li, Y. Currency and Checking Deposits as Means of Payment. Manuscript, National Taiwan University, 2008.

Mankiw, G. "Small Menu Costs and Large Business Cycles: A Macroeconomic Model of Monopoly." *Quarterly Journal of Economics* 100 (No. 2 1985): 529-37.

Mester, L. "Changes in the Use of Electronic Means of Payment: 1994-2004." *Federal Reserve Bank of Philadelphia Business Review*

(No. 2 2006): 26-30.

Prescott, E. "A Multiple Means of Payment Model." In W. A. Barnett and K. J. Singleton (eds.), *New Approaches to Monetary Economics*. New York: Cambridge University Press, pp. 42-51, 1987.

Rotemberg, J. "Customer Anger at Price Increases, Changes in the Frequency of Price Adjustment and Monetary Policy." *Journal of Monetary Economics* 52 (No. 4 2005): 829-52.

Smets, F., and Wouters, R. "An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area." *Journal of the European Economic Association* 1 (No. 5 2003): 1123-75.

Telyukova, I., and Wright, R. "A Model of Money and Credit, with Application to the Credit Card Debt Puzzle." *Review of Economic Studies* 75 (No. 2 2008): 629-47.

Tobin, J. "The Interest-Elasticity of Transactions Demand for Cash." *Review of Economics and Statistics* 38 (No. 3 1956): 241-7.

Wallace, N. "Whither Monetary Economics?" *International Economic Review* 42 (No. 4 2001): 847-70.

Williamson, S. "Private Money." *Journal of Money, Credit and Banking* 31 (No. 3 1999): 469-91.

Williamson, S., and Wright, R. New Monetarist Economics. Manuscript, Washington University in St. Louis, 2009.

Woodford, M. *Interest and Prices: Foundations of a Theory of Monetary Policy*. New York: Princeton University Press, 2003.

Yun, T. "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles." *Journal of Monetary Economics* 37 (No. 2 1996): 345-70.

Zbaracki, M., Ritson, M., Levy, D., Dutta, S., and Bergen, M. "Managerial and Customer Costs of Price Adjustment: Direct Evidence from Industrial Markets." *Review of Economics and Statistics* 86 (No. 2 2004): 514-33.